IMPLICIT AND EXPLICIT STANCES IN LOGIC

Johan van Benthem, http://staff.fnwi.uva.nl/j.vanbenthem

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Abstract We identify a pervasive contrast between `implicit' and `explicit' stances in logic design. To study new topics, implicit stances change meanings of logical constants and consequence, explicit stances extend classical logic with new vocabulary. We discuss the contrast in intuitionistic vs. epistemic logic, default reasoning, information dynamics, and, tentatively, in hyperintensional logic, discuss translations and merges between stances, and give concrete consequences of our perspective.

1 Explicit and implicit stances in logical analysis

The agenda of logic keeps growing: from ontology to cognitive activity. New topics keep generating new logics. *Explicit approach*: add new (modal) operators to fixed base logic. Information about the new topics: new laws in a richer language on a classical base. *Implicit approach*: reinterpret logical constants and/or consequence, information about the new topics in deviations to or even absence of classical logical laws. Our plan: we explain the contrast by example, and discuss what it means, taking both sides seriously.

2 Information, knowledge, and epistemic logic

'Semantic information': range of candidates for the actual situation. Agent *knows that* φ:

> $K\varphi$ true at a current world s if φ is true in all worlds in the current range of s,
> the 'epistemically accessible' worlds from s, given by a binary relation $s \sim t$.

Base language: propositional logic, modalities $K\varphi$. Laws: *S5* for each agent (or *S4*, *S4.2*), no substantial laws relating knowledge of different agents. Deeper axioms: group knowledge. Many debates about relation to knowledge: $K(\varphi \to \psi) \to (K\varphi \to K\psi)$, $K\varphi \to KK\varphi$ – but even more sophisticated modern alternatives (Dretske, Lewis, Nozick, Holliday) do not abandon the explicit style of analysis – the semantic truth conditions just get more complex.

> The explicit operator approach is quite typical for many areas of philosophical logic.

3 Intuitionistic logic

We do not add knowledge operators, but encode behavior of knowledge in failures of the laws of classical consequence: $\varphi \vee \neg\varphi$, or in modified laws that stay valid: $\neg\varphi \leftrightarrow \neg\neg\neg\varphi$. Motivation: proof theory – or partially ordered stages of inquiry, models $\boldsymbol{M} = (W, \leq, V)$:

> $\boldsymbol{M}, s \models \neg\varphi$ iff *for no* $t \geq s$, $\boldsymbol{M}, t \models \varphi$, likewise for the conditional as modal entailment

'Meaning loading': negation is impossibility over time, likewise for implication. Persistence: if $\boldsymbol{M}, s \models \varphi$ and $s \leq t$, then also $\boldsymbol{M}, t \models \varphi$. Greater variety of non-equivalent logical constants.

4 The explicit/implicit contrast: epistemic logic versus intuitionistic logic

Epistemic logic	explicit, conservative language extension of classical logic
Intuitionistic logic	implicit, meaning change old language, non-classical logic

Both logics have a distinguished track record. What should we make of this coexistence?

5 Connecting logical systems by translations

Gödel's *translation* from IL to modal *S4*: intuitionistic $\neg\varphi$ as modalized classical $\Box\neg\varphi$, etc.

Fact $IL \vdash \varphi$ iff $S4 \vdash \boldsymbol{t}(\varphi)$, for all propositional formulas φ.

This is a faithful interpretation of IL into S4, not just embedding. Strong `theory reduction'.

Standard wisdom: *S4* is a more general logic, including also non-persistent information, allowing not just accumulation of knowledge but also revision. But there is also a converse, less known, faithful embedding of S4 into IL (Fernandez, Goré & Thomson).

Discuss: What do these mutual faithful embeddings say?

What is equivalence of logical systems?

6 Dynamic logic of information change

Agency: inference, observation, social interaction: strategies, aligning preferences, etc. Simple pilot system for update. *Public events* $!\varphi$ of hard information, learning with total reliability that φ is the case eliminates all current worlds with φ false: from $\boldsymbol{M}$ to $\boldsymbol{M}|\varphi$. Public announcement logic *PAL* with modal operators for model change:

$$\boldsymbol{M}, s \models [!\varphi]\psi \quad \text{iff} \quad \text{if } \boldsymbol{M}, s \models \varphi, \text{ then } \boldsymbol{M}|\varphi, s \models \psi$$

Epistemic *S5*, obvious axioms for Boolean compounds after update, key recursion law:

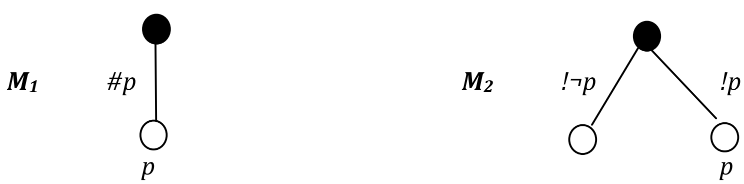
$$[!\varphi]K\psi \leftrightarrow (\varphi \rightarrow K(\varphi \rightarrow [!\varphi]\psi))$$

One 'dynamifies' a given static logic, making its underlying actions explicit and defining them as suitable model transformations. Dynamic logic: compositional analysis of post-conditions for the key actions via recursion laws. Conservative extensions of the base logic, though recursion laws sometimes force some redesign of the base language.

Many notions can be treated in this style: beliefs, inferences, issues, preferences.

[7] Implicit dynamics in intuitionistic logic

Dynamics remains hidden in intuitionistic models: `observation', 'awareness raising'.



Intuitionistic models register *two* notions of information on a par: factual information about how the world is, procedural information about our current investigative process. This changes the standard translation view of how intuitionistic and epistemic logic connect, since *S4* models now look like temporal 'protocol models' for epistemic *S5*-stages.

8 Dynamic semantics, meaning as information change potential

More prominent 'implicit' counterpart to dynamic logics of information like *PAL*: *dynamic semantics* of natural language. Meaning of an expression is its potential for changing information states (and whatever else is relevant) of someone who accepts the information conveyed. This takes informational action seriously in natural language – not with new operators, but by loading meanings of classical vocabulary with dynamic features.

Our case study: Update Semantics on information states as sets of atomic valuations, each propositional formula φ induces a *state transformation* $[[\varphi]]$ by the following recursion:

$[[p]](S) = S \cap [[p]]$, $[[\varphi \vee \psi]](S) = [[\varphi]](S) \cup [[\psi]](S)$, $[[\varphi \wedge \psi]](S) = [[\psi]][[\varphi]](S)$,
$[[\Diamond\varphi]](S) = S$, *if* $[[\varphi]](S) \neq \emptyset$, and $\emptyset$, *otherwise*

Dynamic consequence. After processing the premises, conclusion has no further effect:

$\varphi_1, \ldots, \varphi_n \models \psi$ iff for every information state X in any model, $\varphi_n(\ldots(\varphi_1(X))$ is a fixed point for $[[\psi]]$: this set stays the same under an update $[[\psi]]$

Behaves differently from classical consequence, deviations encode key features of update, such as its sensitivity to ordering of premises, or to numbers of repetitions of a premise.

Dynamic semantics: many notions of meaning, state change, dynamic consequence.

9 The contrast returns: dynamic semantics vs. dynamic logic of information

Dynamic semantics keeps actions implicit, gives the old language richer 'dynamic meanings' supporting new notion of consequence, with technical theory different from standard logic

Dynamic epistemic logic makes actions explicit, provides them with explicit recursion laws, extends the old base language with its old meanings, and works with standard consequence

10 Co-existence and translation for implicit and explicit dynamic logics

There are several simple translations, not yet well-known in the dynamics community.

Fact There is a faithful translation from update-validity into the modal logic *S5*.

There is a recursive map *tr* from formulas φ in update semantics to modal $tr(\varphi)(q)$, with q a fresh proposition letter, such, for all models $\boldsymbol{M}$ whose domain is denoted by q:

$[[\varphi]](S) = \{s \in S \mid \boldsymbol{M}, s \models tr(\varphi)(q)\}$

As a consequence, for dynamic update consequence, we have that

$\varphi_1, \ldots, \varphi_n \models_{US} \psi$ iff $\models_{S5} [tr(\varphi_1 \wedge \ldots \wedge \varphi_n)/q]tr(\psi) \leftrightarrow tr(tr(\varphi_1 \wedge \ldots \wedge \varphi_n))$

Fact There is also a faithful translation from *S5*-validity into update validity.

Discuss: Is *US* the same system as *S5*? Our translations recursively trace the workings of one framework inside another, step by step. Yet intensional differences remain. Likewise:

Fact There are faithful translations between *PAL*-validity and modal *S5*.

But, going via the static logic *S5* misses the dynamic character. Direct translation to actions:

$Tr(p) = !p$, $Tr(\neg\varphi) = !\neg\langle Tr(\varphi)\rangle T$, $Tr(\varphi \vee \psi) = Tr(\varphi) \cup Tr(\psi)$
$Tr(\Diamond\varphi) = !\Diamond\langle Tr(\varphi)\rangle T$, $Tr(\varphi \wedge \psi) = Tr(\varphi) ; Tr(\psi)$

Fact For models $\boldsymbol{M}$ whose domain is the set S, $[[\varphi]](S) = \{s \in S \mid \boldsymbol{M}, s \models \langle Tr(\varphi)\rangle T\}$.

Public announcements are closed under sequential composition, and thus, the effect of *US*-processing of φ is the same as that of publicly announcing just one associated *S5*-formula.

Question Is there a direct converse translation from *PAL* actions into *US* updates?

Benefits: Results about *PAL* as road signs for *US*: group knowledge, high complexity PAL^*.

Challenge: Do the translations still work after model change, say, on *PAL* protocol models?

11 Dynamic logics of soft information

Our discussion also applies to logics of belief. Epistemic-doxastic models for belief order epistemic ranges by `relative plausibility' $\leq xy$ between worlds x, y:

$\boldsymbol{M}, s \models B\varphi$ iff $\boldsymbol{M}, t \models \varphi$ for all $t \sim s$ maximal in the order $\leq$ on $\{u \mid u \sim s\}$

$\boldsymbol{M}, s \models B^{\psi}\varphi$ iff $\boldsymbol{M}, t \models \varphi$ for all $\leq$-maximal t in $\{u \mid s \sim u$ and $\boldsymbol{M}, u \models \psi\}$

Richer repertoire of epistemic notions includes 'safe belief' and 'strong belief'.

There is a complete explicit dynamic logic of belief change under hard information:

$$[!\varphi]B\psi \leftrightarrow (\varphi \rightarrow B^{\varphi}[!\varphi]\psi)$$

Other updates. Soft information changes plausibility order. E.g.,

Radical upgrade $\Uparrow\varphi$

from $\boldsymbol{M}$ to $\boldsymbol{M}\Uparrow\varphi$, all φ-worlds better than all $\neg\varphi$-worlds; within these zones, old order remains

Dynamic logic of $[\Uparrow\varphi]\psi$ and other soft modalities can again be axiomatized completely.

[12] Implicit intuitionistic versions of doxastic logic

Introducing belief in intuitionistic Kripke models as based on sets of current `conjectures' that generate plausibility order among future states (not pursued here).

13 Another tack: nonmonotonic consequence relations as implicit devices

Circumscription in AI: Conclusion only true in all most plausible models of premises. Structural rules: failure of monotonicity or cut – but retain cautious monotonicity, etc.

Explicit alternative. Problem solving can also be seen as belief formation.

Circumscriptive consequence $\varphi_1, \ldots, \varphi_n \Rightarrow \psi$ ~ explicit dynamic formula $[!\varphi_1]\ldots[!\varphi_n]B\psi$.

This translation explains the usual 'deviations' of non-monotonic logic from classical logic.

nonstandard consequence	old classical language, deviant rules of reasoning
explicit dynamic reanalysis	new language with belief and action modalities, consequence is just classical consequence

On the explicit approach, non-standard reasoning is a mixture of classical reasoning and further features of basic informational actions, not a family of radical alternatives.

14 Comparisons and translations: two-way traffic

Explicitize: Given a non-standard notion of consequence, one can tease out informational or other events motivating it intuitively, and write their explicit dynamic logic.

Vice versa, given a dynamic logic of informational events, one can package structure in new consequence relations, and study those per se. E.g., new kinds of circumscription:

soft-weak $[\Uparrow\varphi_1]\ldots[\Uparrow\varphi_n]B\psi$, soft-strong $[\Uparrow\varphi_1]\ldots[\Uparrow\varphi_n]K\psi$, and others

However, a more radical view of peaceful links: 'deep logical pluralism' is not needed!

15 Examples of explicitization keep appearing

From Quantum Logic (implicit, alternative logic) to explicit dynamic logic of measurement. E.g., famous failure of the distribution law simple fact of classical process or game algebra.

Fact Fine's truth maker semantics is faithfully translatable into classical modal logic of partial orders with explicit modalities $\langle\leq\rangle, \langle<\rangle, \langle\rangle, \langle sup\rangle, \langle inf\rangle$.

$$\boldsymbol{M}, s \models \langle sup\rangle\varphi\psi \text{ iff there exist } t, u \text{ with } s = sup(t, u) \text{ and } \boldsymbol{M}, t \models \varphi \text{ and } \boldsymbol{M}, u \models \psi$$

Varieties of truth making, convexity modally definable: $\varphi \to [up](\langle up\rangle\varphi \to \varphi)$. Translation:

$$(p)^+ = p^+, (p)^- = p^-, (\neg\varphi)^+ = (\varphi)^-, (\neg\varphi)^- = (\varphi)^+, (\varphi\wedge\psi)^+ = \langle sup\rangle(\varphi)^+(\psi)^+, (\varphi\wedge\psi)^- = (\varphi)^- \vee (\psi)^-$$

What to make of this? Explicit version just a richer theory of a metaphysical universe?

16 Two stances in logic design: implicit versus explicit

Natural stances. No definition, just examples and some recurrent features! Yet, once you acquire a sensibility to the contrast, concrete consequences are all around.

Finding a counterpart. The contrast is a force for new logic design. Implicit inquisitive semantics of questions suggests explicit dynamic logics of inquiry. Explicit logics of belief change suggest belief extensions of intuitionism, new types of nonmonotonic consequence.

But there are some unresolved challenges, such as proof-theoretic resource logics.

Sliding scale? There need not be a unique implicit or explicit approach to a phenomenon.

Borrowing ideas. Epistemic logic developed rich varieties of group knowledge. Intuitionistic logic can import these, reflecting mathematics as a social activity. Borrowing inside the same stance: counterpart to the BHK interpretation for dynamic semantics?

Translations? Could we define the explicit/implicit contrast technically, using mathematical translation between logics? I see no general definition out of my case studies. But is there a general mathematical method behind the translations presented here? Often, it seems to consist in defining semantic truth conditions in an appropriate sparse formalism.

Merging. Weaker test for a meaningful relationship: the existence of natural merges. Many joint implicit/explicit systems in the literature: sometimes contrived, sometimes natural.

Challenge. Is the contrast mostly semantic in origin, and not proof-theoretic?

17 Conclusion

We have drawn attention to a significant contrast running through modern logic, between (more) implicit and (more) explicit stances. Realizing this works leads to new research questions, and a more coherent picture of the field. Can we find a sharper definition? Perhaps, but more groundwork seems needed on the issue of translations and merges.

Can we fight it out philosophically: claiming that one stance, implicit or explicit, is better conceptually? This may be undesirable. Like any scientific discipline, logic is an evolving practice: the richer that practice is in terms of methodological options, the better.

Coda. Other ways of adjudicating in special cases of the contrast might be fit with empirical reality, such as natural language. But natural language encompasses both 'participating' (implicit) and 'commenting' (explicit) stances, reflecting its universality.

18 Reference

Johan van Benthem, 'Implicit and Explicit Stances in Logic', ILLC, University of Amsterdam.