Statistics/EE 530, Spring Quarter 2018

Problem Set 1

Problem 1 (to be turned in – 10 points). Verify the upper left-hand plot of Figure 62, which shows the Haar wavelet transform for $\{X_{1,t}\}$ (this transform is also shown in the upper plot of overhead II–20). The 16-point time series $\{X_{1,t}\}$ is shown in the left-hand plot of Figure 42 and also in the bottom plot of overhead II–20. The values for this time series are given in the caption to Figure 42 and are also available from the 'Data' portion of the course Web page. To do the verification, take the approach of premultiplying the time series by the 16×16 DWT transform matrix shown in Figure 57 and also shown in overheads II–11 and II–22. (This is Exercise [4.2], p. 156 of the course textbook.)

Problem 2 (to be turned in – 5 points). Verify the discrete Haar wavelet empirical power spectra calculated in Equations (63a) and (63b). (This is Exercise [4.3], p. 156.)

Problem 3 (to be turned in – 15 points). Show that the DFT of the sequence $\{b_t\}$ defined by Equation (26a) is given by the function $B(\cdot)$ defined by Equation (26b). Hint: write $\phi_l^t e^{-i2\pi ft}$ as z^t with $z \equiv \phi_l e^{-i2\pi f}$. (This is Exercise [2.4], p. 39.)

Self-Graded Problem I (*not* to be turned in). Suppose $\{a_t : t = \ldots, -1, 0, 1, \ldots\} \longleftrightarrow A(\cdot)$. Show that the DFT of the infinite sequence $\{a_{2n} : n = \ldots, -1, 0, 1, \ldots\}$ is the function defined by $\frac{1}{2}[A(\frac{f}{2}) + A(\frac{f}{2} + \frac{1}{2})]$. (This is Exercise [23b], p. 23.)

Self-Graded Problem II (*not* to be turned in). Suppose that the mth filter $\{a_{m,t} : t = \ldots, -1, 0, 1, \ldots\}$ in a cascade of M filters is such that $a_{m,K_m} \neq 0$ and $a_{m,K_m+L_m-1} \neq 0$, while $a_{m,t} = 0$ for all $t \leq K_m - 1$ and $t \geq K_m + L_m$ (we assume L_m is a positive integer). Because such a filter is zero outside of L_m contiguous terms, we say that it has a *width* of L_m. Its transfer function is given by

$$A_m(f) = \sum_{t=K_m}^{K_m+L_m-1} a_{m,t} e^{-i2\pi ft}.$$

Show that the width of the equivalent filter $A(\cdot)$ is given by

$$L \equiv \sum_{m=1}^{M} L_m - M + 1.$$

(This is Exercise [28a], p. 28.)

Solutions to Problems 1, 2 and 3 are due Wednesday, April 4, at the beginning of the class.