Statistics/EE 530, Spring Quarter 2018

Problem Set 4

Problem 11 (to be turned in – 5 points). Let $\mathcal{S}_{J_0,t}$ be the tth element of the J_0 level smooth $\mathcal{S}_{J_0}$ for the time series $\mathbf{X}$. Show that

$$\frac{1}{N}\sum_{t=0}^{N-1}\left(\mathcal{S}_{J_0,t}-\overline{\mathcal{S}}_{J_0}\right)^2=\frac{1}{N}\|\mathcal{S}_{J_0}\|^2-\overline{X}^2,$$

where $\overline{\mathcal{S}}_{J_0}$ and $\overline{X}$ are the sample mean of, respectively, $\mathcal{S}_{J_0}$ and $\mathbf{X}$ (the left-hand side is the sample variance for $\mathcal{S}_{J_0}$, and the last two terms in Equation (104c) are the same as the right-hand side). (This is Exercise [4.20], p. 158 of the course textbook.)

Problem 12 (to be turned in – 5 points). If a time series $\mathbf{X}$ has a sample mean of zero, is it true that the sample mean of its DWT coefficients $\mathbf{W}=\mathcal{W}\mathbf{X}$ is also zero? Be sure to thoroughly justify your answer. (This is Exercise [4.21], p. 158.)

Problem 13 (to be turned in – 10 points). Verify that the D(4) wavelet filter as given in Equation (59a) has a squared gain function given by Equation (105b) with $L=4$. Hint: make use the notion of a filter cascade and the solution to Exercise [4.1] (Problem 9). (This is Exercise [4.22], p. 158.)

Problem 14 (to be turned in – 5 points). In the discussion surrounding Figure [76a], we stated that the '3 dB down' point is such that the nominal pass-band for Haar and D(4) wavelets is $1/4 \leq |f| \leq 1/2$. Show that this result extends to all wavelet filters (not necessarily just the Daubechies wavelet filters); i.e., show that $\mathcal{H}(\frac{1}{4})=\frac{1}{2}\mathcal{H}(\frac{1}{2})$. (This is a slightly modified version of Exercise [4.23], p. 158.)

Problem 15 (to be turned in – 5 points). Show that $\|\widetilde{\mathcal{D}}_j\|^2 \leq \|\widetilde{\mathbf{W}}_j\|^2$ and that $\|\widetilde{\mathcal{S}}_j\|^2 \leq \|\widetilde{\mathbf{V}}_j\|^2$ for all j. You might find it useful to note that the frequency-domain orthonormality condition for the DWT, namely, $\mathcal{H}(f)+\mathcal{G}(f)=2$, implies that, for the MODWT, $|\widetilde{H}(f)|^2+|\widetilde{G}(f)|^2=\widetilde{\mathcal{H}}(f)+\widetilde{\mathcal{G}}(f)=1$, where, as usual, $\widetilde{H}(\cdot)$ and $\widetilde{G}(\cdot)$ are the transfer functions for the MODWT filters $\{\tilde{h}_l\}$ and $\{\tilde{g}_l\}$, while $\widetilde{\mathcal{H}}(\cdot)$ and $\widetilde{\mathcal{G}}(\cdot)$ are their squared gain functions. (This is Exercise [5.7], p. 205.)

Self-Graded Problem IX (*not* to be turned in). Show that the filter $\{a_0=-1/\sqrt{2}, a_1=0, a_2=0, a_3=1/\sqrt{2}\}$ is a wavelet filter. Compute its squared gain function, and argue that this wavelet filter is not a reasonable approximation to a high-pass filter. (This is Exercise [105a], p. 105.)

Self-Graded Problem X (*not* to be turned in). Show that the squared gain function for the difference filter $\{a_0 = 1, a_1 = -1\}$ is given by $\mathcal{D}(f) \equiv 4\sin^2(\pi f)$. (This is Exercise [105b], p. 105.)

Self-Graded Problem XI (*not* to be turned in). Show that the transfer function for the advanced filter $\{u_l^{(\nu)}\}$ is given by

$$U^{(\nu)}(f) \equiv e^{i2\pi f\nu} U(f).$$

(This is Exercise [111], p. 111.)

Self-Graded Problem XII (*not* to be turned in). Use Equation (76a), namely,

$$G(f) \equiv \sum_{l=-\infty}^{\infty} g_l e^{-i2\pi fl} = \sum_{l=0}^{L-1} g_l e^{-i2\pi fl} = e^{-i2\pi f(L-1)} H(\tfrac{1}{2} - f),$$

to show that

$$H(f) = e^{-i2\pi f(L-1)+i\pi} G(\tfrac{1}{2} - f).$$

(This is Exercise [112], p. 112.)

Solutions to Problems 11 to 15 are due Wednesday, April 25, at the beginning of the class.