Introduction to Classical Decision and Game Theory

Models and Simulations in Philosophy
October 21st, 2013

Review

Last Week:

- Two Platonic Puzzles: Justice and Meaning
- ABMs vs. Equilibrium Explanations in Classical Economics and Mathematical Biology

We'll return to the first puzzle next week and to ABMs in two weeks.

Today's Class

Today:

1. Why game theory? - An elaboration on last week
2. Basic decision and game theory, which we'll use for the remainder of the semester.
3. More problems with "equilibrium explanations"

Outline

Definitions

You tell me! How are game and strategy profile defined?

Models and Explanations

Why game theory? In short:

- Scientific explanations often employ models.
- Many philosophers use games as parts of their models.

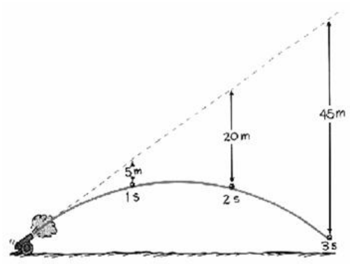
In greater detail ...

Models and Explanations

It will be helpful to compare the use of game theory to a simple example from kinematics.

Models and Explanations

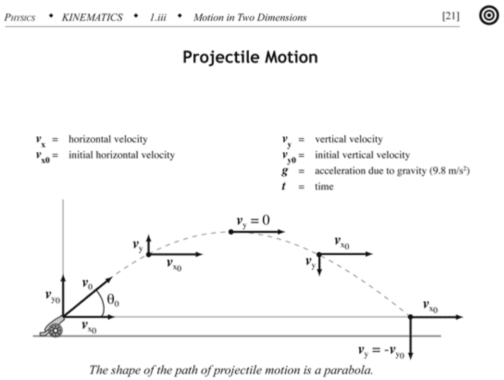
Question: why do projectiles, like cannonballs, follow parabolic-like paths?



How is this question typically answered?

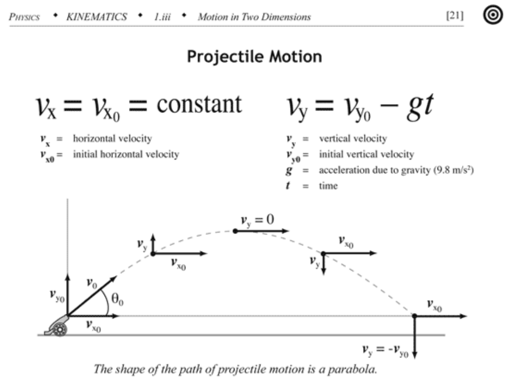
Models and Explanations

Step 1: Represent real objects by mathematical ones, like numbers or lists of numbers.



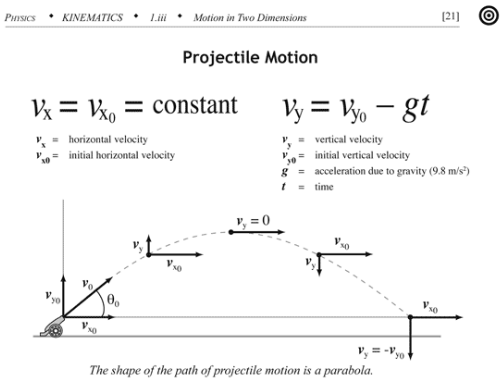
Models and Explanations

Step 2: Represent known relations among real objects by mathematical relations among the corresponding objects. Use equations when possible.



Models and Explanations

Step 3: Represent the phenomenon you would like to explain by some relation or equation.



Models and Explanations

Example: Let x_t and y_t be horizontal and vertical position of the ball at time t, which are represented by the anti-derivatives of $v_x(t)$ and $v_y(t)$ satisfying $x_0 = y_0 = 0$.

Then we are interested in explaining why for all times t:

$$y_t(x_t) = ax_t^2 + bx_t + c$$

for some constants a, b, and c.

Models and Explanations

Step 4: Derive (via proof), use simulations, or some probabilistic fact to show that the mathematical objects in your model satisfy the relation/equation that you wish to explain.

$$v_y(t) = v_{y_0} - gt$$
$$\Rightarrow y_t = v_{y_0} t - \frac{g}{2}t^2 + c$$

and

$$v_x(t) = b$$
$$\Rightarrow x_t = bt + d$$

Substitute $\frac{x_t - d}{b}$ for t in the y_t equation and you get the equation of a parabola.

Game Theory in Models

Philosophers develop models with games to explain human or animal behavior in a similar way:

- **Step 1:** Humans or animals are represented by players in the game.
- **Step 2:** The players' actions over time (i.e., how they interact/relate with/to one another) are determined by decision rules - more on this in a minute.

Game Theory in Models

- **Step 3:** The phenomenon (i.e. cooperation, language, etc.) is represented by a strategy profile in the game.
- **Step 4:** One proves or use simulations to show that players, employing the decision rules of Step 2, will converge on playing the strategy profile representing the phenomenon.

Class Participation

You tell me!

- What games did you read about?
- Which strategy profiles in those games might correspond to moral behavior, social norms, or any of the phenomena we discussed last class that was in need of explanation?

Class Participation

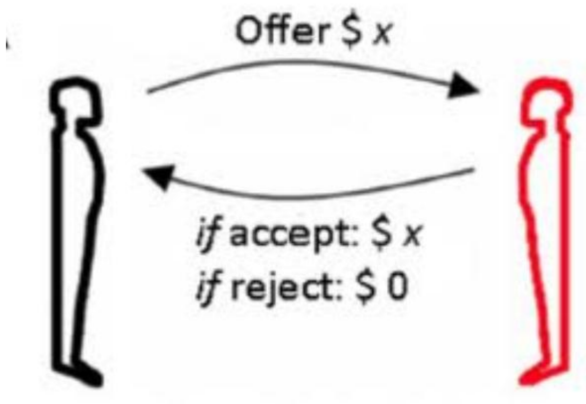
You tell me!

- What is a Nash equilibria?
- What are the Nash equilibria of the games?

Sequential Games

- In all of the games we discussed, players choose actions **simultaneously**.
- Here are two important sequential games in which players choose actions at different times.

Ultimatum Game



Ultimatum Game

The ultimatum game has two players.

- Player 1 is told she has a chance to earn some money, say $10. To earn the money, she must offer some amount of money to Player 2.
  - He can offer any non-negative amount, including $0, $3.5, $5, $10, and so on.
- Player 2 knows how much money Player 1 has to offer.
- Player 2 also knows Player 1's offer and can choose to accept or deny it.
  - If she accepts, then the players split the money in way Player 1 decided it.
  - If she denies it, then no one gets any money.

Ultimatum Game

Some Nash equilibria of this game are bizarre and motivate an alternative equilibrium type called subgame perfection.

We'll talk more about this later, but you can read chapter six of [Osborne, 2004] for more details.

Ultimatum Game

What strategy profile might be important in this game?

How do you think people behave when this game is conducted in laboratory settings?

Signaling Games

Example: Paul Revere's Ride

- Sender: Lighthouse attendant who observes if the British are coming by sea or land. He can show one lantern or two.
- Receiver: Paul Revere, who must ride to warn the colonists whether the British are coming by land or sea.
- The attendant and Revere both want to make the right warning, but
- They haven't agreed upon how many lanterns to use for each situation!

Signaling Games

Signaling games also have two players: sender and receiver.

- Sender observes some state of the world (e.g., by land or by sea).
- She then sends a signal to receiver (e.g., one lantern or two).
- The receiver then chooses an action (e.g. shout "by land" or shout "by sea" as you ride)
- The payoff that both receive depends upon the world and the receiver's action.

Signaling Games

Formally, in cooperative signaling games:

- There are finite sets of states of the world W, a finite number of signals S, and finitely many actions A.
- Sender's actions: A function from worlds W to signals S.
- Receiver's actions: Functions from signals to acts.
- The payoffs to sender and receiver are the same, and they are determined by the state of the world and the action taken by the receiver.

Equilibria

We'll talk a bit more about sequential games later in the course, what counts as an equilibrium, and what the equilibria are of these two games.

Why Nash Equilibria?

Let's return to the simultaneous games about which you read.

Why should one expect humans (or animals) to employ the strategies in the Nash equilibria of the strategic games?

- One standard justification for interest in Nash equilibria is that rational individuals will employ strategies in Nash equilibria.
- What does it mean to be rational?

Decision Matrices

	Sun	Rain
Read	2	3
Biergarten	4	-2
Watch "Glee"	-10	-10

Decision Matrices: Like game matrices, except one of the "players" is "Nature", which is responsible for the state of the world (generally, in columns).

Decision Matrices

	Sun	Rain
Read	2	3
Biergarten	4	-2
Watch "Glee"	-10	-10

Decision Matrices: Payoffs to the decision-maker depend upon the unknown state of nature and what action she chooses (in the rows).

Decision Rule

A decision rule is a method for choosing an action given a decision matrix and one's beliefs about likelihood of various states of the world.

Formally, it is function that

- takes as input (i) a decision matrix and (ii) a probability distribution over states of the world, and
- outputs an action from the decision matrix.

Dominance

	Sun	Rain
Read	2	3
Biergarten	4	-2
Watch "Glee"	-10	-10

Dominance: If the outcome of some action a_1 (e.g., Watch Glee) is worse than that of another a_2 (e.g., Read) regardless of the state of the world, do not choose a_1.

Worst-Case

	Sun	Rain
Read	2	3
Biergarten	4	-3

Worst-Case: Each action has a worst-case payoff. E.g., For Read, it's 2. For Biergarten, it's -3.

Minimax

	Sun	Rain
Read	2	3
Biergarten	4	-3

Minimax: Pick the action with the best worst-case payoff. Here, it's Read.

Decision Matrices

- But suppose you look outside, and it's a beautiful spring day in Munich.
- You read the weather forecast, which claims the chance of rain is .5%.
- Minimax ignores the probability of rain.
- We'd like some decision rule that simultaneously considers payoffs/losses and probability.

Decision Matrices

Suppose you fully believe the weather forecast, which claims the chance of rain is .5%.

	Sun	Rain
Read	2	3
Biergarten	4	-3

The expected utility of Biergarten is:

$$\begin{aligned} \text{SEU}(\textit{Biergarten}) &= p(\textit{Sun}) \cdot 4 + p(\textit{Rain}) \cdot -3 \\ &= 995 \cdot 4 + .005 \cdot -3 \\ &= 3.965 \end{aligned}$$

Decision Matrices

Suppose you fully believe the weather forecast, which claims the chance of rain is .5%.

	Sun	Rain
Read	2	3
Biergarten	4	-3

In contrast, expected utility of Read is:

$$\begin{aligned} \text{SEU}(\textit{Read}) &= p(\textit{Sun}) \cdot 2 + p(\textit{Rain}) \cdot 3 \\ &= 995 \cdot 2 + .005 \cdot 3 \\ &= 2.005 \end{aligned}$$

Three Decision Rules

- Maximize (subjective) expected utility (SEU)
- Dominance
- Minimax

Rationality and Expected Utility

- The Standard in Economics: An agent is rational if she acts **as if** she were maximizing expected utility.
- That is, the agent may not act **with the intent** of maximizing expected utility. She may happen to do maximize utility accidentally or unconsciously (due to practice and training, or genetic predisposition).
- There are a number of arguments for the claim that expected utility maximization is the **unique** rational decision rule; we won't discuss them here.

Three Decision Rules

Here are three simple observations about the relationship between these three rules ...

Three Decision Rules

Observation 1: Dominance and minimax are well-defined decision rules even if

- One does not assign states of the world probabilities; in fact, neither rule requires even the qualitative comparison of the likelihood of outcomes.
- One does not assign outcomes numerical payoffs; the decision rule makes sense even if payoffs are only qualitatively ordered.

Three Decision Rules

Observation 2:

Theorem

Suppose a is a dominant action. Then a is a minimax action and also maximizes subjective expected utility.

Dominance

Dominant actions maximize expected utility:

	Sun	Rain
Frisbee	5	-1
Biergarten	4	-2

Suppose one believes the probability of rain is p. Then:

$$\text{SEU}(\textit{Frisbee}) = (5 \cdot p) + (-1 \cdot (1-p))$$
$$\text{SEU}(\textit{Biergarten}) = (4 \cdot p) + (-4 \cdot (1-p))$$

Each term in the sum of Frisbee is bigger than the corresponding term for Biergarten

Three Decision Rules

Observation 3:

Theorem

Suppose a is **not** *a dominant action. Then there is some probability distribution under which a does not maximize expected utility.*

Decision Matrices

	Sun	Rain
Read	2	3
Biergarten	4	-3

Example: If you believe it will rain with probability one, then Read maximizes SEU.

If you believe the sun will shine with probability one, then Biergarten is better.

Outline

Dominance in Game Theory

	Cooperate	Defect
Cooperate	2,2	0,3
Defect	3,0	1,1

- In game theory, one player's strategies are his opponent's states of the world.
- So dominance says that, if the outcome of employing a_1 is better than that of a_2 for each possible strategy employed by one's opponent, then one should not choose a_2.

Dominance in One Shot Prisoners' Dilemmas

A classical prisoner's dilemma has the following structure:

	Cooperate	Defect
Cooperate	2,2	0,3
Defect	3,0	1,1

From the standpoint of row player, is there a dominant action? What about column player?

Dominance in One Shot Prisoners' Dilemmas

- In a Prisoner's Dilemma, the dominant action is to defect.
- By the first remark, it is also an SEU maximizing action.
- So according to the classical economic view, rational actors will defect in a prisoner's dilemma.
- By the second remark, rational actors will defect **regardless** of the
  - Numerical payoffs in the outcomes and
  - Likelihood that their opponent employs a particular strategy.

Group Work

Group Work: Analyze the remaining games we have discussed. Are there any dominated or dominant actions? Minimax actions? SEU maximizing actions (given your beliefs)?

Are the actions picked by these decision rules the ones employed in Nash equilibria?

Are the actions picked by these decision rules the ones employed in the strategy profiles we wished to explain?

Decision Theory and Nash Equilibria

Moral: In some games (the PD being an exception), rational (i.e. SEU maximizing) agents may not play those strategies that lead to Nash equilibria nor those in the strategy profiles we wish to explain.

We need to assume more about

- Individual' beliefs
- How individuals learn strategies over time
- With whom individuals interact over time

Summary

In sum:

1. Game theory will be useful in modeling human and animal behavior:
   - We wish to use some parts of game theory to explain why individuals, represented as players in game, end up in particular strategy profiles.
2. Nonetheless, we need to consider more than just what the equilibria of the game are for three reasons:
   - Experimentally: Humans play strategies out of equilibrium. I.e. we wish to explain non-equilibrium strategy profiles.
   - Multiple equilibria: only some of which are played.
   - Rational agents, in the absence of additional assumptions, may not play the equilibria.

References I

Osborne, M. J. (2004). *An introduction to game theory*. Oxford University Press New York.