Mathematical Methods for Philosophy:
Problem Set 3

Assignment: This problem set is due November 5th, 2013 at the beginning of class. Complete the following exercises, as well as the following exercises from *Logic and Proofs*.

Logic and Proofs: Take the quiz at the end of Chapter 7.

Exercise 1 *Let n be a natural number. Show that if a formula φ contains n many different propositional variables $p_1, p_2, \ldots p_n$, then φ contains at least $n-1$ many occurrences of binary connectives. For example, the formula $p \vee (q \vee r)$ contains three different propositional variables and two binary connectives.*

Exercise 2 *Let Γ be any set of formula of propositional logic.*

- *A formula φ is said to be* ***independent*** *of Γ if $\Gamma \nvdash \varphi$ and $\Gamma \nvdash \neg\varphi$. Show that $p \to q$ is independent of $\Gamma = \{p \vee q, q \vee r, p\&s\}$.*
- *Suppose that $\Gamma \models \varphi\&\neg\varphi$ for some formula φ. Show that, for every well-formed formula ψ, there is a proof of ψ from Γ.*

Exercise 3 *In this exercise, you will fill in the steps of the proof of completeness theorem in the* Logic and Proofs *textbook. You should read the proof in the text carefully. Recall, a formula ψ is in conjunctive normal form if it is equal to a conjunction $\psi_1\&\psi_2\&\ldots\&\psi_n$, where each ψ_i is a disjunction of literals.*

1. *Suppose that $\psi \equiv \alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n$ is a disjunction of literals. Prove that if ψ is valid, then there is a propositional variable p and two disjuncts α_i and α_j such that α_i is p and α_j is $\neg p$.*
2. *Using structural induction, prove that for every valid propositional formula φ, there is a validity ψ in conjunctive normal form such that $\vdash \varphi \leftrightarrow \psi$.*
3. *Conclude that propositional logic is complete.*