Propensity Theories

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Philosophy of Probability
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- A problem for infinite frequentism: randomness
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- Six versions of finite frequentism and criticisms [Hájek, 1996].
- A problem for infinite frequentism: randomness
    - Defining randomness can also be a problem for certain versions of finite frequentism, but one must define probabilities in terms of sequences in the infinite case. That's why the problem is most visible there.
    - For criticisms of hypothetical frequentism more generally, see [Hájek, 2009]. Like [Hájek, 1996], this paper investigates whether hypothetical frequentism matches certain "pre-theoretic intuitions."

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- Popper [1959] argues that hypothetical frequentists are already committed to a "propensity" view.
- Roughly, his argument is that in order to pick which sequence to use in the definition of probability, one appeals to certain physical facts about an experiment. Here's his example ...

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- The sequence of flips of **both** coins together.
- The sequence of flips of the second, biased coin.

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Commitments of Hypothetical Frequentism

- Intuitively, many want to say the second sequence is the "correct" one to use.
- The limiting frequencies of the even and odd sequences are different because different physical properties of the two coins are different.
- One ought to define the probability of an event as a the limiting relative frequency in a sequence of repeated experiments, where experiments are repeated if the physical properties are relevantly similar.

So it looks like the hypothetical frequentism is really the conjunction of the following thesis and definition:

Let E be an experiment.

- **Thesis:** If E were repeated a large number of times, the relative frequency of some events will approach a limiting in virtue of the properties (physical, chemical, etc.) of E.

So it looks like the hypothetical frequentism is really the conjunction of the following thesis and definition:

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- **Thesis:** If E were repeated a large number of times, the relative frequency of some events will approach a limiting in virtue of the properties (physical, chemical, etc.) of E.
- **Definition:** The probability of an event in E is this limiting value.

But this is what the propensity theory of probability asserts Popper [1959].

The properties (or "generating conditions" or "causes") of the experiment are called propensities.

Why give this view a new name? Why does Popper not just say that he has clarified hypothetical frequentism?

There are two reasons . . .

Reason 1: Many frequentists claimed probability must be measurable.

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- In many circumstances, we don't know the relevant "generating conditions" in the sequence that produce limiting relative frequencies. So we can't measure them.
- Actual (not hypothetical) frequentists wanted to avoid defining probability in terms of counterfactuals precisely because we can't observe what does not happen.

Reason 2: Many frequentists argue that, if an experiment is not repeated, it is meaningless to talk about the probability of an event.

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- But if the limiting relative frequencies arise due to properties of the experiment, one can define the probability of an event were it repeated even if it is not.
- This is what Popper [1959] does.

The Propensity Interpretation

Just as there are several different frequency theories, there are several different "varieties" of propensity theories.

We'll distinguish them at the end of the class.

Outline

1. Review
2. Popper's Motivation
3. Suppes on Representation Theorems
4. References

Recall Salmon [1967]'s first criterion of an interpretation of probability is as follows:

Admissibility: An interpretation should satisfy the axioms of probability, hopefully, Kolmogorov's.

Admissibility in the Frequentist and Propensity Interpretations

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- Admissibility is mathematically trivial for frequentists: they define probability as a limiting relative frequency in sequences in which such limits exist.
- Propensity theorists define probability in terms of properties of an experiment, not a sequence. This raises the question:
  - What experimental setups produce limiting relative frequencies? That is, which experiments (if any) will be amenable to application of probability theory?

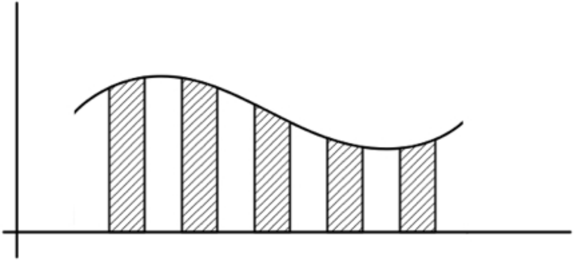
Suppes [1987] and Suppes [2002] try to characterize four types of experiments in which one should expect mathematical probabilities to emerge.

Suppes' Representation Theorems

One of Suppes [1987]'s theorems is structually similar to an argument given by Poincare, Reichenbach, and now Strevens.

See Glymour and Eberhardt [2012] for references.

Strike Ratios



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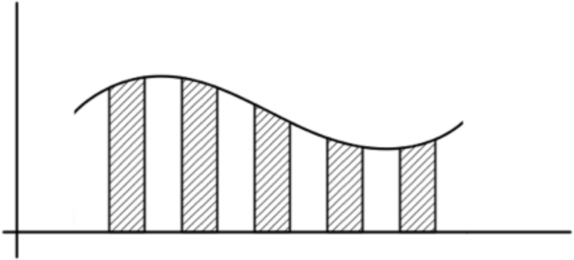

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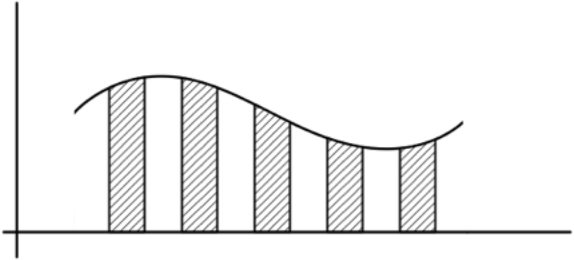
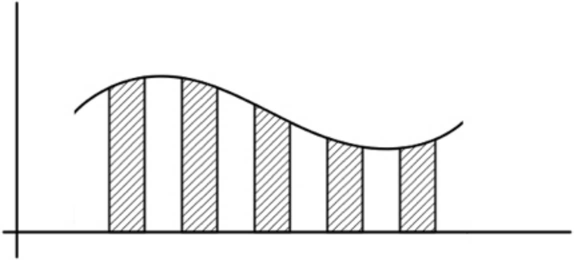


- Imagine a coin is tossed and that whether it lands heads or tails depends exclusively on its angular velocity.
- Imagine that small differences between angular velocities correspond to changes in the outcome of the toss, and
- The intervals corresponding to heads and tails tosses alternate and are of the same width.



- Then whatever the frequency distribution over angular velocities is, the coin will land heads $\frac{1}{2}$ of the time.

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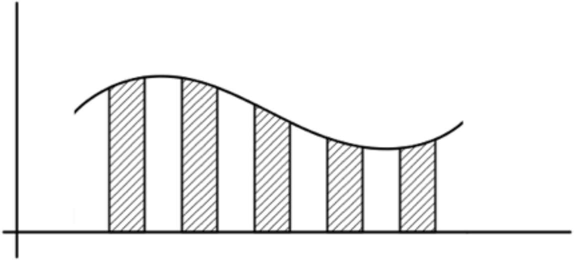

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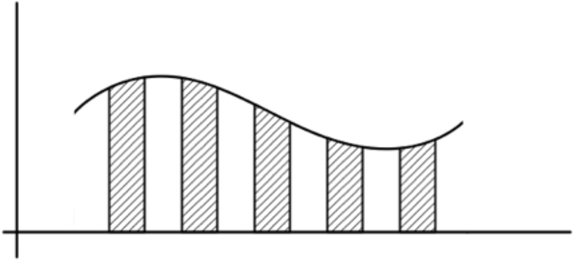

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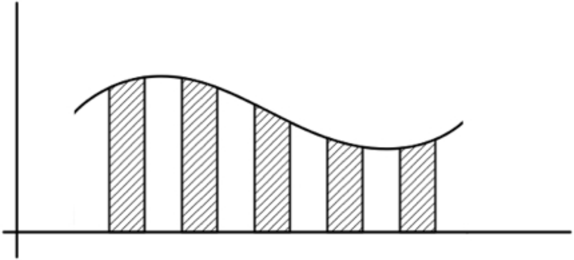

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This is one of four (!) representation theorems that Suppes offers. This raises a number of question . . .

About Suppes [2002]:

- Why doe Suppes prove four theorems rather than one? Does he think there is a general reason to believe propensities can be represented as probabilities? Why or why not?
- Does Suppes prove there is a unique probability measure for each of the four physical situations he describes? Why or why not? In what ways does Suppes claim his theorems differ from those concerning subjective probability?

Discussion Questions

About Gillies [2000]:

- Describe at least two distinctions that Gillies' draws among different propensity theories. How do the various propensity theories fair with respect to Salmon [1967]'s three criteria for an interpretation of probability?
- Do Suppes' theorems support the various propensity theories that Gillies describes, and if so how?
- What is "Humphreys' paradox"? Explain in what ways it is intended to be a challenge to the propensity interpretation of probability.

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