

## PROGRAMS FOR PLATO: INTRO. TO AGENT-BASED MODELING IN PHILOSOPHY

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## MODELS AND SIMULATIONS

This is an atypical philosophy course.

- Computer simulations? In Philosophy?
- You might say, "I've read Plato. He was a better-than-fair philosopher. He didn't need a computer."
- How can computer simulations help us in answering questions about justice, about the nature of mind, about free will, and so on? How can simulations answer any of the core questions in philosophy?

## OUTLINE

- 1 TWO PLATONIC PUZZLES
  - Justice
  - Meaning
- 2 ABMS VS. POPULATION MODELS
  - Population models
  - ABMS
- 3 GAME THEORY
  - Basic Concepts
  - Equilibria Explanations
- 4 BACK TO PLATO'S PUZZLES
- 5 ABMS VS. ALTERNATIVE EXPLANATIONS
- 6 REFERENCES

Puzzle 1: Justice

## THE REPUBLIC



In Plato's most famous work, [Republic](#), Socrates is faced with two very difficult questions:

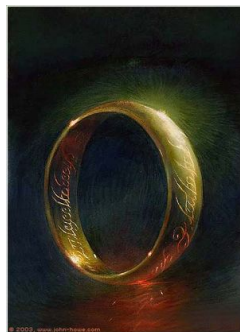
- What is justice?
- Why do individuals, for the most part, behave justly, even when it seems to be counter to their interests?

## GLAUCON'S CHALLENGE

### Glaucon's Challenge:

*First, I'll state what kind of thing people consider justice to be and what its origins are. Second, I'll argue that all who practice it do so unwillingly, as something necessary, not as something good. Third, I'll argue that they have good reason to act [unjustly], for the life of an unjust person is, they say, much better than that of a just one.*

## THE RING OF GYGES



## GLAUCON'S ARGUMENT

Glaucon defines "justice" and explains how it arises as follows:

*This, they say, is the origins and essence of justice. It is intermediate between the best and the worst. The best is to do injustice without paying the penalty; the worst is to suffer it without being able to take revenge. Justice is a mean between the two extremes.*

Plato. Book II. [Republic](#)



## GLAUCON'S ARGUMENT

**Problem:** When modeled formally, we'll see Glaucon's solution about why people behave justly is incomplete ...

We'll say nothing about Plato's solution.

## Puzzle 2: Meaning

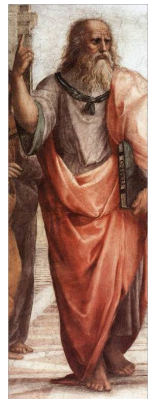
## PLATO ON THE ORIGINS OF LANGUAGE

- In his dialogue *Cratylus*, Plato studies how noun phrases can acquire **meaning**.
  - For instance, why does "table" (or "Tisch") refers to a flat surface supported by legs?
- Roughly, Plato thinks there is a difficulty in explaining how we agree that "table" should refer to a table **without already having a language**.
- His solution is rather ingenious ...

## PLATO ON THE ORIGINS OF LANGUAGE

*Cratylus says, Socrates, that there is a correctness of name for each thing, one that belongs to it by nature. A thing's name isn't whatever people agree to call it – some bit of their native language that applies to it – but there is a natural correctness of names, which is the same for everyone, Greek or foreigner.*

Plato. *Cratylus*



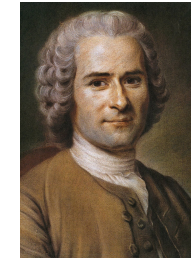
## PLATO ON THE ORIGINS OF LANGUAGE

*So Cratylus is right in saying that things have natural names, and that not everyone is a craftsman of names, but only someone who looks to the natural name of each thing and is able to put its form into letters and syllables.*

Plato. [Cratylus](#)



## ROUSSEAU ON THE ORIGINS OF LANGUAGE



*Whether there is a natural language, common to all mankind, has long been a matter of investigation. Without doubt there is such a language, and it is the one that children utter before they know how to talk.*

Rousseau. [Emile](#).

## PLATO'S PUZZLES

**Question:** How can we address Plato's puzzles?

**Idea:** Questions of justice and language deal with

- Multiple individuals who
- Engage in complex interactions with one another and
- Whose well-being depends upon the outcomes of everyone's actions, how well they coordinate their actions, and so on.

## AGENT-BASED MODELS

**Agent-based models** (ABMs) are a recent and powerful tool for studying complex social interactions.

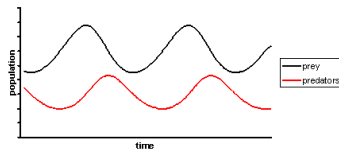
**Idea:** Perhaps we can employ ABMs to start attacking Plato's puzzles.

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Population models describe the aggregate properties of a group.

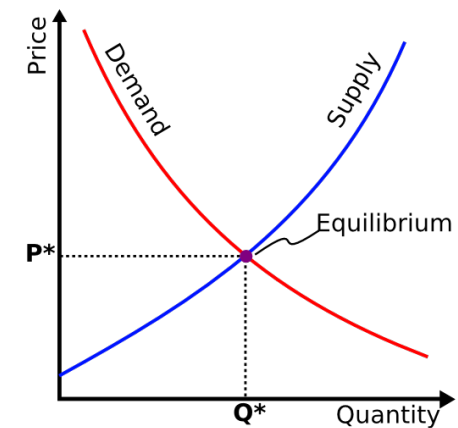
## LOTKA-VOLTERRA



**Example:** The Lotka-Volterra equations describe the change in the size  $P$  of the population of some predator and that of its prey  $p$ :

$$\begin{aligned}\frac{\partial P}{\partial t} &= p(g - aP) \\ \frac{\partial p}{\partial t} &= P(abp - dP)\end{aligned}$$

## SUPPLY AND DEMAND



**Example:** Supply and demand curves predict the price of a good.

## POPULATION MODELS

**Question:** Why are these population models?

- **Lotka-Volterra:** Equations do not describe particular animals. They only describe the total number of predator and prey.
- **Supply-Demand:** Curves do not describe individual consumers. They only describe aggregate demand and supply.

**Agent-based models** (ABMs) describe the interactions among many individuals in a large population.

## EXAMPLE ABM

## COMPONENTS OF ABMS

Agent based models (ABMs) have the following components:

- Agents with properties (e.g., location, preferences, beliefs)
- Environment (e.g. a terrain)
- Initial Conditions for agents and environment
- Rules specifying how agents interact with one another and the environment

## LIMITATIONS OF POPULATION MODELS

Why use an ABM rather than a population model?

- **Advantage 1:** By describing how agents causally interact, ABMs are often thought to be **explanatory** in a way that population models are not.
- **Advantage 2:** For mathematical reasons, population models are often used to describe **large** populations; ABMs can be used to describe small ones.

See Epstein and Axtell [1996], Alexander [2007], and Grimm and Railsback [2005] for further discussions of why ABMs are useful.

## APPLICATIONS?

**Question:** In what disciplines have ABMs been employed?

**Answer:** Almost all sciences ...

## ABMs ACROSS DISCIPLINES

- **ARCHAEOLOGY:** Growth and migration of the Anasazi tribe
  - Dean et al. [2000]
- **BIOLOGY:** Spatial structure and dominance in primates
  - Bryson et al. [2007]
- **ECONOMICS:** Chaotic behavior of stock prices in response to traders
  - Hommes [2001]

## ABMs ACROSS DISCIPLINES

In philosophy, ABMs have been employed to study

- Evolution of meaning
  - [Skyrms, 2010]
- Evolution of morality
  - [Alexander, 2007]
- Acquisition of norms
  - [Bicchieri, 2006], [Muldoon et al., 2014]
- Division of labor in science
  - [Weisberg and Muldoon, 2009], [Zollman, 2010].

And more ...

# AGENT-BASED MODELS AND GAME THEORY

The ABMS we'll build will employ some game theory.

- This is a common, but not universal feature of ABMS.

- This is a common, but not universal feature of ABMs.

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# Game Theory

# GAMES

	Rock	Paper	Scissors
Rock	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$
Paper	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$
Scissors	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$

A **game** has three parts:

- A set of players  $I$ .
- A set of actions  $A_i$  for each player  $i$ .
- A preference relation  $\preceq_i$  over outcomes of the game for each player  $i$ .
  - We'll define an outcome in a moment.

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# GAMES: RPS

	Rock	Paper	Scissors
Rock	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$
Paper	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$
Scissors	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$

## STRATEGY

Roughly, a **strategy** for a player is either (i) an action or (ii) a randomly chosen action.

Examples:

- “Rock” is a strategy in the game Rock, Paper, Scissors.
- Rolling a three-sided die to choose rock, paper, or scissors.

STRATEGY

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## MIXED STRATEGIES



### MIXED STRATEGY

A **mixed strategy** for player  $i \in I$  is a probability distribution over her actions  $A_i$ .

## MIXED STRATEGIES



### Notation:

- $\langle p_1, p_2, p_3 \rangle$  with represent the mixed strategy in which a player chooses rock with probability  $p_1$ , paper with probability  $p_2$ , and scissors with probability  $p_3$ .
- Example:  $\langle \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$  represents the mixed strategy in which a player chooses rock half the time, and paper and scissors both one-quarter of the time.

## PURE STRATEGIES



**Rock**

### PURE STRATEGY

If a strategy assigns probability one to some action, it is called a **pure strategy**.

E.g., "Rock" is a pure strategy in Rock, Paper, Scissors.

## BEST RESPONSE

### STRATEGIC PROFILE

A **strategic profile** is a list of the strategies employed by each player in a game.

### Examples:

- $\langle \text{Rock}, \text{Paper} \rangle$  represents a situation in which Player 1 chooses Rock, and Player 2 chooses Paper.
- $\langle \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle, \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle \rangle$  represents the situation in which both players choose rock, paper, and scissors with equal probability.

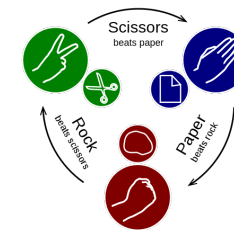
## GAMES

	Rock	Paper	Scissors
Rock	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$
Paper	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$
Scissors	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$

A game has three parts:

- A set of players  $I$ .
- A set of actions  $A_i$  for each player  $i$ .
- A preference relation  $\preceq_i$  over outcomes of the game for each player  $i$ .
  - Outcome = **strategic profile**.

## BEST RESPONSE



### BEST-RESPONSE

Let  $i \in I$  be any player and  $s_{-i}$  describe the strategies played by every player other than  $i$ . A **best-response** for player  $i$  is what it sounds like, i.e., it is any strategy  $s$  such that the strategic profile resulting from adding  $s$  to  $s_{-i}$  is the best according to the player's preference relation  $\preceq_i$ .

## BEST RESPONSE

	Rock	Paper	Scissors
Rock	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$
Paper	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$
Scissors	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$

Best Response to Rock/Paper/Scissors = Paper/Scissors/Rock

Best Response to  $\langle \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$  = Paper.

## NASH EQUILIBRIA

### NASH EQUILIBRIA

A strategic profile is called a **Nash equilibrium** if each player's strategy is the best response to the remaining players' strategies.

That is, if players are in a Nash equilibrium, then no one has any incentive to change his or her strategy: each player is doing the best given how other players are acting!



## NASH EQUILIBRIA

	Rock	Paper	Scissors
Rock	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$
Paper	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$	$\langle -1, 1 \rangle$
Scissors	$\langle -1, 1 \rangle$	$\langle 1, -1 \rangle$	$\langle 0, 0 \rangle$

**Nash equilibrium:** Both players pick rock, paper, and scissors with equal probability.

This equilibrium is **unique**.

## EQUILIBRIA IN GAME THEORY

*The basic concept of traditional game theory is the idea of a Nash equilibrium ... In traditional game theory, a Nash equilibrium is interpreted as ... a situation from which no rational player will unilaterally deviate. Thus, we can **predict** that rational players in a game will end up at a Nash equilibrium. [my emphasis]*

Okasha and Binmore [2012], pp. 4.

## PLATO ON THE ORIGINS OF LANGUAGE

Do real people (who are sometimes rather irrational) act in such a way that Nash equilibria are obtained?

- The research is mixed ...
- It's generally recognized that individuals do **not** act in **exactly** the way that is predicted by Nash equilibria, but in many games, they **approximate** the behavior [Kline, 2012].

## FISHER'S PRINCIPLE

- Equilibria explanations are also common in biology.
- E.g., The statistician R.A. Fisher asked, "Why are there equal numbers of males as females in the populations of many sexually reproductive animals?"
- Fisher's explanation, now called "Fisher's Principle", is one of the most celebrated pieces of reasoning in evolutionary biology.

## FISHER'S PRINCIPLE

Here's how Hamilton [1967] explains it.

- ① *Suppose male births are less common than female.*
- ② *A newborn male then has better mating prospects than a newborn female, and therefore can expect to have more offspring.*
- ③ *Therefore parents genetically disposed to produce males tend to have more than average numbers of grandchildren born to them.*
- ④ *Therefore the genes for male-producing tendencies spread, and male births become more common.*
- ⑤ *As the 1:1 sex ratio is approached, the advantage associated with producing males dies away.*
- ⑥ *The same reasoning holds if females are substituted for males throughout. Therefore 1:1 is the equilibrium ratio.*

Break for In-Class Exercises

Back to Plato's Puzzles

## PLATONIC EQUILIBRIA?

Can equilibria explanations help us to solve Plato's puzzles?

Let's take a closer look at the game described by Glaucon.

## THE PRISONER'S DILEMMA

*This, they say, is the origins and essence of justice. It is intermediate between the best and the worst. The best is to do injustice without paying the penalty; the worst is to suffer it without being able to take revenge. Justice is a mean between the two extremes.*

	Justice	Injustice
Justice	$\langle -1, -1 \rangle$	$\langle -3, 0 \rangle$
Injustice	$\langle 0, -3 \rangle$	$\langle -2, -2 \rangle$

## THE PRISONER'S DILEMMA

Games with the following payoff structure are called **prisoners' dilemmas**.

Here's the standard story:

	Silent	Confess
Silent	$\langle -1, -1 \rangle$	$\langle -3, 0 \rangle$
Confess	$\langle 0, -3 \rangle$	$\langle -2, -2 \rangle$

## THE PRISONER'S DILEMMA

What are the Nash equilibria of a Prisoner's dilemma?

	Silent	Confess
Silent	$\langle -1, -1 \rangle$	$\langle -3, 0 \rangle$
Confess	$\langle 0, -3 \rangle$	$\langle -2, -2 \rangle$

**Answer:** In the unique Nash equilibrium, both players confess.

## PREDICTIONS IN PRISONER'S DILEMMAS

- So, if Nash equilibria were used to make predictions when prisoner's dilemmas arise, we would expect little cooperation.
- But experiments show that people do cooperate in many prisoner's dilemma scenarios.
- So Glaucon's solution to Plato's first puzzle is deficient:
  - Although people do cooperate (act "justly") in one-shot prisoner's dilemma,
  - Doing so is strictly dominated, and hence, irrational.

## PLATONIC EQUILIBRIA?

What about Plato's second puzzle?

## PREDICTIONS IN SIGNALING GAMES

- In the last 45 years or so, philosophers, biologists, and economists have investigated games that represent communication or primitive language [Skyrms, 2010].
- Some equilibria in the game represent the case in which “signals” (think words for now) acquire meaning, e.g., where we reach agreement that “table” refers to tables.
- **Problem:** These games have **several** equilibria, not one.
  - So it's hard to use them for predictions ...

## THE STAG HUNT

The most common example of a game with multiple equilibria is called the **stag hunt**.

	Stag	Hare	
Stag	$\langle 2, 2 \rangle$	$\langle 0, 1 \rangle$	
Hare	$\langle 1, 0 \rangle$	$\langle 1, 1 \rangle$	

**Question:** What are the Nash equilibria of the stag hunt?

**Answer:**  $\langle \text{Stag}, \text{Stag} \rangle$  and  $\langle \text{Hare}, \text{Hare} \rangle$ .

## EQUILIBRIA IN SIGNALING GAMES

**Question:** Why do the games used to explain the evolution of meaning typically have multiple equilibria?

**Answer:** There's no reason why “table” rather than “Tisch” ought to be used to describe the object on which you serve food.

## GAME THEORY AND PLATO'S PUZZLES

So we've seen two problems for giving standard equilibria explanations to Plato's puzzles:

- **Justice**: Some times individuals don't play the Nash equilibria (e.g., Prisoner's dilemmas)
- **Meaning**: Some times the equilibria are not unique.

But there are other limitations of equilibria explanations as well . . .

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## LIMITATIONS OF EQUILIBRIA EXPLANATION

Equilibria explanations, like Fisher's, typically also do not discuss

- **How long** it takes to get to equilibria
  - E.g., How long before the sex ratio is equal?
  - E.g., How long will it take before two player start randomizing in RPS?
- The **path** to equilibria

Some question other underlying features of game-theory, and not just the pattern of equilibria explanations.

## LIMITATIONS OF GAME THEORY

Limitations of classical game theory:

- In economics: Agents (firms or consumers) are presumed to be **rational**.
- Agents (e.g., firms or organisms) are often assumed to be **homogeneous** (e.g., in quality of products, in reproductive ability).
- Agents (e.g., firms or organisms) interact **globally** rather than locally.

See Epstein and Axtell [1996], Alexander [2007], and Grimm and Railsback [2005] for other contrasts.

## LIMITATIONS OF ABMS?

**Question:** Are ABMs the answer to all of life's problems? Do they have any drawbacks?

**Answer:** Of course, they have drawbacks.

- Population models aim to answer certain types of questions; ABMs another.
- ABMs are typically more computationally intensive and harder to analyze mathematically than either population models or more classical game theoretic ones.

## GOALS OF THIS COURSE

**Goals:** By the end of the course, students should be able to

- 1 Compare and contrast ABMs and population-level models
- 2 Describe ABMs used to address Plato's puzzles.
- 3 Implement an ABM explaining the evolution of cooperation.

## STRUCTURE OF THE COURSE

How we're going to meet those goals:

	Lecture	Tutorial
1	Intro to ABMs; Some Game theory	Data types
2	ABMs of cultural evolution	Loops and procedures
3	Plato's Puzzles Revisited	Agent Commands
4	Group model building	

Click on the links below (current as of July 24th, 2014):

- [Download NetLogo](#)
- [Tutorials](#)
- [NetLogo Dictionary](#)

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