Phil. 373: Predicate Logic Exercises

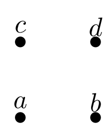
January 11th, 2017

Readings:

- C. Shields. "Aristotle". In: *The Stanford Encyclopedia of Philosophy.* Ed. by E. N. Zalta. Winter 2016. Metaphysics Research Lab, Stanford University, 2016.
- Mayo-Wilson. "A Somewhat Quick Introduction to Predicate Logic."

Goals: By the end of class, students should be better able to translate mathematical claims (in English) into formulae of first-order predicate logic with identity (FOL). Because (i) my notes contain examples of how to translate arithmetical claims into FOL whereas (ii) geometrical proof was the model of rigor for much of mathematical history, the exercises below focus on geometry, rather by arithmetic. Some of the claims below are Tarski's axioms of plane geometry.

Instructions: In the first two sections, suppose the domain of discourse is all points in the plane. Suppose a, b, c and d represent the points below. Next, let $C(w, x, y, z)$ be a four-place relation representing the claim that "the line segment with endpoints w and x is congruent to the line segment with endpoints y and z." Finally, let $B(x, y, z)$ be a three-place relation representing the claim "y lies on the line segment with endpoints x and z."



Quantifier-Free Fragment

In this section, none of your answers ought to contain quantifiers.

1. What do the following formulae represent? Which are true?
   (a) $\neg B(a, b, c)$
   (b) $C(a, b, c, d) \& B(a, b, c)$
   (c) $C(a, b, b, a)$
   (d) $C(a, b, c, d) \rightarrow C(c, d, a, b)$
   (e) $C(a, a, b, d) \vee C(a, a, b, b)$

(f) $B(a, b, c) \to B(b, a, c)$

2. Translate the following sentences into predicate logic:

   (a) Points a and b are not the same.

   (b) If a is between b and c, then c is not between a and b.

   (c) If c is between a and b, then c is between b and a.

   (d) If a is between b and b, then the line segment with endpoints a and b is congruent to the line segment with endpoints a and a.

   (e) If a is between b and b, then a and b are the same point.

Quantifiers

In this section, your formula may contain quantifiers.

1. What do the following formula of predicate logic represent?

   (a) $(\exists x)(\exists y)\neg x = y$

   (b) $(\forall x)(\forall y)C(x, y, y, x)$

   (c) $(\exists x)(\neg B(a, x, b) \& C(a, b, b, x))$

      - On the above diagram, draw at least one point guaranteed to exist by this formula.

   (d) $(\forall x)(\forall y)\,(\neg x = y \to (\exists z)(B(x, z, y) \& \neg w = y \& \neg w = z))$

      - On the above diagram, draw at least one point guaranteed to exist by this formula.

2. Translate the following sentences into predicate logic with identity.

   (a) The plane contains at least three points.

   (b) The plane contains *exactly* two points.

   (c) The plane contains at least three non-collinear points. [Hint: Three points are not collinear if none of the points lie between the other two].

   (d) Transitivity of congruence: If one line segment is congruent to a second, and the second is congruent to a third, then the first and third are also congruent.

Aristotelian Logic

Formalize your examples of Darii and Festino in predicate logic. Make sure to specify the domain of discourse and the intended interpretation of the predicate symbols. How do the formula in your syllogisms differ from the formulae you've written in previous sections?