Philosophy of Probability

Criteria for Interpretations
April 15th, 2013

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- What is the probability that there is a god?

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- What is the probability that human beings have landed on the moon?
- What is the probability that Obama is not the current president of the United States?
- My friend John is a conspiracy theorist. He thinks the chance that people have landed on the moon is 12%. Is his assessment reasonable?

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- What is the probability that Angela Merkel will be re-elected?
- I have a friend in Pittsburgh who wrote me an email this morning. What is the probability that my friend is Mexican? What is the probability my friend is a woman?
- My friend's name is Ruben Sanchez-Romero. What is the probability that my friend is Mexican? A woman?

Ubiquity of Probability Judgments

Discussions of likelihood, chance, and probability are everywhere:

- **Observable** events: Throws of dice, FC Bayern matches, the weather in Munich.
- **Un-observable** Events: Extinction of dinosaurs.

Ubiquity of Probability Judgments

Discussions of likelihood, chance, and probability are everywhere:

- **One-time** events: Merkel's re-election in a particular year.
- **Repeatable** events: Throws of dice.

Ubiquity of Probability Judgments

Discussions of likelihood, chance, and probability are everywhere:

- **Events**: Throws of dice, FC Bayern's next match, Merkel's next re-election.
- **Facts or Propositions**: The existence of God

Objectivity of Probability Judgments

Judgments of probability sometimes seems objective and sometimes subjective:

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Judgments of probability sometimes seems objective and sometimes subjective:

- **Subjective**: If different persons can reasonably disagree about the probability of an event (the exact chance of FC Bayern winning), then probability must be subjective in some way.
- **Objective**: If some assessments of probability are unreasonable (low probabilities of moon landings) or are consequences of physical facts (symmetrical weighting of dice), then probability must be objective in some way.

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Strength of Evidence

Judgments of probability sometimes **change**: You likely thought it was more likely that my friend was Mexican, and less likely that he is female upon learning his name.

In particular, the **change** in probability seems to reflect the **strength of evidence** that learning my friend's name provides for his nationality.

In short: We use probability everywhere.

- Events vs. Facts
- Observable vs. Unobservable Events
- One-Time vs. Repeatable Events
- Subjective vs. Objective
- Strength of Evidence

Your assessments of probability likely obeyed certain mathematical axioms.

- Were most of your answers numbers?
- Did you provide **single** numbers? (e.g. 90% instead of 85 – 90%)

Numbers and probabilities: **Comparisons**

- Numbers can be compared: either $x > y$, or $y > x$, or $x = y$.
- Can you always make comparisons of probability? E.g., What's more likely that the temperature of the earth will rise 2°C by the year 2100 or that China will fight a war with some other major world power?

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- Compare. Clearly, Batman is more awesome than Superman. Is he at least 6.3 times as awesome?
- Can assessments of probability be made so precise?

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- Compare: The length of objects can be assigned a number, but there is no maximum length.
- What is it that there is some maximum degree of certainty? And some minimum degree of confidence?

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- Compare. John's IQ is 120, and Jane's is 130. Do John and Jane have a combined IQ?
- Moreover, why didn't you multiply the two numbers? What's so special about addition?

Properties of Probability

In short:

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- We assume it has particular mathematical properties.

Outline

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1. Probability: Uses and Properties

2. The Central Philosophical Question

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Properties of Probability

The central philosophical question: What is probability?
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Salmon [1967] argues that any adequate answer is really an answer to at least three other questions:

- Why should probability have particular mathematical properties?
- How do we determine or measure probabilities?
- Most importantly, why and when is probability useful (especially in the sciences)?

An answer to these questions is called an **interpretation** of probability.

Interpretations of Probability

- **Frequency**:
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  - To say that the probability that a person is male is 49% is to say that about one-half of people are men.
  - Probabilities are just frequencies in some population.
  - Sometimes populations are infinite (e.g. as in the flips of a coin).
- **Propensity**: Probabilities are properties of objects that cause the objects to behave in certain ways.
  - E.g., An nucleus has a propensity to decay, even if it's decay cannot be repeated in considered in some larger population of nuclei.

Logical: Probability is measure of strength of an argument.

- Some arguments logically entail their conclusion. E.g.
  - Assumption 1: If John is married, then he's not a bachelor
  - Assumption 2: John is married.
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- Some arguments logically entail their conclusion. E.g.
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  - Assumption 2: John is married.
  - Conclusion: So he's not a bachelor.
- Some arguments only make the evidence for the conclusion stronger. E.g.,
  - Assumption: John works at Google.
  - Conclusion: So he knows a lot about computers.
- Probability quantifies how strong the evidence is for a conclusion given the premises of the argument or some data.

Subjective: Probability is simply a measure of how strongly we believe particular propositions.

- E.g., I believe that Munich is in southern Germany. I also believe that Hilary Clinton will not run for President of the United States. But I believe that Munich is in southern Germany much more strongly than I believe that Clinton won't run.
- Probability is what quantifies the difference in strength of my beliefs.

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Interpretations of Probability

- Some philosophers think there is only one interpretation of probability.
- Some don't: they think we use the word probability to mean several different things, and different interpretations of probability are useful in different ways.

Goal of Course: To study various interpretations of probability and when (if ever) they are useful.

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- E.g. $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$
- 1 is said to be a **member** or **element** of A.
- If x is a member of A, then we will write $x \in A$.

UNIONS

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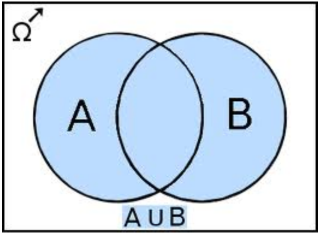

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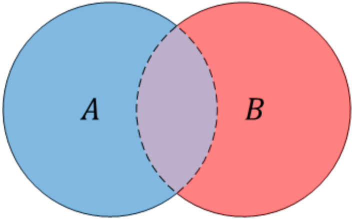
- If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then
- If $A \cap B = \{3\}$.

Two data sets A and B



The Sample Space

In probability, there is generally some large set Ω called the **sample space** that is intended to represent all the possible outcomes of an experiment. For instance:

- Experiment 1: Roll a die.
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- Experiment 1: Roll a die.
- $\Omega = \{1, 2, 3, 4, 5, 6\}$.
- Experiment 2: Flip a coin twice.
- $\Omega = \{\langle H, H\rangle, \langle T, T\rangle, \langle H, T\rangle, \langle T, H\rangle\}$

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- Experiment 1: Roll a die.
- The event that it lands on an odd number is $A = \{1, 3, 5\}$.
- Experiment 2: Flip a coin twice. The event it lands heads exactly once is:

$$A = \{\langle H, T \rangle, \langle T, H \rangle\}$$

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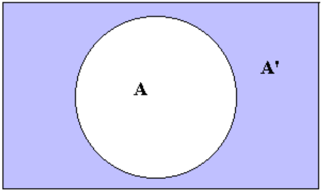
A^c will indicate the set that contains all the elements of the sample space that are **not** in A.

- If $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$, then
- then $A^c = \{2, 4, 6\}$.



For technical reasons that we probably won't discuss, it turns out that, in some cases, it's hard to let every subset of the sample space count as an event.

Accordingly, define an **algebra** $\mathcal{E}$ to be a collection of subsets of Ω satisfying the following properties.

- If A is a member of $\mathcal{E}$, then so is its complement A^c.
- If A and B are members of $\mathcal{E}$, then so is their union $A \cup B$.
- The empty set $\emptyset = \{\}$ is a member of $\mathcal{E}$.

The members of an *algebra* will be called **events**.

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- Let $\mathcal{E} = \{\emptyset, A, B, \Omega\}$. Then $\mathcal{E}$ is an algebra.

Suppose $\mathcal{E}$ is an algebra (i.e. a collection of events).

A **probability** measure assigns every event A in $\mathcal{E}$ some number $P(A)$ between 0 and 1 (inclusive) such that

- $P(\emptyset) = 0$.
- If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.

Probability Axioms

These are Kolmogorov's axioms for probability. Throughout the semester, we'll see that different authors propose different mathematical assumptions about probability.

References I

Salmon, W. C. (1967). *The Foundations of Scientific Inference*, volume 28. University of Pittsburgh Press.