Mathematical Methods for Philosophy:
Problem Set 1

Assignment: Complete the following exercises, as well as lab problems 3, 5-9, and 13 from chapter three of the *Logic and Proofs* textbook. This problem set is due October 22nd, 2013 at the beginning of class.

Exercise 1 *For each of the following English sentences, first choose propositional variables to represent the independent clauses. Then, using said propositional variables, write a well-formed formula in the language of sentential logic that represents the English sentence.*

- *Berlin is an exciting city, but Munich is the most beautiful city in Germany.*
- *It will rain tomorrow or on Thursday.*
- *Provided it does not rain, we will go hiking this weekend.*
- *Either Suzy or Rebecca won the competition.*
- *If Jane lost the competition, then Mark won.*
- *Jane loves brussel sprouts and ice cream.*
- *Jane loves gin and tonics.*

Example: Suppose the sentence is "Mary built the hard drive, and Jim built the monitor." Then let p represent "Mary built the hard drive", and q represent "Jim built the monitor." The above English sentence is represented by $p\&q$.

Exercise 2 *In class, we discussed three sentential connectives, namely, the conjunction ($\&$), the disjunction ($\vee$), the conditional ($\rightarrow$). We also wrote down truth-tables that defined the truth-conditions for these three connectives. Fill in the truth-tables for two additional connectives $\leftrightarrow$ and $|$, which are intended to represent "if and only if" and "neither ... nor" respectively. For example, let p represent "Mary built the hard drive", and q represent "Jim built the monitor." The $p \leftrightarrow q$ represents the sentence "Mary built the hard drive if and only if Jim built the monitor" and $p|q$ represents "It is neither the case that Mary built the hard drive nor that Jim built the monitor." Explain, perhaps by using example sentences, why you filled in each row of the truth-table in the way that you did.*

Table 1: Biconditional

p	q	$p \leftrightarrow q$
F	F	?
F	T	?
T	F	?
T	T	?

Table 2: Neither nor

p	q	$p \mid q$
F	F	?
F	T	?
T	F	?
T	T	?

Exercise 3 *Use a truth-tree to show that the following two formula are tautologies:*

$$((p \to q) \to p) \to p$$

$$\neg(p \to q) \to (p \& \neg q)$$

Exercise 4 *Represent the following argument in sentential logic. Use a truth-tree to show that is valid.*

- *Either the butler or the cook is the murderer.*
- *If the butler were the murderer, then his hands would be bloody.*
- *If the cook committed the murder, then a kitchen knife would be missing.*
- *The butler's hands are not bloody.*
- ***Conclusion:** A kitchen knife is missing.*