Reading Assignment 14: Cauchy and the arithmetization of analysis

Required Readings

- Berkeley 1992. *The Analyst.* Articles 1-8 and 13.
- Grabiner 2012. Introduction and Chapter 1.

Questions

1. In no more than a paragraph, compare one of Berkeley's arguments against the existence of infinitesimals in Sections 1-6 with his arguments against the existence of Lockean abstract ideas you read previously. Are there any obvious similarities and/or differences?
2. In your own words, summarize the following argument:

   > *For when it is said, let the Increments vanish, i.e. let the Increments be nothing, or let there be no Increments, the former Supposition that the Increments were something, or that there were Increments, is destroyed, and yet a Consequence of that Supposition, i.e. an Expression got by virtue thereof, is retained. Which, by the foregoing Lemma, is a false way of reasoning. Certainly when we suppose the Increments to vanish, we must suppose their Proportions, their Expressions, and every thing else derived from the Supposition of their Existence to vanish with them.*

   In particular, Berkeley is criticizing Newton's proof of what theorem? Which steps of Newton's derivation does Berkeley claim rely on the assumption that the ``increments were something'' and which rely on the assumption that ``the increments be nothing''? It may help to continue reading until the end of Section 16.
3. According to Grabiner, what and was one central distinction between the goal of eighteenth and nineteenth century analysts? Was nineteenth century analysis a continuation of eighteenth century analysis? Why or why not?
4. According to Grabiner, what did nineteenth century analysts mean when they discussed ``rigor?''
5. Given a real-valued function $f : \mathbb{R} \to \mathbb{R}$ and two real numbers $x_0, L \in \mathbb{R}$, we write $\lim_{x \to x_0} f(x) = L$ if for all all $\epsilon > 0$, there is some $\delta > 0$ such that if $|x - x_0| < \delta$, then $|f(x) - L| < \epsilon$. The number L is called the *limit* of f as x approaches x_0. Grabiner notes that Cauchy is attributed with this definition, even though it never appears in his work. In what two ways did Cauchy's *use* of limits show that his concept of a ``limit'' was closer to the contemporary definition than were the concepts of eighteenth century analysts?

References

[1] G. Berkeley. *De Motu and The Analyst.* Ed. by D. M. Jesseph. Vol. 41. The New Synthese Historical Library. Springer Science+ Business Media, 1992.

[2] J. V. Grabiner. *The origins of Cauchy's rigorous calculus.* Courier Corporation, 2012.