 Name ______________________ Student ID ______________

**Thermal Physics, Physics 224** **Midterm exam 2** **March 1, 2003**
**Winter 2003**
Instructor: David Cobden **4 pages (2 sheets printed on both sides)**

You have 50 minutes to answer two questions. **Move on to question 2 after 25 minutes!** Begin and end on the buzzer. Be sure to write your name and ID on every page. Answer all questions. Write all your working on these question sheets. Watch the blackboard for corrections during the exam. This is a closed book exam. You are allowed one sheet (two sides) of notes. You are allowed a simple calculator. *Throughout this exam, you will get credit for giving algebraic answers wherever possible as well as numerical answers.*

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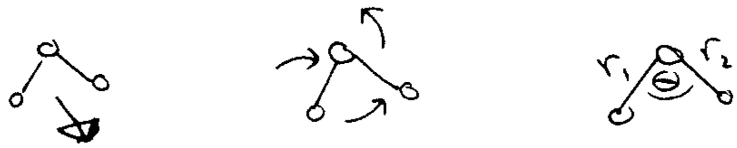
**Question 1**. [50 points total]

List and count the $q$ quadratic degrees of freedom of a free carbon dioxide ($CO_2$) molecule. [9]



$v_x \; v_y \; v_z$ translational kinetic; $\omega_1 \; \omega_2 \; \omega_3$ rotational kinetic; $r_1 \; r_2 \; \theta \; \dot{r}_1 \; \dot{r}_2 \; \dot{\theta}$ vibrations; $q = 12$

Say why at room temperature ($T_0$ = 300 K) only $q_{eff} = 6$ of these are thermally excited. [2]

The ones with high frequency, and thus large quantization energy, are "frozen out" at low temperatures

What is the internal energy of one mole of $CO_2$ at $T_0$ = 300 K? The gas constant $R$ = 8.3 J/mol/K. [3]

$$U = N_A \frac{q_{eff}}{2} kT = \frac{6}{2} RT$$
$$= 3 \times 8.3\ \text{J/mol/K} \times 300\ \text{K}$$
$$\doteq 7500\ \text{J} = 7.5\ \text{kJ}$$

What are its molar isochoric and isobaric specific heats, $c_v$ and $c_p$, at 300 K? [6]

$$c_v = \frac{q_{eff}}{2} R = 3 \times 8.3\ \text{J/mol/K} \doteq 25\ \text{J/mol/K}$$

$$c_p = \frac{q_{eff}+2}{2} R = 4 \times 8.3\ \text{J/mol/K} \doteq 33\ \text{J/mol/K}$$

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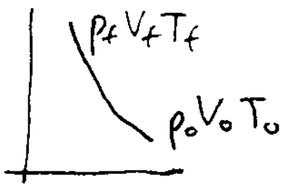
One mole of $CO_2$ has initial volume $V_0$ at pressure $p_0 = 1.0\times10^5$ Pa and temperature $T_0 = 300$ K. It is then compressed adiabatically and reversibly to half its initial volume. Given that an adiabatic can be characterised by $TV^{\gamma-1}$= constant, deduce the final temperature [6]

$$TV^{\gamma-1} = \text{const} \quad \therefore T_f V_f^{\gamma-1} = T_0 V_0^{\gamma-1} \qquad V_f = \frac{V_0}{2}$$




$$\therefore T_f = T_0\left(\frac{V_0}{V_f}\right)^{\gamma-1} = 2^{\gamma-1} T_0$$

$$= 2^{(4/3-1)} \times 300\text{K} = 378\text{ K}$$

What is the change in its internal energy, $\Delta U$, during the process? [6]

$$\Delta U = U_f - U_0 = \frac{g_{eff}}{2} R(T_f - T_0) = 3 \times 8.3\ \text{J/mol/K} \times (378-300)\text{K}$$

$$= 1.94\ \text{kJ}$$

What is the total work done by the gas, $W$, during the process? [3]

$$\Delta U = \underset{\uparrow\text{zero}}{Q} - W \qquad \therefore W = -\Delta U = -1.94\ \text{kJ}$$

What is the total change in entropy, $\Delta S$, of the gas during the process? [3]

$$\Delta S = \int \frac{dQ}{T} = 0 \quad \text{as no heat flows}$$

If the diameter of a $CO_2$ molecule is about 0.4 nm, estimate the mean free path at pressure $p_0$ and temperature $T_0$. [8]

$$L_m \doteq \frac{1}{4\pi\sqrt{2}\, r^2 (N/V)} \qquad p = \frac{N}{V}kT \quad \therefore \frac{N}{V} = \frac{p}{kT}$$

$$4r^2 = d^2 \quad d = 0.4\text{nm}$$

$$\therefore L_m \doteq \frac{1}{\pi\sqrt{2}\, d^2 \cdot \frac{p}{kT}} = \frac{kT}{\pi\sqrt{2}\, d^2 p} = \frac{1.38\times10^{-23}\ \text{J/K} \times 300\text{K}}{\pi \cdot \sqrt{2} \cdot (0.4\text{nm})^2 \times 10^5\ \text{Pa}}$$

$$\doteq 58\ \text{nm}$$

If the diffusion constant is $D = 2\times10^{-5}$ m$^2$/s, what is the typical time taken for a molecule of $CO_2$ to diffuse across the 1 cm gap between the two panes of a cavity window? [4]

$$L \doteq \sqrt{Dt} \qquad \underset{\text{characteristic time}}{t_{Diff}} = \frac{L^2}{D} = \frac{(0.01\text{m})^2}{2\times10^{-5}\text{m}^2\text{s}^{-1}} = \frac{10^{-4}\text{m}^2}{2\times10^{-5}\text{m}^2\text{s}^{-1}}$$

$$\doteq 5\ \text{sec}$$

**Question 2**. [50 points total]

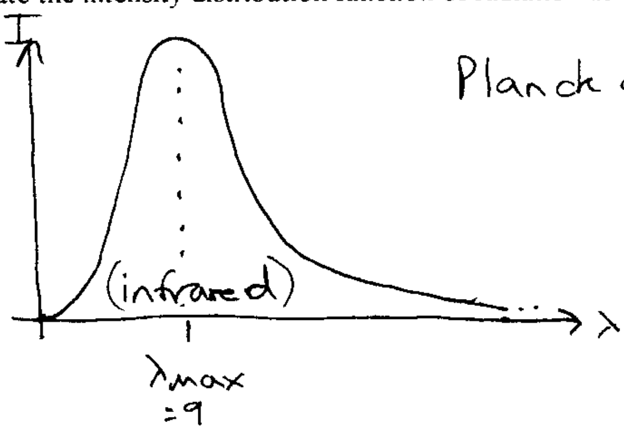
Estimate the characteristic wavelength of radiation emitted by living human skin in a dark room. [5]

Wien's : $\lambda_{max} = \frac{2.90\times10^{-3}\,m.K}{T_{skin}(K)} = \frac{2.90\times10^{-3}\,m.K}{310\,K}$

$T_{skin} \simeq 310\,K$

$\therefore \lambda_{max} \simeq 9\ \mu m$

A dark, thermally insulated room is full of (live) students having some kind of party. Sketch and annotate the intensity distribution function of radiation as a function of wavelength inside the room. [5]




Planck distribution

They get excited for some reason, and the temperature rises by 3° C. Be approximately what fraction does the total intensity of the radiation inside the room increase? [7]

$\Delta T = 3K$

$T = T_{skin} \simeq 310\,K$

Total intensity $\propto T^4$ (Stefan's law)

$\therefore \frac{\text{New power}}{\text{Old power}} = \frac{(T+\Delta T)^4}{T^4} = 1 + 4\frac{\Delta T}{T} \ldots\ldots$

$\therefore$ fractional increase $\simeq 4\frac{\Delta T}{T} = 4 \times \frac{3K}{310K} \simeq 4\%$

State the Clausius form of the Second Law of thermodynamics. [3]

Heat never flows from a colder to a hotter body spontaneously.

Also, state precisely the Second Law in terms of entropy. [3]

The total entropy of the universe never decreases.

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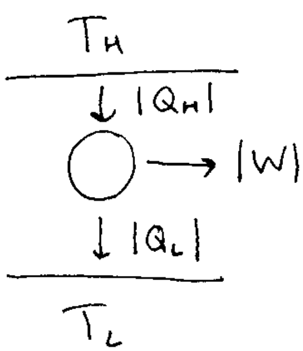
What limits does the Second Law put on the efficiency $e$ of any heat engine operating between reservoirs at temperatures $T_H$ and $T_L$? [3]

$$\frac{|W|}{|Q_H|} = e \leq 1 - \frac{T_L}{T_H}$$

A Carnot engine operates between a hot reservoir at $T_H$ = 500° C and a cold reservoir at $T_L$ = 100° C. If heat is deposited in the cold reservoir at a rate of 1000 kW, what is the rate at which the engine produces useful work? [12]




We need $\frac{|W|}{|Q_L|} = \frac{|Q_H| - |Q_L|}{|Q_L|}$

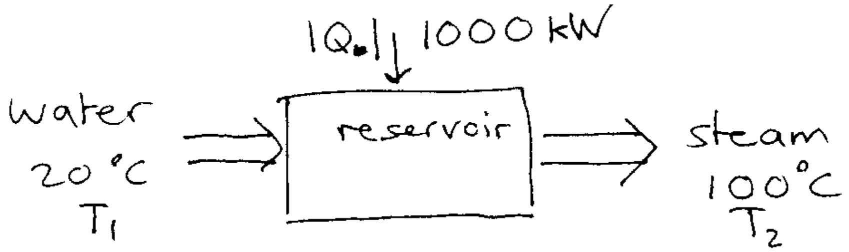
$$= \frac{|Q_H|}{|Q_L|} - 1 = \frac{T_H}{T_L} - 1$$

$\therefore$ rate of work $= \frac{|W|}{|Q_L|} \times$ (rate of heat into cold reservoir)

$$= \left(\frac{T_H}{T_L} - 1\right) \times 1000 \text{ kW}$$

$$= \left(\frac{773\text{K}}{373\text{K}} - 1\right) \times 1000 \text{ kW} \doteq 1070 \text{ kW}$$

To keep the cold reservoir at $T_L$ it is cooled by flowing water through it. The water enters as liquid at 20° C and leaves as steam at 100° C. How many liters per second of liqid water must be used? (Specific heat of water $c_p$ = 4.2 J/g/ °C; latent heat of vaporization $L_v$ = 2260 J/g). [12]




$\Delta T = 80°C$

In 1 second, water absorbs $10^6$ J $= |Q|$

$|Q| = c_p M \Delta T + L_v M$ M = mass of water in one second

$$\therefore M = \frac{|Q|}{c_p \Delta T + L_v}$$

$$= \frac{10^6 \text{ J}}{4.2 \text{ J/g/°C} \times 80°\text{C} + 2260 \text{ J/g}} = 385 \text{ g}$$

$\therefore$ mass flow = M per second = 0.39 kg/s