Aristotle's Logic

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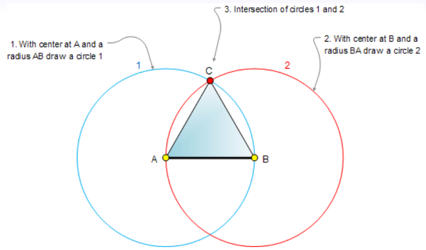
Review

The Generality Problem: How do we know Euclid's proofs apply/work for all triangles, circles, etc., rather than just for the objects in the particular diagrams? [Mumma, 2010]

Small Group Review: Last class, we discussed three reasons why Euclidean diagrams are necessary. What were they?

Necessity of the diagram



There are at least three reasons why Euclid's diagrams are necessary:

1. Objects may be "underspecified" [Netz, 1999].
2. The considered case cannot be recovered from the text alone.
3. The existence of some objects (e.g., intersection point in I.1) is not guaranteed by the postulates alone.

Euclid's Diagrams and Generality

These three reasons raise the generality problem because it is unclear whether

1. Euclid's proofs work when objects are specified differently (than in the diagram),
2. The proof in omitted cases is similar (and valid!), and
3. All purportedly constructed objects exist if the diagram is drawn differently,

Euclid's Diagrams and Generality

Readers unfamiliar with the cultural and mathematical context in which the **Elements** was written may not know how to answer the following questions:

1. Is there a method for eliminating underspecification?
2. What information can be inferred from the diagram?
3. Which are cases left for the reader?

So the generality of Euclid's proofs **could have** been a concern for Greek and modern philosophers . . .

Generality of Mathematics

- The generality of Euclid's proofs **could have** been a concern for Greek and modern philosophers . . .
- But it wasn't: few doubted Euclid's proofs were general.
- Until the 19th century, the question was not **if** Euclid's proofs were general, but rather **why**.

Logical Solution to Generality Problem

In hindsight, we might try to propose the following solution.

Proposed Solution: Euclid's proofs are general because one can fill in the logic by

1. Specifying objects completely in proofs.
2. Making appropriate case distinctions.
3. Adding postulates that guarantee existence of geometric objects, and
    - Following appropriate rules of inference.

Logical Solution to Generality Problem

Today: Why the logical solution is not so straightforward.

1. Until the 19th century, logic was Aristotlelian logic, and
2. Aristotle's logic is limited in several ways that we'll explore.

Where We're Going

Future Classes: These limitations make Aristotle's logic insufficient for mathematical reasoning, and most philosophers realized this. They provided alternative reasons for the generality of mathematical proofs:

1. Locke's theory of abstract ideas and demonstration
2. Leibniz's extension to logic,
3. Kant's forms of pure intuition

Outline

Syntax

Below, think of a and b as representing sets of things, like humans, mammals, brown objects, etc.

- *Aab* represents the sentence "All b's are $a's$."
- *Eab* represents the sentence "No b's are $a's$."
- *Iab* represents the sentence "Some b's are $a's$."
- *Oab* represents the sentence "Some b is not a."

Syntax

If $b =$ humans and $a =$ mammals, then

- *Aab* represents "All humans are mammals."
- *Eab* represents "No humans are mammals."
- *Iab* represents "Some humans are animals."
- *Oab* represents "Some humans are not mammals."

Syllogisms

- **Syllogisms** (or deductions) have two premises/assumption and one conclusion.
- Example: *Camestres*
  - P1: All humans are mammals (Aab)
  - P2: No reptiles are mammals (Eac)
  - Conclusion: No reptiles are human (Ebc)

Group Exercise

Group Exercise: Compare your answers to the second question on the reading assignment. Pick your best example of one type of syllogism, and come write your answer on the board.

Mnemonics

- The letters A, E, I, and C were used by medieval scholars to create **mnemonics**.
- Syllogisms were given different names (e.g., Barbara, Celarent, etc.) to aid memorization.

Logical Mnemonics

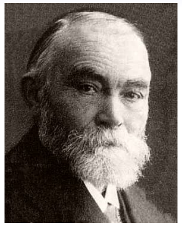
- E.g., Barbara is the syllogism with two "A" premises and one "A" conclusion.
  - Aab and $Abc \vdash Aac$
- E.g., Camestres is the syllogism with an "A" and "E" premise, and an "E" conclusion:
  - Aab and $Eac \vdash Ebc$

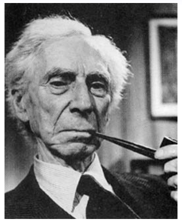
Exercise

Exercise: Without looking at the reading, guess the structure of the syllogisms called "Darii" and "Festino," and give an example of each.

Try this first on your own. Then compare your answers with your group members.

Predicate Logic





Advertisement: You should really take Phil 120 or Phil 470 to learn more about predicate logic.

Today, we'll discuss only how to represent sentences symbolically in predicate logic. Then we'll compare it with Aristotelian logic.

Exercise

Group Exercise: Write your examples of Darii and Festino in the language of predicate logic.

Monadic Predicates

- In modern parlance, Aristotle's logic contains only monadic predicate symbols, i.e., symbols denoting properties that apply to exactly one object.
  - E.g., Aristotle's logic could represent statements "A is a point" or "L is a line."
- However, many geometric statements are relational, e.g., "A lies on line BC."

Multiple Quantifiers

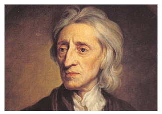
- In modern parlance, Aristotle's logic contains sentences with only one quantifier.
  - E.g., Aristotle's logic allows one to say "Everything is awesome."
- However, many geometric statements involve multiple nested quantifiers
  - $(\forall x)\,(P(x) \rightarrow (\exists y)(L(y)\&O(x,y)))$ represents "Every point lies on some line" if the domain of discourse is points and lines, $P(x)$ represents "x is a point," $L(x)$ represents "x is a line," and $O(x,y)$ represents "x lies on y."

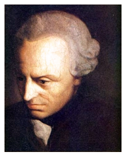
No constant symbols

- In modern parlance, Aristotle's syllogisms contains no sentences with constant symbols
  - E.g., Aristotle's logic allows one to say "Not everyone is special" but not "Joey is special."
- However, many geometric statements involve sentential connectives of precisely this sort:
  - K is a circle.

Upshot 1: It is hard to **translate** Euclid's postulates, theorems, etc. into an Aristotelian framework, let alone **prove** the theorems using Aristotle's methods.

Where We're Going



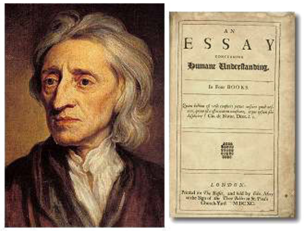


Upshot 2: Many modern philosophers argued, therefore, that we have additional **cognitive abilities** that allow us to prove mathematical theorems, even if Aristotelian logic is insufficient:

1. Locke's theory of abstract ideas and demonstration
2. Descartes' theory of rational insight
3. Leibniz's extension to logic,
4. Kant's forms of pure intuition

Up Next

Where We're Going



- Locke's Solution: abstract ideas.
  - You'll also see how Locke's theory of demonstration is based upon the above ideas about showing equality of geometric magnitudes.
- Why Locke's solution is/was unpopular

Today's Response Question

Response Question: Explain two reasons that Aristotelian syllogisms are insufficient for reconstructing the reasoning in the elements.

References I

Mumma, J. (2010). Proofs, pictures, and Euclid. *Synthese*, 175(2):255–287.

Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge University Press Cambridge.