Reading Assignment 7: Probability and the Dutch Book Theorem

Assigned Reading

- Instructors notes on probability and set theory
- J. B. Kadane. *Principles of uncertainty*. Vol. 92. Chapman & Hall, 2011, p. 1-5.

Technical Requirements

Answer questions one, three, five, six, and eight below. Together, your answers should not be longer than a single typed page. Remember to provide page numbers indicating which passages you are paraphrasing. For the remaining optional questions, please write down the page numbers on which the authors address the question.

1. Pick two of the "assumptions" about probability that I discuss that are most interesting to you, and describe the assumptions in your own words.
2. Imagine five horses will compete in an upcoming race; the horses are called Seabiscuit, Kayak II , Whichcee, Wedding Call, and Heelfly. Match each of the following statements with exactly one of the three conditions in the definition of "field of sets." Explain what the sets Ω, E, $\Omega \setminus E$, etc. are in the conditions to which you match the three statements below.
   i. If a probability can be assigned to the event that Seabiscuit winning the race, then a probability can be assigned to the event that Seabiscuit does not win.
   ii. If a probability can be assigned to Whichee winning the race, and a probability can be assigned to the event that Wedding Call wins the race, then one can likewise assign a probability to the event that some horse who names begins with "W" will win.
   iii. At least one of the five horses will win the race.
3. State each of the three probability axioms in your own words, without the use of symbols.
4. Distinguish two ways in which the word "axiom" is used in philosophy and mathematics.
5. In Hajek's Stanford Encyclopedia article on "Pascal's Wager", Pascal is accused of making both of the"common misunderstandings" discussed in my notes. Briefly summarize which of Pascal's arguments contain which misunderstanding and why.
6. Give examples of two English sentences in which the word "probable" (or "probability") should be interpreted differently. Which of the two interpretations best describes the use of the word "probable" in each of the two sentences?
7. In the assigned reading, Kadane asks you, the reader, to imagine you are asked to bet on events in the future. When he asks you for a "price", what exactly is he asking you for?
8. What does it mean to be a "sure loser"?
9. Kadane explains via several examples why, if your prices do *not* satisfy the probability axioms, then you are a sure loser. Give your own example illustrating how you can be made a sure loser if your price on $A \cup B$ exceeds the sum of your prices on A and B. In other words, (1) specify what the events A and B are; (2) name some prices on the events A, B, and $A \cup B$ such that that price of the third exceeds the sum of the prices of the previous two, (3) specify what bets a person would need to make against you to ensure that you're a sure loser, and (4) specify how much money that person would make against you if A occurs, if B occurs, and if neither occurs

10. **Challenge:** The statement of the previous question is missing an assumption that Kadane makes about the relationship events A and B. What assumption is missing? Is that assumption necessary to show you can be made a sure loser if your price on $A \cup B$ exceeds the sum of your prices on A and B? Explain.