Subjective Probability: Criticisms

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Philosophy of Probability
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Review: Four Interpretations of Probability

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- Frequentist: Probability is just a relative frequency.
- Propensity: Probability is a tendency towards an outcome.
- Logical: Probability is the measure of the degree to which a set of sentences support a conclusion.
- Subjective: Probability is the strength of a (rational?) individual's degree of belief.

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- **Applicability:**

Subjective Probability

Purported Virtues of Subjective Probability:

- **Admissibility:** Several representation theorems that indicate that degrees of belief are (or ought to be) represented by probabilities.
- **Ascertainability:** Those same theorems often suggest a way to measure degree of belief.
- **Applicability:** Subjective probability is one component (the other is utility) in the most widely applied theory of rational-decision making: subjective expected utility theory.

Today: Real experiments and thought experiments challenging subjective probability as either (i) a descriptive theory or (ii) a normative theory

Outline

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Disclaimer

Disclaimer:

- Empirical science is hard.
- There are always several hypotheses compatible with any scientific experiment.
- The following experiments are often thought to suggest theories of subjective probability are not descriptively accurate.
- There are, however, many folks who think the experiments are not decisive.

The Linda Problem

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Conjunction Fallacy

Kahneman and Tversky found around 90% chose the second statement as more probable, even though, on first glance, it seems to violate the following rule of probability:

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Probability Elicitation

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- How are personal probabilities elicited according to Dutch book arguments? According to Savage?
- What does Savage think about asking individuals for their qualitative judgments about the relation "more likely than"?

Savage: Behavior vs. Intuitions

> *Even if the concept (of "more probable than") were so completely intuitive . . . what could such interrogation have to do with the behavior of the person in the face of uncertainty, except of course for his verbal behavior under investigation?*

Savage [1972] pp. 27

Experimental Evidence

Charness et al. [2010] repeated Tversky and Kahneman's experiments with several variations. I'll mention three.

- The original experiment.
- The original question. Participants were told there is a correct answer and that they would receive $4 if they answered correctly.
- The original question. Participants discussed the question in groups of three, and then answered independently.

Experimental Evidence

Charness et al. [2010] found the following:

- The original experiment: 85% commit conjunction fallacy.
- With \$4 Incentive: 33% commit conjunction fallacy.
- Groups of three with \$4 Incentive: 10% commit the conjunction fallacy
  - My Note: If subjects decided independently and voted according to majority rule, one should expect $\sim$ 26% error rate given the individual results.

Experimental Evidence

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In addition to subject indifference to the experiment (which drives a lot of survey results), there are lots of explanations. Let me note one thing ...

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- Subjects are still asked to answer a question rather than choose an action or name a price.
- So a better question in this regard would be the following.

The Linda Problem

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Choose one of the following.

1. I will ask Linda if is a bank teller. If she answers "Yes", then I will give you \$1.
2. I will ask Linda if is a bank teller and if she is active in the feminist movement. If she answers "Yes" to both questions, then I will give you \$1.

Experimental Evidence

I will bet that very few subjects will choose option 2. Someone should get some grant money and try it.

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I will bet that very few subjects will choose option 2. Someone should get some grant money and try it.
The options also eliminate ambiguity in the original question concerning the meaning of the sentences (and of "probable") ,for example ...

> *. . . some apparent biases might occur because the specific words used, or linguistic convention subjects assume the experimenter is following, convey more information than the experimenter intends. In other words, subjects may read between the lines. The potential linguistic problem is this: in the statement "Linda is a feminist bank teller," subjects might think that this statement "Linda is a bank teller" tacitly excludes feminists; they might think it actually means "Linda is a bank teller (and not feminist)." If subjects interpret the wording this way none of the statements are conjunctions of others and no probability rankings are wrong.*

Camerer [1995], pp. 598.

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- One was proposed by Ellsberg; he did not conduct it.
- Subsequent empirical work: MacCrimmon and Larsson [1979] and Slovic and Tversky [1974]

Ellsberg Paradox

- Imagine an urn with 90 balls.
- 30 are red.
- 60 are either yellow or black.
- You do not know the proportions of black vs. yellow balls.

Ellsberg Paradox

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You are offered the following choice:

- R: A prize of \$1 if a red ball is drawn.
- B: A prize of \$1 if a black ball is drawn.

Ellsberg Paradox

Poll: Which would you prefer? For which would you pay a higher price?

Ellsberg Paradox

Suppose I am about to pull a ball from the urn.

You are offered the following choice:

- $R \vee Y$: A prize of \$1 if a red ball or a yellow ball is drawn.
- $B \vee Y$: A prize of \$1 if a black ball or a yellow ball is drawn.

Ellsberg Paradox

Poll: Which would you prefer? For which would you pay a higher price?

Ellsberg Paradox

Unless this class is exceptionally strange, here are your preferences:

- $B \prec R$ and $Pr(B) < Pr(R)$
- $R \vee Y \preceq B \vee Y$ and $Pr(B \cup Y) \leq Pr(R \cup Y)$

where Pr is the price you would offer for the bets above.

Ellsberg Paradox

According to the Dutch Book arguments, you think:

- $P(B) \leq P(R)$
- $P(R \cup Y) \leq P(B \cup Y)$

where P is the probability that you assign to the events.

Since $R \cap Y = B \cap Y = \emptyset$:

$$\begin{aligned} & P(R \cup Y) \leq P(B \cup Y) \\ \Rightarrow \quad & P(R) + P(Y) \leq P(B) + P(Y) \\ \Rightarrow \quad & P(R) \leq P(B). \end{aligned}$$

which is a contradiction.

Ellsberg Paradox

Your preferences also contradict Savage's Sure-Thing principle.

- If Yellow:
  - R agrees with B (both lose), and
  - $R \vee Y$ agrees with $B \vee Y$ (both win).

Ellsberg and the Sure-Thing Principle

- If Yellow:
  - R agrees with B (both lose), and
  - $R \vee Y$ agrees with $B \vee Y$ (both win).
- If Not Yellow:
  - R agrees with $R \vee Y$ (both win iff Red is drawn)
  - B agrees with $B \vee Y$ (both win iff Black is drawn)

Ellsberg and the Sure-Thing Principle

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- If Not Yellow:
  - R agrees with $R \vee Y$ (both win iff Red is drawn)
  - B agrees with $B \vee Y$ (both win iff Black is drawn)
- You prefer R to B.

By Savage's Sure-Thing principle, you ought to prefer $R \vee Y$ to $B \vee Y$.

Experimental Results

So experimental results seem to indicate (though not as strongly as you might think) that DB and Savage's postulates are not descriptive of behavior.

Savage on Normativity

In what ways does Savage think his axioms are normative?

Savage on Normativity

Suppose someone says to me, "... I behave in flagrant disagreement with your postulates because they violate my personal taste, and it seems to me more sensible to cater to my taste than to a theory arbitrarily concocted by you/" I don't see how I could really controvert him, but I would be inclined to match his introspection with my own. I would, in particular, tell him that, when it is explicitly brought to my attention that I have shown a preference for **f** *when compared with* **g**, *for* **g** *when compared with* **h**, *and for* **h** *when compared with* **f**, *I feel uncomfortable in much the same way that I do when it is brought to my attention that some of my beliefs are logically contradictory.*

Savage [1972], pp. 21.

Savage on Normativity

In other words, a postulate is normative if you'll switch your preferences when shown that preferences violate it.

Question: Is there any experimental way of testing the normativity of the Sure-Thing principle without really confusing subjects?

Discussion Questions

- Explain two potential problems for the theory of personal probability according to Savage. How does Savage resolve these problems?
- Distinguish four interpretations of theories of personal probability according to Kyburg.
- Explain Kyburg's argument for the claim that the DB argument, as he originally presents it, is invalid. What additional assumptions are necessary? And why, according to Kyburg, are those assumptions false?
- Does Kyburg's argument show similar problems for Savage's representation theorem? Why or why not?

Explain, defend, and criticize Kyburg's argument for the following conclusion:

> *If we suppose we begin with a full preference ranking among acts, there are two possibilities. Either the preference ranking is coherent, or it is not. If it is coherent, we are all set - we merely follow the dictates of our preference ranking with no further analysis. If it is not, then something must be changed; but as Savage never tired of pointing out, the subjectivistic theory will not tell you what to change. Subjectivistically interpreted, Normative Bayesian Decision Theory, whatever its heuristic virtues, is either philosophically vacuous or impotent.*

Kyburg [1978], pp. 171.

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