Bayesian Coherentism

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1 A Puzzle About Bayesian Updating

> Bayesian conditioning: You should condition your beliefs on certainties.
>
> Jeffrey conditioning: You should condition your beliefs on evidence that you hold with any degree of confidence.

An Inconsistent Triad:

1. If one norm is a special case of another, then they should have the same normative structure.
2. Bayesian conditioning is a special case of Jeffrey conditioning.
3. Bayesian conditioning and Jeffrey conditioning have different normative structures.

The Degeneracy Problem for Bayesians: The special case of Bayesian conditioning is *normatively* different from the more general case of Jeffrey conditioning.

2 Bayesianism

- Probabilism: Your credence function should be a probability function.
- Rigidity: Your belief revisions should preserve those conditional probabilities that are conditional on your evidence.

3 Two Kinds of Accounts of Evidence

- Substantive Accounts of Evidence: accounts of evidence defined of in terms of commitments Bayesians needn't accept.
- Formal Accounts of Evidence: accounts of evidence defined of in terms of commitments Bayesians must accept.

An Inconsistent Triad (Restated):

1. If one norm is a special case of another, then they should have the same normative structure.
2. Bayesian conditioning is a special case of Jeffrey conditioning.
3. Bayesian conditioning, though not Jeffrey conditioning, imposes a *foundationalist* constraint on evidence.

4 Bayesian Foundationalism

Bayesian conditioning: If the strongest evidence you get raises your credence in B to one, then your new degree of belief in A, for any A, should be $p'(\mathrm{A})=p(\mathrm{A}|\,\mathrm{B})$, where A and B are propositions.

Three Hallmarks of Bayesian Foundationalism:

1. Certainty: Our evidence is, by definition, a proposition of which we are certain.
2. Justification from Basic States: The values of all of our non-evidential beliefs are justified in virtue of the relation they bear to our evidence.
3. Evidence is Non-Inferentially Justified: Since our evidence cannot later receive the same sort of inferential support that it offers, its value isn't justified by its relations to other cognitive states.

5 Traditional Foundationalism vs. Bayesian Foundationalism

Traditional foundationalism: f_S: P $\rightarrow \{0, 1\}$

(1 if p $\in$ S, and 0 otherwise),

where P is the content of a belief and S is some basic state.

Bayesian foundationalism: f_E: UP $\rightarrow \{0, 1\}$

(1 if up $\in$ E, and 0 otherwise),

where UP is the value of an update and E is some piece of evidence.

6 Bayesian Foundationalism Undermined

Jeffrey conditioning: If experience directly changes your credences over a partition $\{\mathrm{B}_i\}$ from $p(\mathrm{B}_i)$ to $p'(\mathrm{B}_i)$, then your new degree of belief in A, for any A, should be $p'(\mathrm{A})=\sum_i p(\mathrm{A}|\,\mathrm{B}_i)p'(\mathrm{B}_i)$.

1. Certainty Undermined: If our evidence is uncertain, then our evidence is uncertain.
2. Evidence Can be Inferentially Justified: If our evidence is uncertain, its value *can* later receive the same sort of inferential support that it offers.
3. No Justification from Basic States: Bayesianism does not countenance an inferential relation between our experiences and our beliefs. So, experience isn't a basic state either.

7 Bayesian Coherentism

7.1 Commutativity as a Norm for Evidence:

Commutativity for Bayesians: Two updates commute under the Bayesian framework just in case reversing their order does not change the agent's final credence distribution.

A different way of thinking of the importance of commutativity: Instead of taking the non-commutativity of

Jeffrey conditioning to be an inessential bad-making feature of it, why not take the commutative property to *ground* updates?

That is: why not maintain that a set of updates is diachronically coherent to the extent that they commute.

7.2 From Commutativity to Coherentism:

- Probabilistic Coherence
- Logical Coherence
- Evidential Coherence

Traditional Evidential Coherence: A measure of the degree to which some proposition confirms each other belief in the set to which it belongs. It is a measure of the degree to which every proposition in a set is *evidence for* every other proposition in the set.

Bayesian Evidential Coherence: We can triangulate on an account of Bayesian evidential coherence from the descriptions of Bayesian foundationalism and traditional coherentism we already have:

- From **Bayesian foundationalism**, we borrow the idea that the members of this set are updates:

$$f_E\text{: UP} \rightarrow \{0, 1\}$$

- From **traditional coherentism**, we borrow the idea that this structure of justification entails a relation of consistency:

$$f\text{: } \{\text{ P}_1\text{ ,P}_2\text{....P}_n\} \rightarrow \mathbb{R}^+.$$

- The ideal for **Bayesian coherentism** is a consistent set of updates or, equivalently, it is the requirement that one's evidence be treated consistently:

$$f\text{: } \{\text{UP}_1\text{ ,UP}_2\text{,....UP}_n\} \rightarrow \mathbb{R}^+$$

This is just the requirement that updates commute.

8 Bayesian Foundationalism vs. Bayesian Coherentism

Bayesian foundationalism: A probabilisitic belief transition ought to be such that:

(a) There is a partition, $\{\text{E}_i\}$, that our belief transition is conditional on, and

(b) Some E_i is held with certainty.

Bayesian coherentism: A sequence of probabilisitic belief transitions ought to be such that:

(a) There are partitions, $\{\text{E}_i\}$, $\{\text{F}_j\}$, that each of our belief transitions is conditional on, and

(b) The updates on these partitions are minimally non-commutative.

9 The Final Piece of the Puzzle

- Updates on uncertain evidence are, in general, capable of commuting. But updates on uncertain evidence aren't capable of being updates on certain evidence.
- Since updates on uncertain evidence, qua updates on uncertain evidence, are capable of commuting, then where they fail to commute, we just have the failure of some state to conform to the evaluative norm in question.
- By contrast, since updates on uncertain evidence, qua updates on uncertain evidence, aren't capable of being updates on certain evidence, then where they fail to be updates on certain evidence, this is because there is no norm in place.
- Finally, since coherence comes in degrees, all updates can be held to conform to Bayesian coherentism to at least some degree.

A Consistent Triad:

1. If one norm is a special case of another, then they should have the same normative structure.
2. Bayesian conditioning is a special case of Jeffrey conditioning.
3. Both Bayesian conditioning and Jeffrey conditioning impose a coherentist constraint on evidence.