Philosophy of Statistics

Course Introduction
April 7th, 2014

Question: What is philosophy of statistics?

Cheap Advertisement

I'm going to delay "answering" this question and show you funny pictures instead.

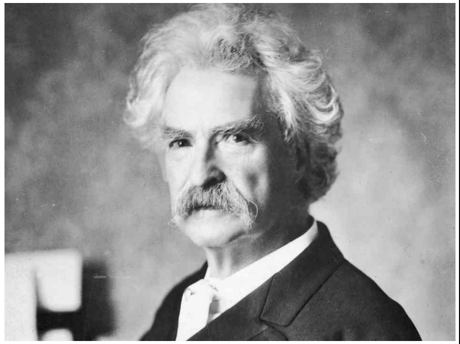
Maybe you'll tell your friends to take this class.

Here are two popular views about statistics.

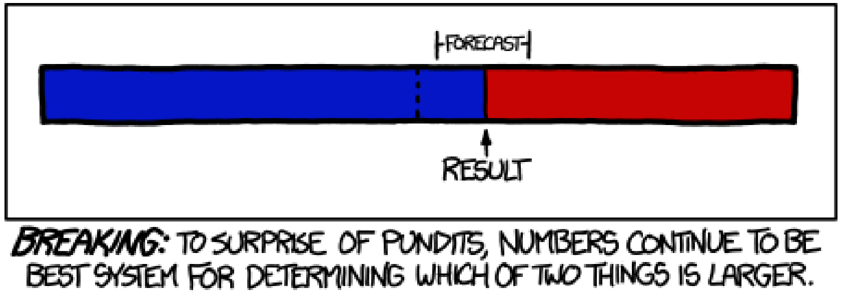
Lies, Damned Lies, and Statistics

There are three kinds of lies: lies, damned lies, and statistics.

Mark Twain



Statistics = The Answer



Source: `xkcd.com/1131/`

Moral:

- Some are rather distrustful of statistics.
- Others engage in blind worship: statistics is a black-box for making predictions, and it yields unequivocal answers.
- The truth is obviously in between:
  - There are cases in which virtually all philosophers and statisticians agree about the interpretation of an experiment.
  - However, there are substantial disagreements about how to interpret statistical data: statistics is not a black box.

Question: What is philosophy of statistics?

Philosophy of Statistics

My Answer: Philosophy of statistics is the subject that attempts to clarify those fundamental debates/questions about experimental design and inference.

Philosophy of Statistics: Questions

Here's an incomplete list of such questions/debates:

- To what degree should an experimenter's "prior" beliefs about a space of hypotheses influence her interpretation of (or inferences from) the results after she collects the data?
- Do principles of rational belief-updating conflict with the maxim that one ought to acquire (asymptotically) true belief?
- Should an experimenter's intention to collect a sample confirming (or refuting) a hypothesis affect our interpretation of the results? In particular, how should we interpret the result of an experiment in which a scientist collects data until she obtains a "statistically significant" result?
- What is the value of random sampling or random assignment of treatments?

And more.

Philosophy of Statistics

Philosophy of statistics = a quantitative branch of epistemology.

- Central question: What ought I believe in light of the outcome of an experiment or observational trial? How ought I conduct an experiment to acquire knowledge?

Philosophy of Statistics

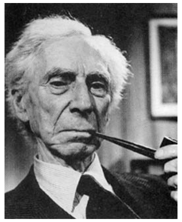
Question: Why not just study standard epistemology?

My Answer: Mathematical statistics allow us

- To draw distinctions that natural language cannot,
- (hence) To ask questions that are more precise than one can ask in natural language, and
- To give mathematically rigorous arguments in response.

[Formal] Logic ... has become the great liberator of the imagination, presenting innumerable alternatives which are closed to unreflective common sense.

Bertrand Russell. Problems of Philosophy.

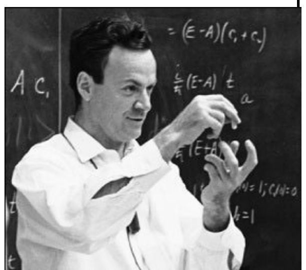


But do statisticians need philosophers at all?

Leave it to the birds

Philosophy of science is as useful to scientists as ornithology is to birds.

Richard Feynman.



Philosophy of Statistics

Question: Do statisticians need philosophers at all?

My Answer: Clarifying and resolving questions about the foundations of statistics requires a combination of philosophical analysis and mathematical rigor.

I don't care whether it's philosophers, statisticians, or ortnithologists (or birds) who do it.

Philosophy of Statistics

For this reason, you'll notice that the syllabus contains readings from many different journals in many different disciplines:

- Philosophy: *Philosophy of Science*, *The Monist*, *British Journal for Philosophy of Science*
- Statistics: *Journal of American Statistical Association*, *Annals of Mathematical Statistics*, *Journal of Statistical Planning and Inference*,
- Medicine: *Annals of Internal Medicine*,
- Game Theory: Von Neumann and Morgenstern's classic book,
- Symbolic Logic: *Journal of Symbolic Logic*

Answers to questions about foundations of statistics often are closely related to one's interpretation of probability.

Course Structure

Course Outline:

- First month: subjective and propensity interpretation of probability
- "Classical/Frequentist" vs. Bayesian methodology
- Central Topics: Stopping Rules, Randomization, Exchangeability, and More

Outline

1. Course Introduction
2. Criteria for Interpretation
3. Subjective Probability
   - Belief and the probability axioms
   - Measuring Belief
   - Use of Probability
4. Plan for this Month

Properties of Probability

The central philosophical question: What is probability?
Salmon [1967] argues that any adequate answer is really an answer to at least three other questions:

- Why should probability have particular mathematical properties?
- How do we determine or measure probabilities?
- Why and when is probability useful (especially in the sciences)?

Interpretations of Probability

An answer to these questions is called an interpretation of probability.

Interpretations of Probability

Frequency:

- To say that the probability that a person is male is 49% is to say that about one-half of people are men.
- Probabilities are just frequencies in some population.
- Sometimes populations are infinite (e.g. as in a sequence of flips of a coin).

Interpretations of Probability

Propensity: Probabilities are properties of objects that cause the objects to behave in certain ways.

- E.g., An nucleus has a propensity to decay, even if it's decay cannot be repeated in considered in some larger population of nuclei.

Interpretations of Probability

Logical: Probability is measure of strength of an argument.

- Some arguments logically entail their conclusion. E.g.
  - Assumption 1: If John is married, then he's not a bachelor
  - Assumption 2: John is married.
  - Conclusion: So he's not a bachelor.
- Some arguments only make the evidence for the conclusion stronger. E.g.,
  - Assumption: John works at Google.
  - Conclusion: So he knows a lot about computers.
- Probability quantifies how strong the evidence is for a conclusion given the premises of the argument or some data.

Interpretations of Probability

Subjective: Probability is simply a measure of how strongly we believe particular propositions.

- E.g., I believe that Munich is in southern Germany. I also believe that Hilary Clinton will not run for President of the United States. But I believe that Munich is in southern Germany much more strongly than I believe that Clinton won't run.
- Probability is what quantifies the difference in strength of my beliefs.

Interpretations of Probability

- Some philosophers think there is only one interpretation of probability.
- Most don't: they think we use the word probability to mean several different things, and different interpretations of probability are useful in different ways.
  - Example: Direct inference (i.e. the principal principle) only makes sense if there are two types of probability.

Interpretations of Probability

This is not obviously the prevalent attitude in statistics:

- Bayesian methodology makes perfect sense if there are no objective chances.
- "Classical" techniques make no (explicit) reference to subjective degrees of belief.

Applying the Criteria

To see how Salmon's criteria work, let's consider the subjective interpretation.

- Do degrees of belief satisfy the probability axioms?
- How do we determine or measure degrees of belief?
- Why and when are degrees of belief useful (especially in the sciences)?

Properties of Probability

Probabilities are always comparable.

- Numbers can be compared: either $x > y$, or $y > x$, or $x = y$.
- Are degrees of belief always comparable? E.g., What's more likely that the temperature of the earth will rise 2°C by the year 2100 or that China will fight a war with some other major world power?

Properties of Probability

Probabilities are always quantitatively comparable.

- Comparisons of numbers can be **quantified**. E.g. 12.6 is 6.3 times greater than 2.
- Compare. Clearly, Batman is more awesome than Superman. Is he at least 6.3 times as awesome?
- Can assessments of degrees of belief be made so precise?

Properties of Probability

Probabilities are bounded (between 0 and 1)

- Compare: The length of objects can be assigned a number, but there is no maximum length.
- Why is it that there is some maximum degree of certainty? And some minimum degree of confidence?

Properties of Probability

Probabilities are additive.

- Compare: Your attitude towards both ice cream and Schweinebraten is positive. Can your positive feelings towards ice-cream and Schweinebraten be meaningfully added?
- Moreover, why didn't you multiply the two numbers? What's so special about addition?

Properties of Probability

We will see answers to this question are closely related to the second question about measurement . . .

Measuring Probability

How can degrees of belief be measured?

- Betting behavior (with small sums)
- Forced choice over actions
- Verbal comparisons of likelihood

Measuring Probability

Each measurement procedure yields a different justification for believing that degrees of belief do (or should) satisfy the probability axioms:

- Betting behavior (with small sums) $\Rightarrow$ Dutch Book
- Forced choice over actions $\Rightarrow$ Savage's theorem
- Verbal comparisons of likelihood $\Rightarrow$ Representations for qualitative probability

Measuring Probability

Why is probability useful?

- One of two components of rational choice; the other is utility.
- Decisions to accept or reject scientific hypotheses can be viewed as applications of general decision theory.

Interpretations of Probability and Statistics

This Month:

- Subjective interpretation, which is intended to support Bayesian methodology.
- Propensity interpretation, which is the basis of frequentist/classical methodology

Frequentist Methodology and The Frequency Interpretation

Note: Classical/frequentist statisticians often think it is the frequency interpretation of probability that supports their methodology.

Frequentist Methodology and The Frequency Interpretation

I think this is wrong for several reasons, both technical and philosophical.

- Many classical techniques assume probability is countably additive. This is not true under the standard understanding of the frequency interpretation.
- Classical statisticians do frequently attach probabilities to events that will not (and cannot?) be repeated. This is questionable or incoherent under a frequency view.
- Frequency theories often require theories of randomness, which play little to no role in classical statistics.

Moral: I'm going to teach you about propensity theories, but I'd be happy to give you canonical references about the frequency interpretation.

See [Hajek, 2003] for more on interpretations of probability.

References I

Hajek, A. (2003). Interpretations of probability.

Salmon, W. C. (1967). *The Foundations of Scientific Inference*, volume 28. University of Pittsburgh Press.

Basic Set Theory

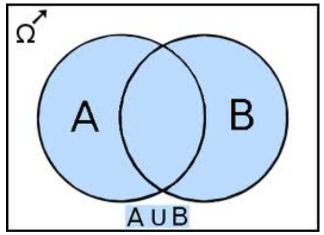
Let's talk about sets baby ...

- We'll use curly brackets $\{\}$ to indicate members of sets, which will generally be numbers.
- E.g. $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$
- 1 is said to be a **member** or **element** of A.
- If x is a member of A, then we will write $x \in A$.

Unions

$A \cup B$ will indicate the set that contains all of (and only) the elements of **either** A or B.

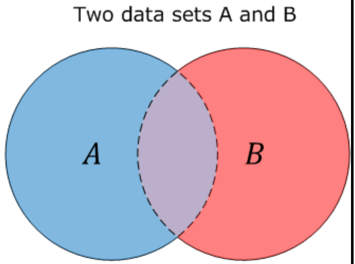
- If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then
- If $A \cup B = \{1, 2, 3, 4, 5\}$.



Intersection

$A \cap B$ will indicate the set that contains all of (and only) the elements of **both** A or B.

- If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then
- If $A \cap B = \{3\}$.



The Sample Space

In probability, there is generally some large set Ω called the **sample space** that is intended to represent all the possible outcomes of an experiment. For instance:

- Experiment 1: Roll a die.
- $\Omega = \{1, 2, 3, 4, 5, 6\}$.
- Experiment 2: Flip a coin twice.
- $\Omega = \{\langle H, H\rangle, \langle T, T\rangle, \langle H, T\rangle, \langle T, H\rangle\}$

The Sample Space

Very roughly, subsets of the sample space are called events:

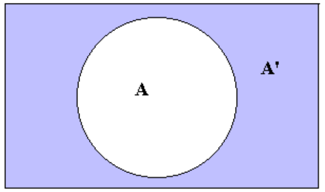
- Experiment 1: Roll a die.
- The event that it lands on an odd number is $A = \{1, 3, 5\}$.
- Experiment 2: Flip a coin twice. The event it lands heads exactly once is:

$$A = \{\langle H, T\rangle, \langle T, H\rangle\}$$

Complements

A^c will indicate the set that contains all the elements of the sample space that are **not** in A.

- If $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$, then
- then $A^c = \{2, 4, 6\}$.



The Sample Space

For technical reasons that we probably won't discuss, it turns out that, in some cases, it's hard to let every subset of the sample space count as an event.

The Sample Space

Accordingly, define an **algebra** $\mathcal{E}$ to be a collection of subsets of Ω satisfying the following properties.

- If A is a member of $\mathcal{E}$, then so is its complement A^c.
- If A and B are members of $\mathcal{E}$, then so is their union $A \cup B$.
- The empty set $\emptyset = \{\}$ is a member of $\mathcal{E}$.

The members of an *algebra* will be called **events**.

Algebras

Example 1 of an algebra:

- Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ represent possible outcomes of rolling a die.
- Let $\mathcal{E} = \{\emptyset, \Omega\}$. Then $\mathcal{E}$ is an algebra.

Algebras

Example 2 of an algebra:

- Let $\Omega = \{\langle H, H\rangle, \langle T, T\rangle, \langle H, T\rangle, \langle T, H\rangle\}$ be all possible outcomes if a coin is flipped twice.
- Define:
  - $A = \{\langle H, H\rangle\}$ is the event that the no tails are observed.
  - $B = \{\langle T, T\rangle, \langle H, T\rangle, \langle T, H\rangle\}$ be the event that at least one tail is observed.
- Let $\mathcal{E} = \{\emptyset, A, B, \Omega\}$. Then $\mathcal{E}$ is an algebra.

Probability

Suppose $\mathcal{E}$ is an algebra (i.e. a collection of events).

A **probability** measure assigns every event A in $\mathcal{E}$ some number $P(A)$ between 0 and 1 (inclusive) such that

- $P(\emptyset) = 0$.
- If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.

Probability Axioms

These are Kolmogorov's axioms for probability. Throughout the semester, we'll see that different authors propose different mathematical assumptions about probability.