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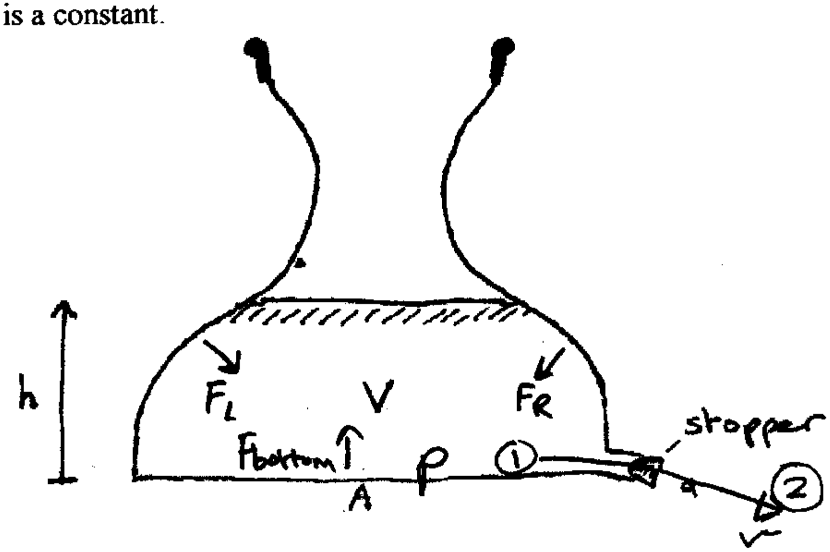
Thermal Physics, Physics 224 **Midterm exam 1** **February 1, 2002**
Winter 2002
Instructor David Cobden **5 pages**

50 minutes. Begin and end on the buzzer.
Answer all questions. Each is worth a total of 30 points.
Write all your working on these question sheets.
Please write your name on every page.

This is a closed book exam.
You are allowed one side of a page of notes.
You are allowed a calculator but it is not necessary.

Question 1. (This question is entirely algebraic.) The splendid shape of the flask shown below is such that when it contains a volume V of liquid, the level h of the liquid is given over a wide range by $h = \alpha V^2$, where α is a constant.



A volume V_0 of water (of density ρ) is poured into the flask at room temperature.
(a) What is the total weight W of the water? [1]

$W = \rho V_0 g$

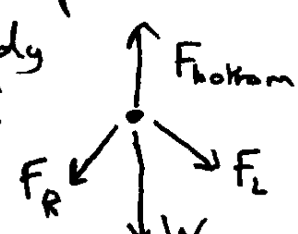
(b) What is the total force F_{bottom} exerted on the bottom of the flask (area A) by the water? [2]

Force due to gauge pressure $F_{bottom} = p.A = \rho g h.A = \rho g \alpha V_0^2 A$

(c) Explain why W and F_{bottom} different. In what type of container would they be the same? [4]

You have to allow for the vertical components of the force on the sidewalls.

free-body diagram for water



$F_{bottom} = W + \left(\begin{array}{c}\text{downward}\\ \text{force from}\\ \text{sides}\end{array}\right)$

This is not the case for a container with vertical sidewalls.

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The water is now warmed so that its density decreases by an amount $\delta\rho$.

(d) What is the change in F_{bottom}? [3]

Mass is conserved so $\rho V_0 = M = \text{const}$

$$F_{bottom} = \rho g \alpha V_0^2 A = g\alpha \frac{M^2}{\rho} A$$

$$\therefore \delta F_{bottom} = g\alpha M^2 A . \left(\frac{1}{\rho - \delta\rho} - \frac{1}{\rho}\right) = +g\alpha M^2 A \frac{\delta\rho}{\rho^2} = +g\alpha V_0^2 A\, \delta\rho$$

Explain how F_{bottom} can increase while the weight of the water stays the same. [2]

As ρ decreases, h increases and the vertical components of the sidewall forces increase.

The water is allowed to cool back to room temperature. The stopper is now removed so that the water can flow out freely through the spout at the bottom, whose cross-sectional area is a.

(e) For a volume V of water, at what speed is water ejected from the spout? [5]

Apply Bernoulli's equation between points ① and ②:

$$p_0 + \rho g h + \tfrac{1}{2}\rho . 0^2 = p_0 + \tfrac{1}{2}\rho v^2$$ (taking velocity to be zero at ①)

$$\therefore \rho g h = \tfrac{1}{2}\rho v^2 \qquad \therefore v^2 = 2gh = 2g\alpha V^2$$

$$\therefore v = \sqrt{2g\alpha}\, V$$

(f) What assumptions did you make in this calculation? [2]

(i) Velocity at ① = 0 $\rightarrow$ no turbulence inside

(ii) Zero viscosity $\rightarrow$ Bernoulli's equation is valid

(g) Show that the total time taken for half the water to run out is $\ln 2/\{a(2g\alpha)^{1/2}\}$. ~~[4]~~ [3]

By continuity equation, $\frac{dV}{dt} = av$ (flux out)

rate of decrease of amount of water in flask

flux out

$$\therefore \frac{dV}{dt} = a.\sqrt{2g\alpha}.V \qquad \therefore \int_{V_0}^{V_0/2} \frac{dV}{V} = a\sqrt{2g\alpha}\int_0^{t_{1/2}} dt$$

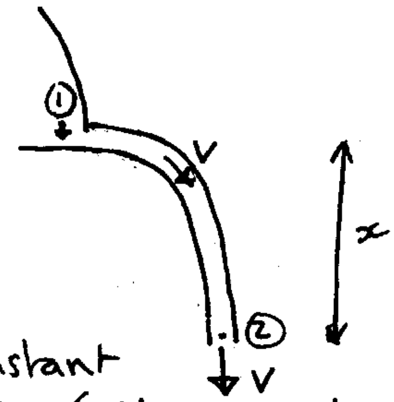
$$\therefore \left[\ln V\right]_{V_0}^{V_0/2} = a.\sqrt{2g\alpha}.t_{1/2} \quad \therefore t_{1/2} = \frac{\ln 2}{a.\sqrt{2g\alpha}}$$

The flask is refilled and a rubber tube is attached to the spout, with its end dangling at vertical distance x below the bottom of the flask.

(h) How fast does the water now emerge from the flask? [2]

As in (e) but now

$$p_0 + \rho g h + \rho g x + \tfrac{1}{2}\rho 0^2 = p_0 + \tfrac{1}{2}\rho v^2 \quad \therefore v = \sqrt{2g(h+x)}$$



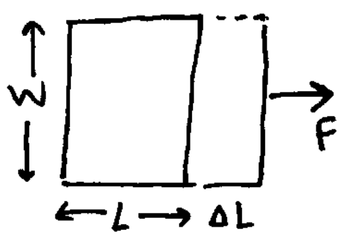
(i) Why is it faster than without the tube? [3]

Energy conservation:
Extra potential energy must be converted to kinetic energy.
Continuity equation → v along tube is constant
Falling water in tube is "sucking" water out of the vessel.

Question 2.
Part 1.

(a) Show that an energy density associated with a free liquid surface leads to a force per unit length exerted by the surface (ie, a surface tension). [4] γ = energy density (J/m²)

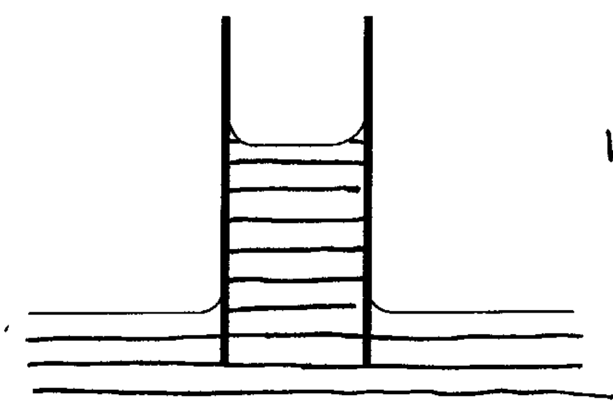
Consider stretching a rectangular piece of surface.



The force F needed along side of length W is given by

$$\underbrace{F\Delta L}_{\text{work done}} = \underbrace{\Delta U}_{\text{change in surface energy}} = \gamma . W \Delta L \quad \therefore \frac{F}{W} = \gamma \quad (N/m)$$

A cylindrical tube of diameter $d = 0.1$ mm is placed vertically with one end in open water:



Contours are horizontal and equally spaced for constant ρ.

(b) Sketch the contours of pressure throughout the water. [3]

$$\frac{dp}{dz} = -\rho g \quad \text{ALWAYS!}$$

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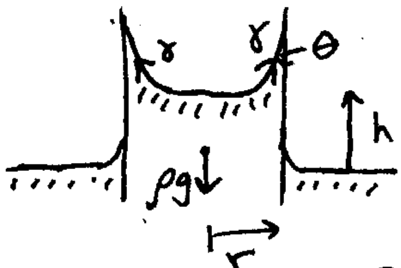
(c) Show that the maximum height to which the water could be raised up the tube by capillary action is 0.3 m. (The surface tension of water at room temperature is γ = 0.073 N/m.) [5]



In equilibrium, F_{st} = weight of column = $\pi r^2 \rho g h$
F_{st} = upward force on water column
$= 2\pi r . \gamma . \cos\theta$ ← contact angle
Maximum for $\cos\theta = 1$, $\theta = 0 \rightarrow F_{st} = 2\pi r \gamma$
$\therefore 2\pi r\gamma = \pi r^2 \rho g h \quad \therefore h = \frac{2\gamma}{\rho g r} = \frac{2 \times 0.073\ N/m}{10^3 kg m^{-3} \times 9.8 m s^{-2} \times 5\times10^{-5} m} = 0.3m$

(d) Discuss whether the pressure in the water column could ever be negative, and if so under what circumstances. [2]

If r is small enough, h can become greater than $\frac{p_0}{\rho g}$ and the pressure can be negative if the water column does not break (by cavitation)

Part 2. A piece of ice is floating in a beaker of water held at 1 degree C.
(e) After the ice melts, is the water higher, lower, or the same as before? Explain. [2] The same

Archimedes: a floating body displaces a mass of water equal to its own mass. It doesn't matter what form that body is in - ie if it changes from ice to water, it still displaces the same amount of water.

(f) Would the answer be different if the ice had air bubbles in it? Explain. [2]

No, by the reasoning above.

(g) What would the answer be if the ice had sand in it? Explain. (Hint: think of the problem of throwing a stone out of a boat in a swimming pool). [4]

Now the level decreases. The sand ends up on the bottom, where it displaces only a volume of water equal to its volume, which is less water than when it floats because the sand is denser than water.

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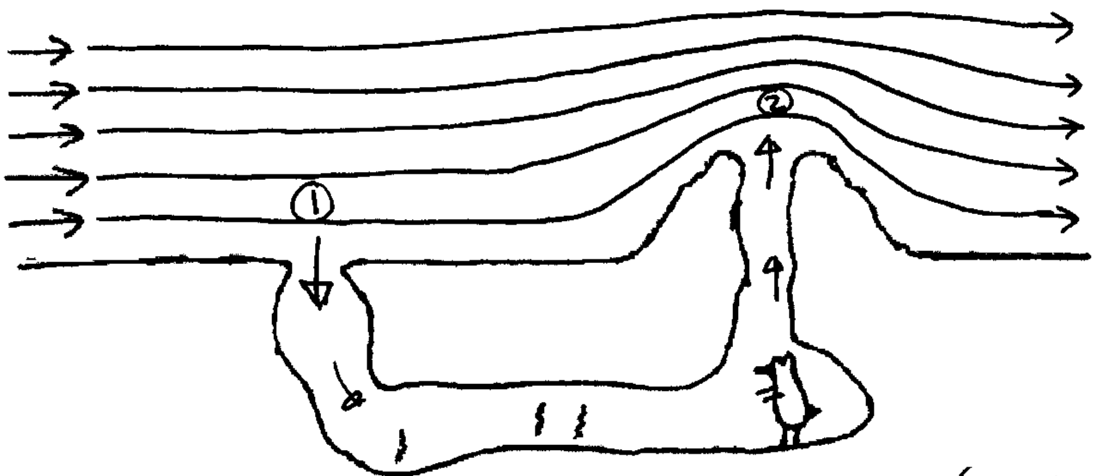
Part 3.

(h) A clever prairie dog knows how to get rid of unpleasant odors in his burrow. He makes one entrance raised above the ground and the other one flush, as indicated below. Sketch streamlines for a steady wind blowing across the landscape, and explain why the burrow remains well ventilated by a steady draft. [5]



(continuity eqn)
Denser streamlines at 2 than at 1 ⇒ greater speed at 2 ⇒ lower pressure at 2 by applying Bernoulli along a streamline from 1 to 2.
∴ Pressure difference drives flow from 1 to 2 along tunnel.

(i) If the diameter of the tunnel is 20 cm, and the average air speed is 1 m/s, will his ventilation system be turbulent? (Take the density of air to be 1 kg/m^3 and the viscosity to be 1.5×10^{-5} Ns/m^2, and use the Reynolds number of 2000 for flow along a pipe.) [3]

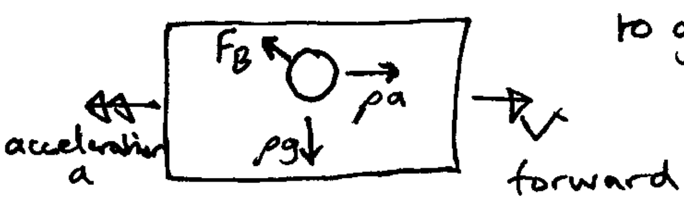
Critical velocity $V_c = \frac{R.\eta}{\rho D} = \frac{2000.1.5.10^{-5}\,Ns/m^2}{1\,kgm^{-3}\times 0.2\,m} = 0.15\,ms^{-1}$

Since $V_c \ll 1$ m/s, flow is turbulent.

Part 4 - (4 bonus marks – don't answer if you don't have time)

(j) A partygoer takes a helium balloon onto a train and lets it float against the ceiling. When the train brakes suddenly as it approaches a hazard, does the balloon float towards the front or the rear of the carriage? Explain your answer.

It moves to the back. Acceleration is equivalent to gravity, pulling everything towards the front of the train.



Buoyancy force F_B opposes both real gravity and this acceleration.

(Think of the air sloshing towards the front)