Avoiding Risk and Avoiding Evidence

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Gathering Evidence

Should you gather available evidence?

Ideal cases:

- Evidence is cost-free,
- Evidence is potentially relevant,
- You're certain you'll process it rationally.

Principles

Look-I

In ideal cases, one is *instrumentally* required to gather the evidence.

- Classical decision theory: ✓ Good (1967)
- Risk aware decision theories: ✗ Wakker (1988); Buchak (2010)

Look-E

In ideal cases, one is *epistemically* required to gather the evidence.

- Classical decision theory: ✓ Oddie (1997)
- Risk aware decision theories: **???** – ✗

Outline

Introduction

Risk awareness
  - Buchak's account
  - Look-I

Epistemic rationality

Accuracy

Avoiding evidence

Summary

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Usual Decision theory

Agents have beliefs c and utilities U

Maximise expected utility.

So any risk attitudes need to be accounted for in U.

But some intuitive preferences are ruled out.

Allais Preferences

	Ticket 1	Ticket 2-11	Ticket 12-100
$L^{\text{Safe}}_{\text{Good}}$	1	1	1
$L^{\text{Risky}}_{\text{Good}}$	0	2	1
$L^{\text{Safe}}_{\text{Bad}}$	1	1	0
$L^{\text{Risky}}_{\text{Bad}}$	0	2	0

$$L^{\text{Safe}}_{\text{Good}} \succ L^{\text{Risky}}_{\text{Good}}, \qquad L^{\text{Risky}}_{\text{Bad}} \succ L^{\text{Safe}}_{\text{Bad}}$$

Intuitive and predicted by Buchak's theory.

Buchak's risk aware decision theory

Agents have beliefs c, utilities U and a risk profile r.
r controls worst-case-scenario reasoning.

General definition

$$\mathrm{Exp}_c \ \ U(A) = U(o_{\text{worst}}) + \ \ c(s_{\text{best}}) \ \ \cdot (U(o_{\text{best}}) - U(o_{\text{worst}}))$$

- If $r(x) = x$ the agent is risk-neutral — get $\mathrm{Exp}_c U(A)$.
- If $r(x) < x$ she cares about worst case scenario more.
- If r is convex then she is risk avoidant. E.g. $r(x) = x^2$.

A risk avoidant agent can have utility vary linearly with money and still take €5 over $\{H : €10;\ Tails : €0\}$.

From now on we assume Buchak's risk-aware agents are rational.

Gathering Evidence

Look-I

In ideal cases, one is *instrumentally* required to gather the evidence.

You have to make a decision. You can either:

- AVOID: Make the decision now.
- GATHER: Gather some free evidence and then decide.

Good (1967): GATHER always has higher expected utility.

Buchak (2010): Sometimes AVOID has higher risk weighted expected utility.

Failures of Look-I

Allais preferences lead to Look-I failures.

	Ticket 1	Ticket 2-11	Ticket 12-100
$L^{\text{Safe}}_{\text{Good}}$	1	1	1
$L^{\text{Risky}}_{\text{Good}}$	0	2	1
$L^{\text{Safe}}_{\text{Bad}}$	1	1	0
$L^{\text{Risky}}_{\text{Bad}}$	0	2	0

$$L^{\text{Safe}}_{\text{Good}} \succ L^{\text{Risky}}_{\text{Good}}, \qquad L^{\text{Risky}}_{\text{Bad}} \succ L^{\text{Safe}}_{\text{Bad}}$$

Outline

Epistemic Rationality of Evidence Gathering

Look-E

In ideal cases, one is *epistemically* required to gather the evidence.

Buchak (2010) suggests Look-E might still hold for the risk-aware.

We'll show that's wrong.

Epistemic Rationality of Actions

One is epistemically required to pursue epistemic utility.

- Epistemic utility: the accuracy of the resultant credences

Focus on credence in a proposition of interest X.

- Pursue: maximise risk-weighted expectation.

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Accuracy

$$\mathrm{Acc} : \underset{\overset{\nearrow}{\text{Credence in } X}}{[0,1]} \times \underset{\overset{\nwarrow}{\text{Truth value of } X}}{\{0,1\}} \to \mathbb{R}.$$

We lay down constraints on Acc.

- Acc is truth directed, i.e.
  - If $x_1 < x_2 < 1$, then $\mathrm{Acc}(x_1, 1) \leqslant \mathrm{Acc}(x_2, 1)$,
  - If $x_1 > x_2 > 0$, then $\mathrm{Acc}(x_1, 0) \leqslant \mathrm{Acc}(x_2, 0)$.
- Acc is 0/1-symmetric, i.e. $\mathrm{Acc}(x, 0) = \mathrm{Acc}(1 - x, 1)$.
- Acc is continuous.
- and. . .

Immodesty

Joyce (2009)

A rational agent should be immodest,
i.e. should think her credences are best at pursuing accuracy.

All probability functions might be rational.

Risk neutral case: pursuit by expected utility.

Definition

Acc is strictly proper if for all probabilistic c, $\mathrm{Exp}_c\mathrm{Acc}(y)$ is uniquely maximised at $y = c(X)$.

So, using a strictly proper scoring rule all probabilistic, risk-neutral agents are immodest.

Risk and Immodesty

But we are assuming some rational risk-aware agents, who will evaluate the goodness of their credences using $\mathrm{RExp}^r_c \operatorname{Acc}(y)$.

Definition

Acc is strictly r-proper if for all probabilistic c, $\mathrm{RExp}^r_c \operatorname{Acc}(y)$ is uniquely maximised at $y = c(X)$.

Using a strictly r-proper scoring rule all probabilistic, r-risk-aware agents are immodest.

Accuracy measures have to be risk-profile-dependent.

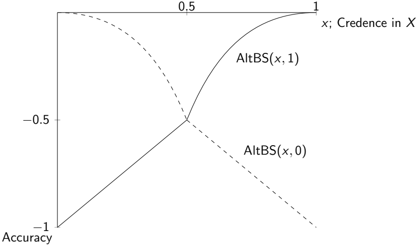
- Subjectivist,
- Objectivist,
- Permissivist.

r-proper measures

There are such strictly proper Acc; at least for r continuous.

For $r(x) = x^2$ one can use the measure:

$$\text{AltBS}(x, v) := \frac{-(v-x)^2}{\max\{x, 1-x\}}$$



General method by Levinstein.

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Look-E

Look-E

In ideal cases, one is *epistemically* required to gather the evidence.

Theorem (Oddie (1997))

If Acc *is strictly proper, then*
$\mathrm{Exp}_c\ \mathsf{Acc}(\textsc{Gather}) > \mathrm{Exp}_c\ \mathsf{Acc}(\textsc{Avoid})$

But for the risk aware?...

An example where one should avoid

Example

$c(X) = 0.7,\ c(X|E) = 0.8,\ c(X|\neg E) = 0.6, \quad r(x) = x^2$

	$E \wedge \neg X$	$\neg E \wedge \neg X$	$\neg E \wedge X$	$E \wedge X$
$c(\cdot)$	0.1	0.2	0.3	0.4
AVOID	$\mathsf{Acc}(0.7, 0)$	$\mathsf{Acc}(0.7, 0)$	$\mathsf{Acc}(0.7, 1)$	$\mathsf{Acc}(0.7, 1)$
GATHER	$\mathsf{Acc}(0.8, 0)$	$\mathsf{Acc}(0.6, 0)$	$\mathsf{Acc}(0.6, 1)$	$\mathsf{Acc}(0.8, 1)$

$$\mathrm{RExp}^r_c\ \mathsf{AltBS}(\textsc{Avoid}) \approx -0.42$$

$$\mathrm{RExp}^r_c\ \mathsf{AltBS}(\textsc{Gather}) \approx -0.48$$

General result

Choice of $r(x) = x^2$ and AltBS weren't important:

Theorem

Suppose r is differentiable and risk avoidant. Then there is c such that for all Acc *satisfying above:*

$$\mathrm{RExp}^r_c\,\mathrm{Acc}(\textsc{Gather}) < \mathrm{RExp}^r_c\,\mathrm{Acc}(\textsc{Avoid})$$

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Look-E

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Assuming:

From the perspective of epistemic rationality, risk-avoidant agents should sometimes avoid freely available and relevant evidence.

Assuming:

- Epistemically required to pursue epistemic utility.
- Risk-avoidant agents should maximise risk-weighted expected epistemic utility.
- Epistemic utility of an action is the accuracy of the anticipated resultant credence in the target proposition.
- Accuracy is strictly r-proper. weakened
- Rational agents expect to update by conditionalisation. weakened

Conclusion

Risk avoidance is in tension with evidence gathering.

We leave the upshot open.

Thanks!

References I

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James Joyce. Accuracy and coherence: Prospects for an alethic epistemology of partial beliefs. Springer, 2009.

Graham Oddie. Conditionalization, cogency, and cognitive value. *British Journal for the Philosophy of Science*, 48:533–541, 1997.

Peter Wakker. Nonexpected utility as aversion of information. *Journal of Behavioral Decision Making*, 1:169–175, 1988.

More generally

Risk weighted expectation

Suppose an act, A, leads to outcomes $o_1, \ldots, o_n$ in states $s_1, \ldots, s_n$ with $U(o_1) \leqslant \ldots \leqslant U(o_n)$. Then:

$$
\begin{aligned}
\mathrm{RExp}^r_c\, U(A) = U(o_1) & \\
& + r(c(s_2) + c(s_3) + \ldots + c(s_n)) \cdot (U(o_2) - U(o_1)) \\
& + \qquad\qquad r(c(s_3) + \ldots + c(s_n)) \cdot (U(o_3) - U(o_2)) \\
& + \ldots \\
& + \qquad\qquad\qquad\qquad\qquad r(c(s_n)) \cdot (U(o_n) - U(o_{n-1}))
\end{aligned}
$$

Back to presentation

Weak propriety

		Weak Look-E		Look-E	
		risk neutral	risk avoidant	risk neutral	risk avoidant
Subjectivist		yes	no	no	no
Objectivist		yes	probably no	no	no
Permissivist	Universal	yes	probably no	no	no
	Pareto	yes	no	yes	no

Table: Look Principles given only Weak Propriety

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Weakening Conditionalisation

Thought about the right way, even these risk weighted accuracy considerations can motivate conditionalization.
But we also only need:

- Continuity: f_E and $f_{\neg E}$ are continuous with respect to changes in the probability of the atoms.
- Certainty: If $c(E) = 1$ then $f_E(c) = c(X)$; and if $c(\neg E) = 1$ then $f_{\neg E}(c) = c(X)$.
- Responsiveness: $f_E(c) = c(X)$ only if $c(X) = c(X|E)$ (i.e. c makes X and E independent); and similarly for $f_{\neg E}$.
- Confidence: There is some t such that if $c(X)$, $c(X|E)$, and $c(X|\neg E)$ are all $> t$ then $f_E(c)$ and $f_{\neg E}(c)$ are both $> 1/2$.

Back to presentation

Proof idea

$$\mathrm{RExp}^r_c\mathsf{Acc}(\textsc{Gather}) = \mathrm{RExp}^r_c\mathsf{Acc}(.8 \text{ if } E,\ .6 \text{ if } \neg E)$$
$$\mathrm{RExp}^r_c\mathsf{Acc}(\textsc{Avoid}) = \mathrm{RExp}^r_c\mathsf{Acc}(.7) > \mathrm{RExp}^r_c\mathsf{Acc}(.8) \quad r\text{-prop}$$

$$\begin{aligned} & \mathrm{RExp}^r_c\mathsf{Acc}(.8 \text{ if } E,\ .6 \text{ if } \neg E) > \mathrm{RExp}^r_c\mathsf{Acc}(.8) \\ \text{iff} \quad & \mathsf{Acc}(.8, 0) + \frac{r(.7) - r(.4)}{r(.9) - r(.4)}(\mathsf{Acc}(.8, 1) - \mathsf{Acc}(.8, 0)) \\ & \quad > \mathsf{Acc}(.6, 0) + \frac{r(.7) - r(.4)}{r(.9) - r(.4)}(\mathsf{Acc}(.6, 1) - \mathsf{Acc}(.6, 0)) \end{aligned}$$

So it suffices to show that $\frac{r(.7)-r(.4)}{r(.9)-r(.4)} > r(.8)$. We will always be able to fiddle the numbers to find some instance of this.