Comments on Beddor's and Goldstein's "Believing Epistemic Contradictions"

Mike Raven (Victoria) / *2017 Formal Epistemology Workshop*

- Beddor & Goldstein (B&G) propose a semantics for belief reports.
  - It's novel for integrating a Lockean/Bayesian approach to belief within update semantics.
  - It solves a puzzle about *epistemic contradictions* ($\phi \wedge \Diamond\neg\phi$).
- I want to explore two issues concerning:

  §1 The link between uncertainty and believing possible.

  §2 Conjunction's noncommutativity in update semantics.

1 Linking Uncertainty and Believing Possible

- The puzzle relies on the principle:

  Fallibility It's sometimes coherent for an agent to believe ϕ and also believe $\Diamond\neg\phi$.

- B&G give three arguments for **Fallibility**, one relying on:

  Uncertainty-Possibility Link If an agent A is coherent, then if A isn't certain that ϕ, A is in a position to believe $\Diamond\neg\phi$.

- **Indeterminacy** poses two potential counterexamples to the **Link**:

<u>**Vagueness**</u> as indeterminacy prevents possible truth or falsity, so in particular B (also $\neg$B) cannot be true or false:

B Homer is bald.

☹ Marge believes this view of indeterminacy. She isn't certain of B but is *not* in a position to believe $\Diamond\neg$B.

<u>**Absolute undecidability**</u> prevents possible truth or falsity, so in particular CH cannot be true or false:

CH There is a bijection between every uncountable subset of real numbers and the real numbers themselves.

☹ Mary believes CH is absolutely undecidable. She isn't certain of CH but is *not* in a position to believe $\Diamond\neg$CH.

- *Is indeterminacy an obstacle to the uncertainty/believing-possible link?*

2 Preserving Commutativity

- An original motivation for update semantics was to allow order of assertions to matter (sometimes $s[\phi][\psi] \neq s[\psi][\phi]$).
  - In particular, the clause for conjunction updates *sequentially*:

    $s[\phi \wedge \psi] = s[\phi][\psi]$

  - So when $s[\phi][\psi] \neq s[\psi][\phi]$, conjunction is **noncommutative**:

    $s[\phi \wedge \psi] \neq s[\psi \wedge \phi]$

- *Even if B&G are correct that the empirical data for/against noncommutativity is inconclusive, is it acceptable* in principle?

2.1 A "minimal" semantics for conjunction

- I want to explore a two-step strategy to preserve commutativity:

  #1: Distinguish two ways of "*taking*" conjuncts ϕ,ψ:

  - **Separately**—apart from their conjunction (ϕ,ψ).
  - **Jointly**—as their conjunction $(\phi \wedge \psi)$.

  #2: Adapt Kleene semantics' idea that the semantic value of a conjunction is the *minimum* of its conjuncts:

<u>**Min-∧**</u> $s[\phi \wedge \psi] = min(s[\phi][\psi], s[\psi][\phi]) = s[\phi][\psi] \cap s[\psi][\phi]$

2.2 Order

- Order is irrelevant when updating by conjuncts *taken jointly* ($s[\phi \wedge \psi] = s[\psi \wedge \phi]$).
- Order is relevant when updating by conjuncts *taken separately* (sometimes $s[\phi][\psi] \neq s[\psi][\phi]$).

  E.g. Let context s have three worlds w, u, v where the house is empty in w and u but not in v:

  A The house is empty.

  Then: $s[A][\Diamond\neg A] = \varnothing$
  $s[\Diamond\neg A][A] = \{w,u\}$

- *Will this do justice to the original motivation to allow order to matter?*

2.3 Idempotence

- An assertion is *idempotent* if repeating it makes no difference:

  Idempotence ϕ is *idempotent* iff for any s, $s[\phi] \vDash \phi$

- B&G notice that reversed epistemic contradictions ($\Diamond\neg\phi \wedge \phi$) are *non-idempotent*:

  E.g. $s[\Diamond\neg A \wedge A] \not\vDash s[\Diamond\neg A \wedge A][\Diamond\neg A \wedge A]$

- **Min-∧** restores idempotence by not updating sequentially.

2.4 Invalidity

- Two main notions of validity in update semantics differing over whether or not the order of the premises matters:

  Unordered⊨ $\phi_1,\ldots,\phi_n \vDash_u \psi$ iff $s \vDash \psi$ for all s where $s \vDash \phi_1,\ldots,s \vDash \phi_n$.

  Ordered⊨ $\phi_1,\ldots,\phi_n \vDash_o \psi$ iff $s[\phi_1]\ldots[\phi_n] \vDash \psi$ for every s.

- **Min-∧** invalidates conjunction introduction for **Ordered⊨**:

  ∧-Intro $\phi_1,\phi_2 \vDash \phi_1 \wedge \phi_2$

Counterexample

$\phi_1 = \Diamond\neg A$
$\phi_2 = A$
$\psi = \Diamond\neg A \wedge A$

Then: $s[\phi_1][\phi_2] = \{w,u\}$
$s[\psi] = \varnothing$
So: $s[\phi_1][\phi_2] \not\vDash_o s[\psi]$

- *Maybe* (?) there is some independent motivation against **Ordered⊨**...

  D The house might not be empty. ($\Diamond\neg A$)
  The house is empty. (A)
  \# The house might not be empty but is empty. ($\Diamond\neg A \wedge A$)

  - Ari's second assertion (A) "supersedes" the first ($\Diamond\neg A$).
    - But why isn't that for Ari to *retract* the first assertion?

2.5 Closure

- B&G's version of *closure of rational belief under logical implication*:

  Multi-Premise Closure If (i) A is rational in believing premises $\phi_1,\ldots,\phi_n$; (ii) $\phi_1,\ldots,\phi_n \vDash \psi$; and (iii) A competently infers ψ from $\phi_1,\ldots,\phi_n$, then A's resulting belief in ψ is rational.

- B&G point out that their resolution to the puzzle provides counterexamples:

  E.g. $\phi_1 = A$
  $\phi_2 = \Diamond\neg A$
  $\psi = A \wedge \Diamond\neg A$

- It is possible to resist B&G's counterexamples:
  - (ii) requires the validity of **∧-Intro**.
  - But **∧-Intro** is invalid, assuming **Ordered⊨** and **Min-∧**.
- *Is closure worth preserving in this particular way (if at all)?*

Thanks to B&G for their excellent paper!