Models and Simulations: Problem Set 3

Exercise 1: Reporters

1. Last week, you wrote a program that flipped a fair coin until three consecutive heads are observed. Turn that program into a reporter called three-heads that reports the total number of coin flips that occurred before three consecutive heads were observed.

2. Write a reporter called remove-first that removes the first occurrence of an element from a list with duplicates and reports the resulting list. For example, suppose mylist is [1 2 1 4 2 2]. Then when applied to mylist and 2, remove-first ought to report the list [1 1 4 2 2].

3. A positive whole number is called **prime** if it is greater than one and is divisible by only one and itself; it is called **composite** otherwise. Write a reporter called, is-prime? that takes a whole number n as input, and returns true if and only if n is a prime number. Otherwise, the program returns false.

   Hint: Last week, you learned to use the mod function, which computes a remainder in division. That is, 10 mod 4 is the remainder left when 10 is divided by 4, namely, 2. So 10 mod 2 is zero precisely because 10 is divisible by 2. In general, n mod m is zero if n is divisible by m. So n is prime precisely if there are no numbers m less than n such that n mod m is zero.

Exercise 2: Calling Reporters

1. Using three-heads, write a program that performs 1000 experiments of the following type. In each experiment, a fair coin is flipped until three consecutive heads are observed, and the number of flips is recorded. Using the 1000 experiments, calculate on average, how many times one needs to flip a fair coin before three consecutive heads are observed. Run your program several times. If your program works, the output should, on average, be around fourteen.

2. Using remove-first, write a program that reports the **numbers** of two cards randomly from a standard deck of cards. To make things easier, you may ignore the suit of the cards, and you can represent face cards by numbers in the following way: a "Jack" is represented by an 11, a "Queen" by a 12, a "King" by 13, and an "Ace" by 1.

   Hint: Given the above simplifications, a deck of cards can be represented by a list of length 52, namely, the list [1 1 1 1 2 2 2 2 . . . 13 13 13 13]. So to draw one card randomly from the deck is to pick an element randomly from this list. Be careful: to draw a second card at random, one should **not** pick another random element from the same list because one could pick the same card twice.

   Optional: Write a program that picks a five card poker-hand at random. To do so, represent a deck of cards by the list [[1 *C*] [1 *D*] [1 *H*] [1 *S*] . . . [13 *C*] [13 *D*] [13 *H*] [13 *S*]], where "C" represents "clubs", "D" represents "diamonds", and so on. Then use a loop and your remove-first reporter to pick five distinct cards.

3. Using the is-prime? reporter you wrote above, write a program that takes any whole number n as input (through the interface) and creates two lists. The first contains all prime numbers less than or equal to n, and the second contains all composite numbers less than or equal to n. Print both lists in the command centers. For example, if $n = 20$, then your program ought to print:
   Output:
   The primes less than or equal to 20 are [2 3 5 7 11 13 17 19].
   The composite numbers less than or equal to 20 are [1 4 6 8 9 10 12 14 15 16 18 20]