Statistics/EE 530, Spring Quarter 2018

Problem Set 2

Problem 4 (to be turned in – 8 points). Let $\{a_t : t = \ldots, -1, 0, 1, \ldots\}$ be an infinite sequence whose DFT is $A(\cdot)$. Find the DFT of the infinite sequence $\{a_{2n+1} : n = \ldots, -1, 0, 1, \ldots\}$ (i.e., the odd indexed variables in $\{a_t\}$) in terms of $A(\cdot)$. (This is Exercise [2.10], p. 39 of the course textbook.)

Problem 5 (to be turned in – 8 points). Let $\{b_t : t = 0, \ldots, 3\}$ be a finite sequence of length $N = 4$, and consider a filter defined by $a_t = \phi^{|t|}, t = \ldots, -1, 0, 1, \ldots$, where ϕ is a real-valued variable satisfying $|\phi| < 1$. Let

$$c_t \equiv \sum_{u=-\infty}^{\infty} a_u b_{t-u \bmod 4}, \qquad t = 0, \ldots, 3,$$

represent the result of filtering $\{b_t\}$ with $\{a_t\}$. Derive the periodized filter of length $N = 4$ from $\{a_t\}$, i.e., the filter $\{a_t^\circ : t = 0, \ldots, 3\}$ such that

$$c_t = \sum_{u=0}^{3} a_u^\circ b_{t-u \bmod 4}.$$

Determine the DFT $A(\cdot)$ of $\{a_t\}$ and the DFT $\{A_k^\circ\}$ of $\{a_t^\circ\}$, and verify explicitly that $A_k^\circ = A(\frac{k}{4})$ for $k = 0, \ldots, 3$. (This is Exercise [2.11], p. 39.)

Problem 6 (to be turned in – 10 points). Using the conditions of Equation (59b), solve for a and b in Equation (60), and show that one set of solutions leads to the values given for h_0, h_1, h_2 and h_3 in Equation (59a). How many other sets of solutions are there, and what values of h_0, h_1, h_2 and h_3 do these yield? (This is Exercise [4.1], p. 156.)

Problem 7 (to be turned in – 4 points). Verify Equation (69d) for the Haar wavelet filter $\{h_0 = 1/\sqrt{2}, h_1 = -1/\sqrt{2}\}$. (This is Exercise [4.5], p. 156.)

Self-Graded Problem III (*not* to be turned in). Suppose $\{a_t : t = \ldots, -1, 0, 1, \ldots\} \longleftrightarrow A(\cdot)$ and that $\{a_t^\circ : t = 0, 1, \ldots, N-1\}$ is $\{a_t\}$ periodized to length N. Let $\{A_k^\circ : t = 0, 1, \ldots, N-1\}$ be the DFT of $\{a_t^\circ\}$. Show that $A_k^\circ = A(\frac{k}{N})$. (This is Exercise [33], p. 33.)

Self-Graded Problem IV (*not* to be turned in). To complete the proof that unit energy and orthogonality to even shifts for $\{h_l\}$ are equivalent to its squared gain function $\mathcal{H}(\cdot)$ satisfying $\mathcal{H}(f) + \mathcal{H}(f + \frac{1}{2}) = 2$ for all f, suppose now that $\{h_l\}$ satisfied Equations (69b), i.e., unit energy, and (69c), i.e., orthogonality to even shifts. Show that Equation (69d), i.e., $\mathcal{H}(f) + \mathcal{H}(f + \frac{1}{2}) = 2$, must be true. (This is Exercise [70], p. 70.)

Solutions to Problems 4 to 7 are due Wednesday, April 11, at the beginning of the class.