

# Reading Assignment 4: von Neumann and Morgenstern's theorem

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**Assigned Reading** Chapter 5 of M. Peterson. *An introduction to decision theory*. Cambridge University Press, 2017

## Technical Requirements

Answer questions four, five, eight, ten, and eleven below. Together, your answers should not be longer than a single typed page. Remember to provide page numbers indicating which passages you are paraphrasing. For the remaining optional questions, please write down the page numbers on which the authors address the question.

1. Peterson claims, "Arguably, utility cannot be directly revealed by introspection." Briefly explain Peterson's reasoning for the claim.
2. Suppose I am a market researcher who is trying to determine your preferences among a list of new albums. If I assume your preferences among the albums satisfy the principle called "Completeness", what am I assuming is the case? What would it mean to say that you find two albums "incomparable", and why is incomparability inconsistent with Completeness? Are there are two pieces of music that you find incomparable?
3. Suppose I am a market researcher who is trying to determine your preferences among a list of new albums. What does it mean to "construct an ordinal utility scale" with respect to your preferences among albums?
4. What type of utility scale do von Neumann and Morgenstern's axioms guarantee? What type of scale does Luce's choice axiom guarantee?
5. What is a lottery? Give an example of a lottery involving music albums that you enjoy.
6. Suppose there are three pieces of fruit in a platter in front of me: an apple, a banana, and a coconut. I prefer the apple to the banana, and I prefer the banana to the coconut. Further, assume I assign a utility equal to 1 to the apple and zero to the coconut. Finally, assume I am indifferent between a banana and a lottery that yields an apple with probability  $2/3$  and a coconut with probability  $1/3$ . If my preferences satisfy von Neumann and Morgenstern's axioms, what utility do I assign to the banana?
7. State von Neumann and Morgenstern's Independence axiom in your own words. Use an example involving your preferences among music albums to illustrate the axiom.
8. What is the "strongest objection" to the Independence axiom according to Peterson?
9. State the continuity axioms in your own words. Use an example involving your preferences among music albums to illustrate the axiom.
10. In no more than two sentences, summarize the objection to von Neumann and Morgenstern's argument that Peterson calls, "Utility without chance."
11. Does Luce's framework assume that individuals have preferences among lotteries? Briefly explain your reasoning.
12. Again, suppose there are three pieces of fruit in a platter in front of me: an apple, a banana, and a coconut. Suppose I assign a utility of 1 to the apple,  $1/2$  to the banana, and  $1/4$  to the coconut. I am offered a choice between the apple and the banana. What will I do if my utility function is interpreted in the way von Neumann and Morgenstern's theory implies? What will I do if utility is interpreted in the way Luce's theory implies?

13. Peterson asserts “Intuitively, it seems plausible to separate the meaning of the term utility from the problem of how to measure it”, and he then motivates that claim by use of an analogy involving mass. Devise your own (different) analogy to illustrate the point that Peterson is trying to make.
14. In the final section of the chapter, Peterson asks, “[W]hat is this core notion of utility, with which operational procedures should be compared?” Briefly summarize how he answers this question.