Phil 373: Philosophy of Mathematics

Generality and Abstraction

January 5th (Mon.) - Introduction to class themes

January 7th (Wed.) - Greek Geometry and the Role of the Diagram
Readings:

- Book 1 of Euclid, *The Thirteen Books of Euclid's Elements: Books I-II*
  - Please read the Common Notions and Postulates (pages 153-155) and proofs or propositions 1, 2, 4, 7, 16, and 47. You may ignore the extensive commentary.
- Sections 1 and 2-2.1.2 of Netz, *The shaping of deduction in Greek mathematics*

January 9th (Fri.) - Aristotelian Logic
Required Readings: Sections 1-5.5 and 6 of Smith, "Aristotle's Logic".

January 12th (Mon.) - Locke on Abstract Ideas and Demonstration
Required Readings: Locke, *An essay concerning human understanding*

- Book II, Chapters 11 and 12.
- Book IV Chapter 2: Sections 1-11.

January 14th (Wed.) - Criticisms of Abstract Ideas
Required Readings:

- Introduction to Berkeley, *A treatise concerning the principles of human knowledge*.
- Section 1.1.7 of Hume, *A Treatise of Human Nature*.
- Pages 67-68 of Shapiro, *Thinking about mathematics*.

Geometry vs. Arithmetic, and Experience vs. Reason

January 16th (Fri.) - Eudoxian Theory of Proportion and Greek Number Theory
Required Readings:

- Book V of Euclid, *The Thirteen Books of Euclid's Elements: Books III-IX*
  - Please read the definitions (pages 113-114) and proofs of propositions 1, 4, and 13. Ignore the commentary.
- Book VII of Euclid, *The Thirteen Books of Euclid's Elements: Books III-IX*
  - Please read the definitions (pages 277-279) and proofs of propositions 1 and 2. Ignore the commentary.
- Book X of Euclid, *The Thirteen Books of Euclid's Elements: Books X-XIII*
  - Please read the commentary on pages 1-3, and the definitions on page 10.

January 19th (Mon.) - No Class

January 21st (Wed.) - Plato's Theory of Recollection
Required Readings: Plato, *Complete works*.

- *Meno*. Translator's introduction (pages 870-871) and Lines 79e - 86a.
- *Phaedo*. Translator's introduction (pages 49-50) and Lines 72e - 77a.

January 23rd (Fri.) - Plato's Philosophy of Mathematics
Required Readings: Republic. Translator's Introduction (pages 971-972) and Book VII. Lines 514 - 530d. In Plato, *Complete works*.

January 26th (Mon.) - Aristotle's Criticisms of Plato
Required Readings: Chapter 3 of Shapiro, *Thinking about mathematics*.

January 28th (Wed.) - Algebra and Geometry in the 14th, 15th, and 16th Centuries
Required Readings:

- Chapter 2, "Controversy and Coefficients" of Pesic, *Abel's proof*

January 30th (Fri.) - Geometric Construction prior to 1637
Required Readings: Bos, *Redefining geometrical exactness*

- Sections 1.1 -1.4, 1.6; Chapter 3 pages 37-41; Sections 3.4-3.5, 3.7; Section 5.1, 5.3; Chapter 6

January 31st (Sat.) - **First paper due by 8PM**

February 2nd (Mon.) - Descartes' Early Philosophy of Mathematics
Required Readings: Rules 2-4, 10, and 12-15 of Descartes, "Rules for Direction of the Mind".

February 4th (Wed.) - Descartes' Geometry
Required Readings:

- Selections from Descartes, *Geometry*
- Pages 317-324 of Kline, *Mathematical thought from ancient to modern times*.

February 6th (Fri.) - Leibniz's Reaction to Descartes
Required Readings:

- "Letter to Queen Sophie Charlotte of Prussia" in Ariew, Garber, and Leibniz, "Philosophical Essays".
- "Critical Thoughts on the General Principles of Descartes." Read Leibniz's criticisms of Articles 1, 5, 26, 31/35, 43/45/46, and 75 in Leibniz, "Philosophical Papers and Letters".

February 9th (Mon.) - Leibniz's Logic
Required Readings:

- "Preface to a universal characteristic" and "Samples of the Numerical Characteristic." Pages 5-18 of Ariew, Garber, and Leibniz, "Philosophical Essays".
- "A Study in the Early Logical Calculus." Pages 371-373 of Leibniz, "Philosophical Papers and Letters".

February 11th (Wed.) - Newton's Philosophy of Mathematics
Required Readings:

- Preface and Book I, Lemmas 1-4, Lemma 28 of Newton, *The Principia*
- Domski, "The Constructible and the Intelligible in Newton's Philosophy of Geometry"

February 13th (Wed.) - Rationalism and Empiricism
Required Readings: Markie, "Rationalism vs. Empiricism"

February 16th (Mon.) - No class

February 18th (Wed.) - Hume
Required Readings: Hume, *A Treatise of Human Nature*

- Book I. Part 1. Sections 1-5, 7.
- Part III. Section 1.

February 20th (Fri.) - Kant
Required Readings: Selections from Kant, *Prolegomena to any future metaphysics*.

The Infinite

February 23th (Mon.) - Aristotle on the Infinite
Required Readings: Aristotle, *The complete works of Aristotle*.

- *Physics*. Book III. Lines 202b30 - 208a25.
- *Physics*. Book VI. Lines 231a18-233a31 and 239b5 - 240b8.
- Book IX: Proposition 20 of Euclid, *The Thirteen Books of Euclid's Elements: Books III-IX*

February 25th (Wed.) The Method of Exhaustion and Infinitesimals
Required Readings:

- Reread the sections from Euclid on the Eudoxian theory of proportions.
- Pages 325-329 (Sections α and β) of Heath, *A history of Greek mathematics*.
- Book XII: Propositions 1 and 2 of Euclid, *The Thirteen Books of Euclid's Elements: Books III-IX*
- Archimedes, *The Works of Archimedes*:
  - "Measurement of the Circle." Proposition 1.
  - "Quadrature of the Parabola." Proposition 16.
- Archimedes, *The Method of Archimedes*. Proposition 1.

February 27th (Fri.) - The Invention of the Calculus
Required Readings: Chapter 17 of Kline, *Mathematical thought from ancient to modern times*.

March 2nd (Mon.) - Berkeley's Critique
Required Readings: Sections 1-15 of Berkeley, "The Analyst".

March 4th (Wed.) - Infinite Divisibility
Required Readings: Part II Sections 1-4 of Hume, *A Treatise of Human Nature*.

March 6th (Fri.) - The Search for Rigor
Required Readings:

- Pages 426-434 of Kline, *Mathematical thought from ancient to modern times*.
- Chapter 18 of Kline, *Mathematical thought from ancient to modern times*.
- Chapter 40. Sections 1, 2, and 7 of Kline, *Mathematical thought from ancient to modern times*.

March 7th (Sat.) - **Second paper due by 8PM**

March 9th (Mon.) - Construction of the reals and the intermediate value theorem
Required Readings: Introduction and Sections 1-5 of "Continuity and Irrational Numbers" in Dedekind, *Essays on the Theory of Numbers*.

March 11th (Wed.) - The Absolute Infinite
Required Readings: Selections from Bolzano, "Paradoxes of the Infinite".

March 13th (Fri.) - Class Wrap-Up and Preview

March 20th (Fri.) - **Third Paper Due at Midnight**

References

[1] Archimedes. *The Method of Archimedes*. Ed. by T. L. Heath. Cambridge: Cambridge University Press, 1912.

[2] Archimedes. *The Works of Archimedes*. Ed. by T. L. Heath. Second. Cambridge: Cambridge University Press, 2010.

[3] "G.W. Leibniz: Philosophical Essays". In: *Indianapolis and Cambridge: Hackett Publishing Company* (1989). Ed. by R. Ariew, D. Garber, and G. W. Leibniz.

[4] Aristotle. *The complete works of Aristotle*. Translation into English from classical Greek. Ed. by J. Barnes. Vol. 2. Princeton, N.J.: Princeton University Press, 1984.

[5] G. Berkeley. *A treatise concerning the principles of human knowledge*. Ed. by J. Dancy. New York: Oxford University Press, 1998.

[6] G. Berkeley. "The Analyst". In: *From Kant to Hilbert Volume 1: A Source Book in the Foundations of Mathematics: A Source Book in the Foundations of Mathematics*. Ed. by W. B. Ewald. Vol. 1. Oxford University Press, 2005.

[7] B. Bolzano. "Paradoxes of the Infinite". In: *From Kant to Hilbert Volume 1: A Source Book in the Foundations of Mathematics: A Source Book in the Foundations of Mathematics*. Ed. by W. B. Ewald. Vol. 1. Oxford University Press, 2005.

[8] H. J. Bos. *Redefining geometrical exactness: Descartes' transformation of the early modern concept of construction*. Springer, 2001.

[9] R. Dedekind. *Essays on the Theory of Numbers*. Dover Publications, June 2012.

[10] R. Descartes. "Rules for Direction of the Mind". In: *The philosophical writings of Descartes*. Ed. by J. Cottingham, R. Stoothoff, and D. Murdoch. Vol. 2. Cambridge University Press Cambridge, 1985.

[11] R. Descartes. *Geometry*. Trans. by D. E. Smith and M. L. Latham. Dover Publications, 1954.

[12] M. Domski. "The Constructible and the Intelligible in Newton's Philosophy of Geometry". In: *Philosophy of Science* 70.5 (Dec. 2003), pp. 1114–1124.

[13] Euclid. *The Thirteen Books of Euclid's Elements: Books I-II*. Ed. by T. L. Heath. Vol. 1. Cambridge University Press, 1908.

[14] Euclid. *The Thirteen Books of Euclid's Elements: Books III-IX*. Ed. by T. L. Heath. Vol. 2. Cambridge University Press, 1908.

[15] Euclid. *The Thirteen Books of Euclid's Elements: Books X-XIII*. Ed. by T. L. Heath. Vol. 3. Cambridge University Press, 1908.

[16] T. L. Heath. *A history of Greek mathematics*. Vol. 1. Oxford, England: Clarendon, 1921.

[17] D. Hume. *A Treatise of Human Nature*. Courier Dover Publications, 2003.

[18] I. Kant. *Prolegomena to any future metaphysics: with selections from the Critique of poor reason*. Translated from the German. Ed. by G. Hatfield. Cambridge: Cambridge University Press, 2004.

[19] M. Kline. *Mathematical thought from ancient to modern times*. Vol. 1. Oxford University Press, 1990.

[20] M. Kline. *Mathematical thought from ancient to modern times*. Vol. 2. Oxford University Press, 1990.

[21] M. Kline. *Mathematical thought from ancient to modern times*. Vol. 3. Oxford University Press, 1990.

[22] G. W. Leibniz. "Gottfried Wilhelm Leibniz: philosophical papers and letters". In: (1969). Ed. by L. E. Loemker.

[23] J. Locke. *An essay concerning human understanding*. English. Ed. by P. H. Nidditch. Oxford: Clarendon Press, 1975.

[24] P. Markie. "Rationalism vs. Empiricism". In: *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Summer 2013. 2013.

[25] R. Netz. *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge University Press Cambridge, 1999.

[26] I. Newton. *The Principia : Mathematical Principles of Natural Philosophy*. English. Trans. by I. B. Cohen, A. Whitman, and J. Budenz. 1st edition. Berkeley: University of California Press, Oct. 1999.

[27] P. Pesic. *Abel's proof*. MIT Press, 2003.

[28] Plato. *Complete works*. Ed. by J. M. Cooper and D. S. Hutchinson. Hackett Publishing, 1997.

[29] S. Shapiro. *Thinking about mathematics: The philosophy of mathematics*. Oxford University Press, 2000.

[30] R. Smith. "Aristotle's Logic". In: *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Spring 2014. 2014.