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**Thermal Physics, Physics 224** **Final Exam** **Thursday March 21, 2002**
Winter 2002
Instructor David Cobden
7 printed sides

Time allowed: 1hr and 50 minutes = 110 minutes. Begin and end on the buzzer.
Attempt all four questions. Remember that each has an equal weight in the final reckoning.
Be careful not to spend more than 20-25 minutes on each question initially – you can come back to the more difficult parts at the end. Also, make sure you take into account how many marks a part is worth and don't spend a disproportionate amount of time on it.

Write all your working on the question sheets.
Please write your name on each page.
This is a closed book exam.
You are allowed three pages of notes.
You are allowed a calculator but it is not essential.

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**Question 1** [33] – Pressure in the atmosphere, adiabaticity, heat flow, Bernoulli

(a) [9] Estimate the pressure $p_{top}$, at the top of a mountain ridge $h$ = 2000 m above sea level, assuming that the atmosphere (on average) has a constant temperature of $T$ = 7 C.

2 $\frac{dp}{dy} = -\rho g \quad p = \frac{\rho}{m} RT$ 1 $\quad \therefore \rho = \alpha p$ where $\alpha = \frac{m}{RT}$

$\therefore \frac{dp}{dy} = -\alpha p g \quad \therefore \int_{p_0}^{p} \frac{dp'}{p'} = -\alpha g \int_0^h dy \quad \therefore p = p_0 e^{-\alpha g h} = p_0 e^{-\frac{\rho_0}{p_0} g h}$ 3

$\rho_0 = 1.2\ \mathrm{kg\,m^{-3}}$

-1 if too accurate! $\quad \therefore p_{top} = (1\ \mathrm{atm}) \times e^{-\left(\frac{1.2\,\mathrm{kg\,m^{-3}}}{10^5\,\mathrm{N m^{-2}}} \times 10\,\mathrm{m s^{-2}} \times 2.10^3\,\mathrm{m}\right)}$

$= e^{-0.24}\ \mathrm{atm} = 0.79\ \mathrm{atm}$ 3

(b) [9] A wind blows up the mountain from the west, and just after passing over the ridge it is at $T_{top}$ = 2° C, which is somewhat colder and denser than the rest of the atmosphere at that height. As a result, it then sinks down into the valley to the east, whose floor is close to sea level. Assuming the air is adiabatically compressed during this process, so that $pV^{\gamma}$ = const, deduce the relation between $T$ and $p$ as it sinks, and hence estimate the temperature $T_{valley}$ reached at the valley floor.

$c_v = \frac{5}{2} \quad c_p = \frac{7}{2} \quad \gamma = \frac{c_p}{c_v}$

$pV^{\gamma} = \text{const} = A \qquad pV = nRT \qquad \gamma = \frac{7}{5}$

$\therefore p\left(\frac{nRT}{p}\right)^{\gamma} = A$ 2 2

3 $\therefore p^{1-\gamma} T^{\gamma} = \frac{A}{(nR)^{\gamma}} = \text{const}. \qquad \therefore T = \frac{\text{const}}{p^{(1-\gamma)/\gamma}}$

$\therefore \frac{T_{valley}}{T_{top}} = \left(\frac{p_{valley}}{p_{top}}\right)^{1-\frac{1}{\gamma}} \qquad \therefore T_{valley} = 275\,\mathrm{K} \times \left(\frac{1}{e^{-0.24}}\right)^{1-\frac{5}{7}} = 295\,\mathrm{K} = 22°\mathrm{C}$ 2

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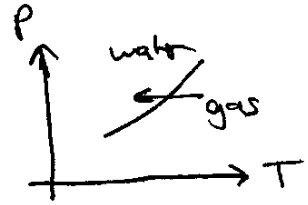
(c) [5] One reason the wind coming down on the east is warmer than the wind going up on the west at the same height is because on the way up the water vapour condenses and precipitates out. Why does it condense, and why does this make the air warmer than it would be otherwise?




1 The air expands adiabatically so it cools.

2 → the water gas turns to liquid

2 → this releases latent heat of water boiling

(d) [5] Another reason is that the flow is not completely adiabatic. List the factors that determine and influence the flow of heat into the rising air.

Heat transfer mechanisms: $\propto T^4$ $\propto$ temp difference

3 Radiation – from ground and surrounding air & sky

1 Conduction – " (probably unimportant).

2 Convection & turbulence within the airstream.

– (much more important than conduction in fluids)

1 viscosity

(e) [5] Why does the wind sink down into the valley at a roughly constant velocity, rather than speeding up rapidly as it flows downhill? (hint: consider Bernoulli's equation)

2 $p + \frac{1}{2}\rho v^2 + \rho g y = \text{const} = A$, indep of height, along a streamline 1

In the surrounding static air, $p + \rho g y = \text{const} = B$

2 $\therefore \frac{1}{2}\rho v^2 = A - B = \text{const}$

$\therefore v = \text{const}$

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**Question 2** [33] – Heat capacities, First Law

(a) The internal energy of an ideal gas is given by $E_{int} = nqRT/2$. Explain what $q$ is in this equation. [3]

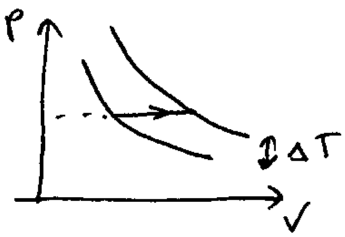
$q$ = number of accessible degrees of freedom per molecule in the gas. 1 2

Calculate the heat $Q_v$ absorbed by $n$ moles of the gas when its temperature is increased by $\Delta T$ isovolumetrically. [4]


1 $\Delta E_{int} = Q_v + W \qquad W = 0$ 1
$= Q_v$

$\therefore Q_v = \frac{nqR}{2}\Delta T = \frac{q}{2} nR\Delta T$ 2

Now calculate the heat $Q_p$ absorbed if the same temperature is instead increased isobarically. [4]




$\Delta E_{int} = Q_p + W = Q_p + p\Delta V \qquad pV = nRT$
$= Q_p + nR\Delta T$ 2

$\therefore Q_p = \frac{nqR}{2}\Delta T + nR\Delta T = \left(\frac{q}{2}+1\right) nR\Delta T$ 2

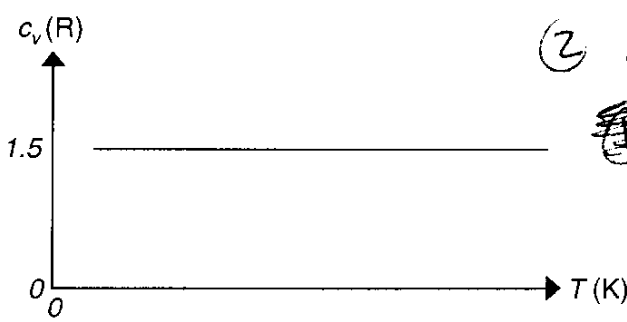
Hence show that $c_p = c_v + R$. [4] These are molar specific heat capacities.

3
$C_p = \frac{Q_p}{n\Delta T}$
$C_v = \frac{Q_v}{n\Delta T}$

$\therefore C_p - C_v = \left(\frac{q}{2}+1\right)R - \frac{q}{2}R = R$ 1

Shown below are sketches of the temperature dependence of the molar heat capacity at constant volume for four different common substances. In each case, deduce the nature of the substance (eg, 'margarine' or 'diatomic gas'), and explain each of the features that is seen.

(a) [3]

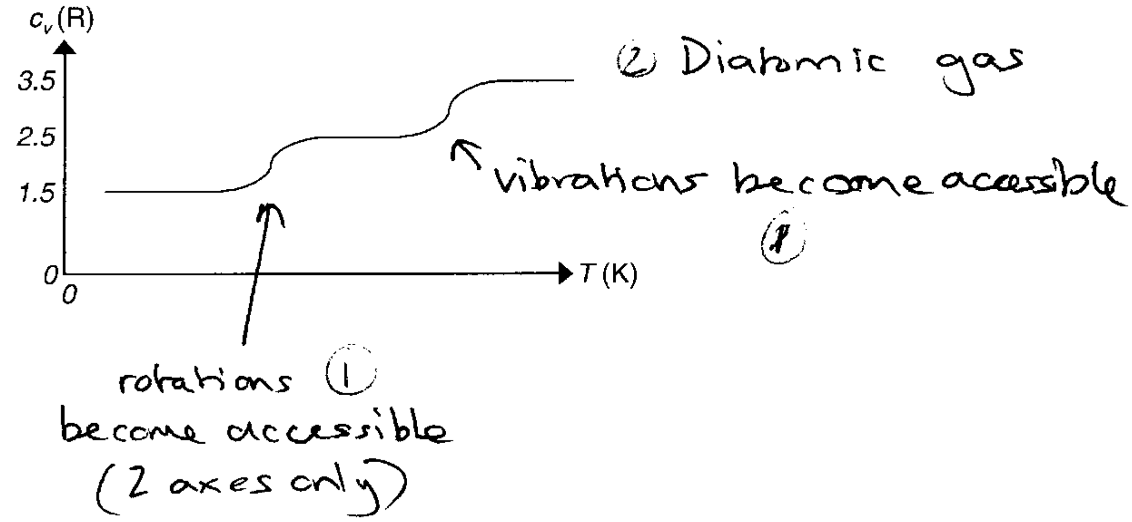



2 monatomic gas
$q = 3$ = constant

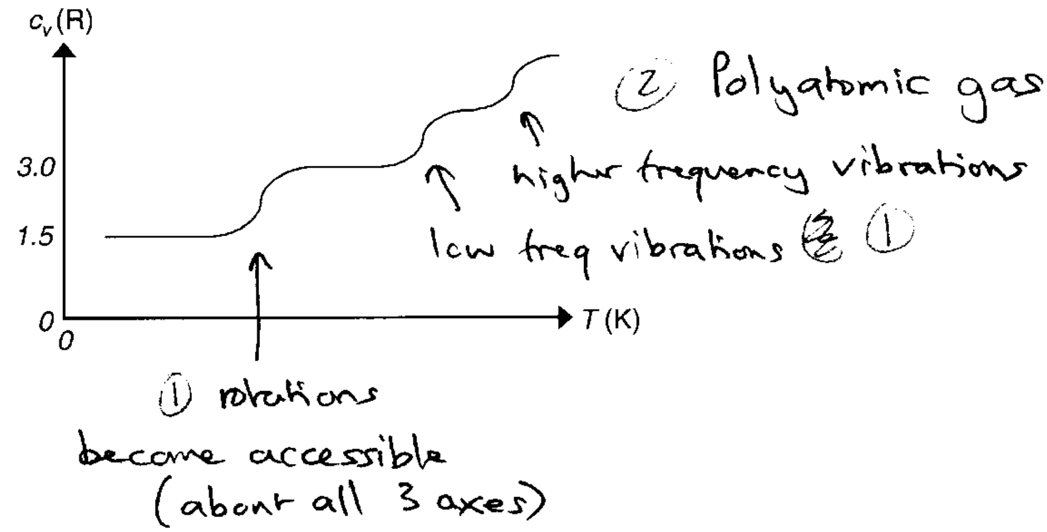
1 no internal degrees of freedom of molecules

 Name Answers

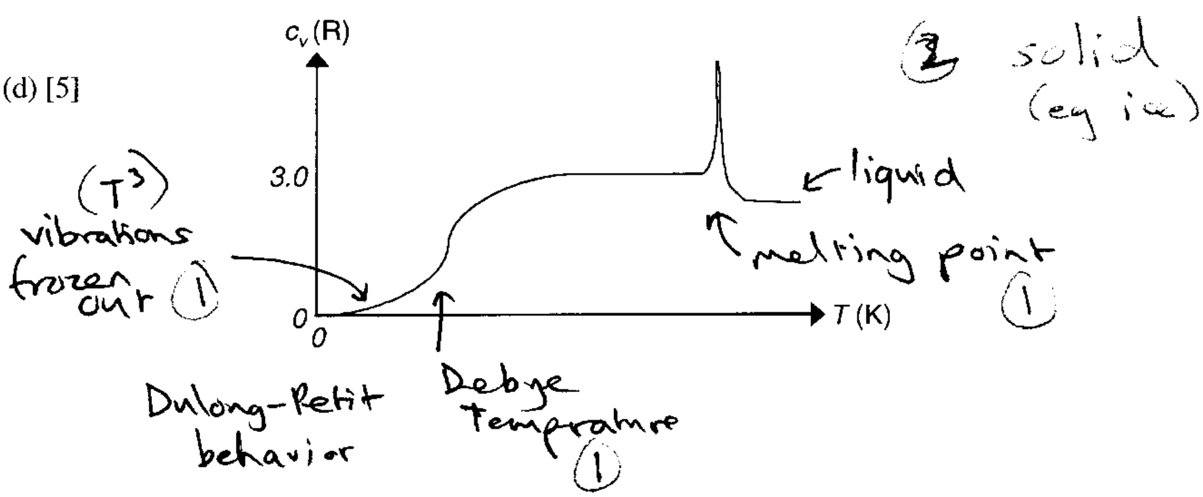
(b) [4]




(c) [4]




(d) [5]




What property of the substance is the area under the spike in (d) equal to? [2]

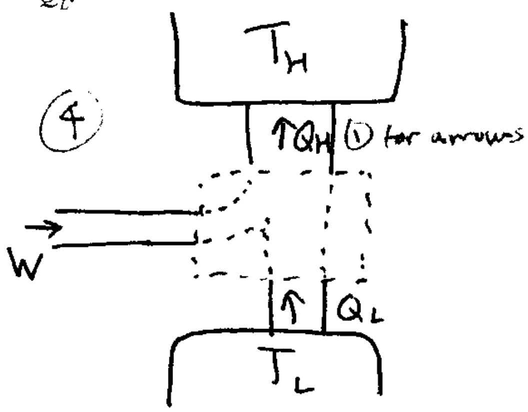
Latent heat of melting 2

 Name answers

**Question 3** [33] Refrigerators and Entropy

(a) [6] Draw a schematic energy flow diagram for a heat pump (ie, refrigerator) working between two reservoirs at temperatures $T_H$ and $T_L$, and define the coefficient of performance $K$ in terms of $W$, $Q_H$ and $Q_L$.

4




$$K = \frac{|Q_L|}{|W|} = \frac{|Q_L|}{|Q_H|-|Q_L|} = \frac{1}{\frac{|Q_H|}{|Q_L|}-1}$$ (2)

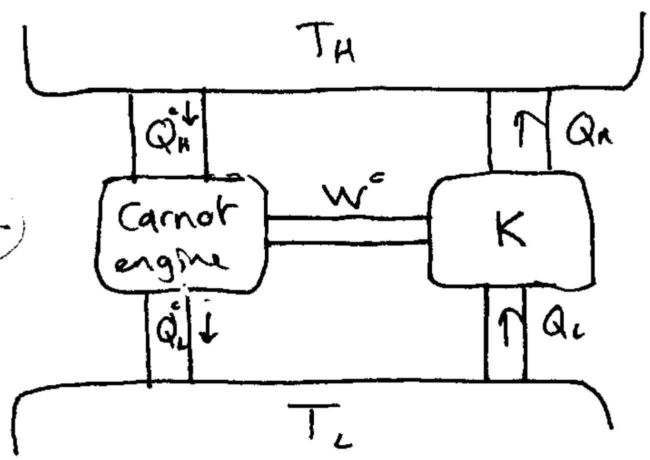
(b) [4] State the second law of thermodynamics in the form (Clausius) that applies to a heat pump.

4 No process is possible whose sole result is the transfer of heat from a colder to a hotter reservoir.

(c) [8] Show that the second law implies $K \le T_L/(T_H-T_L)$ for any real heat pump, by considering driving the heat pump with a Carnot engine.

Consider:

4




$$|Q_L| = K|W^c| \quad \text{(i)}$$

$$\frac{|Q_H^c|}{|Q_L^c|} = \frac{T_H}{T_L} \quad (1)$$

$$\frac{|W^c|}{|Q_L^c|} = \frac{|Q_H^c|-|Q_L^c|}{|Q_L^c|} = \frac{T_H-T_L}{T_L} \quad \text{(ii)}$$

(for Carnot engine)

using (i) and (ii), $|Q_L| = K|W^c| = K.\left(\frac{T_H-T_L}{T_L}\right).|Q_L^c|$ (2)

$|Q_L|$ cannot be greater than $|Q_L^c|$ (1)
or 2nd law would

$T_L$

(d) [8] A heat pump keeps the inside of a house at a temperature $T_H = 22°$ C. For the second reservoir it has two optional settings. It can use either (1) a point deep in the ground, at a constant temperature of 7° C throughout the year, or (2) the outside air, which is at an average temperature of –3° C during the winter and +10° C during the summer. Which is the better setting in summer, and which is better in winter?

define: $T_{L1} = \underset{\text{winter air}}{-3°C} = 270\,K$ $\quad T_H = 22°C = 295\,K$

$T_{L2} = \underset{\text{summer air}}{+10°C} = 283\,K$ $\quad T_{L3} = \underset{\text{ground}}{7°C} = 280\,K$

$\therefore K \leq \frac{T_L}{T_H - T_L}$, (1) For $T_L = T_{L1}$, $K_1 \leq \frac{270}{295-270} = 10.8$ (1)

For $T_L = T_{L2}$, $K_2 \leq \frac{283}{295-283} = 23.6$ (1)

for $T_L = T_{L3}$, $K_3 \leq \frac{280}{295-280} = 18.7$ (1)

$\therefore$ In summer $K_2 > K_3$ $\therefore$ best to use outside air, option (2)

In winter, $K_3 > K_1$ $\therefore$ best to use ground, option (1). 4

(e) [7] The heat pump (which is *real*!) runs with $K = 5$ when using the underground reservoir. What is then the rate of increase of the entropy of the universe due to the heat pump if it takes 1 kW of electric power?

power $P = 1\,kW$
work $|W| = Pt$

$$\Delta S = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L} \quad (2)$$

$$= \frac{|Q_L| + |W|}{T_H} - \frac{|Q_L|}{T_L} = |Q_L|\left(\frac{1}{T_H} - \frac{1}{T_L}\right) + \frac{|W|}{T_H}$$

$$= \left[K.\left(\frac{1}{T_H} - \frac{1}{T_L}\right) + \frac{1}{T_H}\right]|W| \quad (2)$$

$$\frac{\Delta S}{t} = \left[5.\left(\frac{1}{295} - \frac{1}{280}\right) + \frac{1}{295}\right]\frac{|W|}{t}$$

$$= \left(\frac{6}{295} - \frac{5}{280}\right)P$$

$$\therefore \frac{\Delta S}{t} = 2.48\ J K^{-1} s^{-1} \quad (3)$$

**Question 4 [30]**

(a) [10] Determine the rms speeds of $H_2$ and $O_2$ molecules in the earth's atmosphere at 300 K and 1 atm pressure. ($R$ = 8.3 J/K)

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT \quad \therefore v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \quad (4)$$

$M$ = molar mass
$m$ = molecular mass

$$(3) \quad H_2 : M = 2g \quad V_{rms} = \sqrt{\frac{3 . 8.3 J/K . 300 K}{0.002 kg}} = 1.9 \times 10^3 ms^{-1}$$

$$(3) \quad O_2 \quad M = 32g \quad V_{rms} = \sqrt{\frac{3 \times 8.3 J/K \times 300 K}{0.032 kg}} = 4.8 \times 10^2 ms^{-1}$$

(b) [10] The escape velocity of particles on the earth is 11 km/sec. Explain, using the Maxwell speed distribution, why hydrogen can eventually escape the earth's atmosphere, while oxygen almost never does.

Maxwell distribution: $n(v) = A v^2 e^{-\frac{mv^2}{2kT}} = A v^2 e^{-\frac{Mv^2}{2RT}}$ (4)

Estimate exponential part of $n(v)$ at $V = V_{escape} = 11 . 10^3 ms^{-1}$

For $H_2$, $\frac{1}{2}\frac{Mv^2}{RT} = \frac{1}{2}\frac{(2 \times 10^{-3} kg) \times (11 \times 10^3 ms^{-1})^2}{8.3 J/K \times 300 K} \simeq 48$ (2)

For $O_2$, $\frac{1}{2}\frac{Mv^2}{RT} = \frac{32g}{2g} \times 48 = 768$ (2)

$\therefore n(v_{esc})$ for $H_2 \sim e^{-48} \simeq 10^{-21}$ — so if you $N_{Av}$ it's finite.
$n(V_{esc})$ for $O_2 \sim e^{-770} \simeq 10^{-334}$ ie zero, effectively. (2)

(c) [10] Estimate the mean free path $\lambda$ of oxygen molecules in the atmosphere at $T$ = 300 K and sea level. Assume that the diameters of oxygen and nitrogen molecules, which are the main components of air, are both about $d = 2 \times 10^{-10}$ m. How does $\lambda$ vary with altitude above the earth's surface?

$$(3) \quad \lambda = \frac{1}{\sqrt{2}\pi d^2 n_v}$$

ideal gas:
$p = n_v kT$ (2) $n_v$ = molecular density

$$\therefore n_v = \frac{10^5 Nm^{-2}}{1.38 \times 10^{-23} J/K \times 300 K} = 2.4 \times 10^{25} m^{-3} \quad (1)$$

$$\therefore \lambda = \frac{1}{\sqrt{2} . \pi . (2 \times 10^{-10} m)^2 \times 2.4 \times 10^{25} m^{-3}} = 2.3 \times 10^{-7} m = 230 \text{ nm} \quad (2)$$

$\uparrow \lambda_0$

Assuming isothermal atmosphere, $n_v = \frac{p_0 e^{-\frac{\rho_0 g y}{p_0}}}{kT}$

$$\therefore \lambda = \frac{kT}{\sqrt{2}\pi d^2 p_0} e^{\frac{\rho_0 g y}{p_0}} = \lambda_0 e^{-y/L} \quad (3) \qquad L = \frac{p_0}{\rho_0 g} = 10^4 m$$

$\uparrow \frac{Mgy}{RT}$