# Avoiding Risk and Avoiding Evidence

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## *Introduction*

Ideal cases: Evidence is cost-free, potentially relevant, and you're certain you'll process it rationally.

**Look-I** In ideal cases, one is *instrumentally* required to gather the evidence.

- Classical decision theory: ✓ Good (1967)
- Risk aware decision theories: ✗ Wakker (1988), Buchak (2010)

**Look-E** In ideal cases, one is *epistemically* required to gather the evidence.

- Classical decision theory: ✓ Oddie (1997)
- Risk aware decision theories: ✗

## *Risk awareness*

Agents have beliefs $c$, utilities $U$ and a risk profile $r$.[1]

$$\mathrm{RExp}^r_c\, U(A) = U(o_{\text{worst}}) + r(c(s_{\text{best}})) \cdot (U(o_{\text{best}}) - U(o_{\text{worst}}))$$

If $r$ is convex then she is risk avoidant.

Allows for Allais preferences

| | Ticket 1 | Ticket 2-11 | Ticket 12-100 |
|---|---|---|---|
| $L^{\text{Safe}}_{\text{Good}}$ | 1 | 1 | 1 |
| $L^{\text{Risky}}_{\text{Good}}$ | 0 | 2 | 1 |
| $L^{\text{Safe}}_{\text{Bad}}$ | 1 | 1 | 0 |
| $L^{\text{Risky}}_{\text{Bad}}$ | 0 | 2 | 0 |

$$L^{\text{Safe}}_{\text{Good}} \succ L^{\text{Risky}}_{\text{Good}}, \quad L^{\text{Risky}}_{\text{Bad}} \succ L^{\text{Safe}}_{\text{Bad}}$$

Which show failure of Look-I.

[1] Generally: Suppose an act, $A$, leads to outcomes $o_1, \ldots, o_n$ in states $s_1, \ldots, s_n$ with $U(o_1) \leqslant \ldots \leqslant U(o_n)$. Then:

$$\begin{aligned}\mathrm{RExp}^r_c\, U(A) &= U(o_1) \\ &+ r(c(s_2) + \ldots + c(s_n)) \cdot (U(o_2) - U(o_1)) \\ &+ r(c(s_3) + \ldots + c(s_n)) \cdot (U(o_3) - U(o_2)) \\ &+ \ldots \\ &+ r(c(s_n)) \cdot (U(o_n) - U(o_{n-1}))\end{aligned}$$

## *Epistemic rationality*

One is epistemically required to pursue epistemic utility.

- Epistemic utility: the accuracy of the resultant credence in the target proposition.
- Pursue: maximise risk-weighted expectation.

## *Accuracy*

$$\mathsf{Acc} : [0,1] \times \{0,1\} \to \mathbb{R}.$$

- Acc is truth directed, 0/1-symmetric,[2] and continuous.
- Acc is strictly $r$-proper:

for all probabilistic $c$, $\text{RExp}_c^r \, \mathsf{Acc}(y)$ is uniquely maximised at $y = c(X)$.

[2] i.e. $\mathsf{Acc}(x, 0) = \mathsf{Acc}(1 - x, 1)$.

E.g. For $r(x) = x^2$ one can use the measure:

$$\mathsf{AltBS}(x, v) := \frac{-(v - x)^2}{\max\{x, 1 - x\}}$$

## *Avoiding evidence*

**Example** $c(X) = 0.7$, $c(X|E) = 0.8$, $c(X|\neg E) = 0.6$, $\quad r(x) = x^2$

| | $E \wedge \neg X$ | $\neg E \wedge \neg X$ | $\neg E \wedge X$ | $E \wedge X$ |
|---|---|---|---|---|
| Avoid | $\mathsf{AltBS}(0.7, 0)$ | $\mathsf{AltBS}(0.7, 0)$ | $\mathsf{AltBS}(0.7, 1)$ | $\mathsf{AltBS}(0.7, 1)$ |
| Gather | $\mathsf{AltBS}(0.8, 0)$ | $\mathsf{AltBS}(0.6, 0)$ | $\mathsf{AltBS}(0.6, 1)$ | $\mathsf{AltBS}(0.8, 1)$ |

$$\text{RExp}_c^r \, \mathsf{AltBS}(\text{Avoid}) \approx -0.42$$
$$\text{RExp}_c^r \, \mathsf{AltBS}(\text{Gather}) \approx -0.48$$

**Theorem 1.** *Suppose $r$ is differentiable and risk avoidant. Then there is $c$ probabilistic such that for all* Acc *satisfying above:*

$$\text{RExp}_c^r \, \mathsf{Acc}(\text{Gather}) < \text{RExp}_c^r \, \mathsf{Acc}(\text{Avoid})$$

## *Summary*

From the perspective of epistemic rationality, risk-avoidant agents should sometimes avoid freely available and relevant evidence.

Assuming:

- Epistemically required to pursue epistemic utility.
- Risk-avoidant agents should maximise risk-weighted expected epistemic utility.
- Epistemic utility of an action is the accuracy of the anticipated resultant credence in the target proposition.
- Accuracy is strictly r-proper. — weakened
- Rational agents expect to update by conditionalisation.— weakened[3]

[3] Result doesn't hold if one expects to update by following the plan that minimizes risk weighted expected accuracy. But we think the accuracy arguments won't get you this.