Statistics/EE 530, Spring Quarter 2018

Problem Set 5

Problem 16 (to be turned in – 5 points). Equation (176) displays $\widetilde{\mathcal{B}}_2$ for the case $N = 12$ and $L = 4$. What would $\widetilde{\mathcal{B}}_3$ look like for this case? (This is Exercise [5.3], p. 204.)

Problem 17 (to be turned in – 5 points). Show that, for the Haar wavelet filter, we have $\widetilde{H}(f) + \widetilde{G}(f) = 1$. Demonstrate that this resolution of identity can be used to formulate an additive decomposition of a time series based directly on the level $J_0 = 1$ Haar MODWT. (This is a simplified version of Exercise [5.10], p. 205.)

Problem 18 (to be turned in – 5 points for each of 4 parts). This problem looks at the claim that the zephlet transform (discussed on course overheads VI–46 to VI–52) is an orthonormal transform.

Let N be any even positive integer, and let $\mathcal{H}(\cdot)$ be an even nonnegative function defined on the real axis satisfying $\mathcal{H}(\frac{k}{N}) + \mathcal{H}(\frac{k}{N} + \frac{1}{2}) = 2$ for all N and for all integers k. Let $\{\bar{h}_l\}$ be the inverse DFT of the finite sequence $\mathcal{H}^{1/2}(\frac{k}{N})$, $k = 0, 1, \ldots, N-1$; i.e.,

$$\bar{h}_l \equiv \frac{1}{N} \sum_{k=0}^{N-1} \mathcal{H}^{1/2}(\tfrac{k}{N}) e^{i2\pi kl/N}, \quad l = 0, 1, \ldots, N-1.$$

Let $\bar{g}_l \equiv (-1)^l \bar{h}_l$, and define the $\frac{N}{2} \times N$ matrices

$$\mathcal{D}_1 = \begin{bmatrix} \bar{h}_1 & \bar{h}_0 & \bar{h}_{N-1} & \bar{h}_{N-2} & \bar{h}_{N-3} & \cdots & \bar{h}_5 & \bar{h}_4 & \bar{h}_3 & \bar{h}_2 \\ \bar{h}_3 & \bar{h}_2 & \bar{h}_1 & \bar{h}_0 & \bar{h}_{N-1} & \cdots & \bar{h}_7 & \bar{h}_6 & \bar{h}_5 & \bar{h}_4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \bar{h}_{N-1} & \bar{h}_{N-2} & \bar{h}_{N-3} & \bar{h}_{N-4} & \bar{h}_{N-5} & \cdots & \bar{h}_3 & \bar{h}_2 & \bar{h}_1 & \bar{h}_0 \end{bmatrix}$$

and

$$\mathcal{C}_1 = \begin{bmatrix} \bar{g}_0 & \bar{g}_{N-1} & \bar{g}_{N-2} & \bar{g}_{N-3} & \bar{g}_{N-4} & \cdots & \bar{g}_4 & \bar{g}_3 & \bar{g}_2 & \bar{g}_1 \\ \bar{g}_2 & \bar{g}_1 & \bar{g}_0 & \bar{g}_{N-1} & \bar{g}_{N-2} & \cdots & \bar{g}_6 & \bar{g}_5 & \bar{g}_4 & \bar{g}_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \bar{g}_{N-2} & \bar{g}_{N-3} & \bar{g}_{N-4} & \bar{g}_{N-5} & \bar{g}_{N-6} & \cdots & \bar{g}_2 & \bar{g}_1 & \bar{g}_0 & \bar{g}_{N-1} \end{bmatrix}.$$

a. Show that $\bar{h}_l$ is real-valued.

b. Show that $\bar{g}_l$, $l = 0, 1, \ldots N-1$, has a DFT $\overline{G}(\frac{k}{N})$, $k = 0, 1, \ldots N-1$, such that $\mathcal{H}(\frac{k}{N}) + \mathcal{G}(\frac{k}{N}) = 2$, where $\mathcal{G}(\frac{k}{N}) \equiv |\overline{G}(\frac{k}{N})|^2$.

c. To show that the $N \times N$ matrix formed by stacking $\mathcal{D}_1$ on top of $\mathcal{C}_1$ is a real-valued orthonormal matrix, i.e, that

$$\mathcal{D} \equiv \begin{bmatrix} \mathcal{D}_1 \\ \mathcal{C}_1 \end{bmatrix} \text{ is such that } \mathcal{D}^T \mathcal{D} = I_N,$$

formulate and prove the appropriate analogs of Equations (72) and (77d) in the course textbook.

d. Compute and list the 16 zephlet scaling filter coefficients shown in the right-hand plot on course overhead VI–49, clearly describing what steps you took to compute the coefficients. The coefficients for the LA(16) scaling filter shown on the left-hand plot can be obtained from the 'Data' page of the course Web site:

`http://faculty.washington.edu/dbp/s530/data.html`

Self-Graded Problem XIII (*not* to be turned in). Show that, if the filter $\{g_0, g_1, \ldots, g_{L-2}, g_{L-1}\}$ has the transfer function $G(\cdot)$, the reversed filter $\{g_{L-1}, g_{L-2}, \ldots, g_1, g_0\}$ then has a transfer function defined by $\exp\left[-i2\pi f(L-1)\right]G^*(f)$. (This is Exercise [117], p. 117.)

Self-Graded Problem XIV (*not* to be turned in). Show that the sample mean of $\widetilde{\mathbf{V}}_{J_0}$ is equal to $\overline{X}$ and hence that the sample variance of $\widetilde{\mathbf{V}}_{J_0}$ is given by $\frac{1}{N}\|\widetilde{\mathbf{V}}_{J_0}\|^2 - \overline{X}^2$. (This is Exercise [171b], p. 171.)

Solutions to Problems 16, 17 and 18 are due Wednesday, May 2, at the beginning of the class.