 Name ______________________ Student ID ______________

Thermal Physics, Physics 224 **Final exam** **March 20, 2003**
Winter 2003 Instructor: David Cobden

There are four questions, with one double-sided sheet per question. Answer questions. You have 110 minutes. Move on after 25 minutes on each question! Begin and end on the buzzer. Be sure to write your name and ID on every page. Write all your working on these question sheets. Watch the blackboard for corrections during the exam. This is a closed book exam. You are allowed three sheets (six sides) of notes. You are allowed a simple calculator. *Throughout this exam, you will get credit for giving algebraic answers wherever possible as well as numerical answers.* Please circle the algebraic answer and underline the numeric answer.

$R = 8.3$ J/mol; $N_A = 6.02 \times 10^{23}$; $0°$ C = 273 K.

Question 1. [50 points total]
a. A cylindrical bath plug has radius $r = 2$ cm. The bath is rectangular, $L = 2$ m long and $W = 1$ m wide, and is filled to a depth of $h = 40$ cm. No drain pipe is connected to the plug hole. Estimate the force required to remove the plug from the plug hole. [6]

gauge pressure $p = \rho g h$

$\therefore$ force on plug $= p \cdot \pi r^2 = 10^{+3}\,\mathrm{kg\,m^{-3}} \times 9.8\,\mathrm{m\,s^{-2}} \times 0.4\,\mathrm{m} \times \pi \times (0.02\,\mathrm{m})^2$

$= 4.9$ N

b. What assumptions do you need to make to be able to apply Bernoulli's principle to the flow of water out of the plug hole? [4]

- zero viscosity (no friction)
- steady flow, no turbulence
- velocity at surface is negligible.

c. Show that the rate of decrease of the water height is given by $\frac{dh}{dt} = -\frac{\pi r^2}{LW}\sqrt{2gh}$. [6]

volume of water $V = hLW$

Bernoulli: $\rho g h = \frac{1}{2}\rho v^2$

$\therefore v = \sqrt{2gh}$

continuity: $\frac{dV}{dt} = -\pi r^2 v$

$\therefore LW \frac{dh}{dt} = -\pi r^2 \sqrt{2gh}$

d. Hence estimate the time taken for the bath to empty. [4]

$$\int_0^t dt' = -\frac{LW}{\pi r^2}\int_{40\,\mathrm{cm}}^{0} \frac{dh'}{\sqrt{2gh'}} \quad \therefore t = \frac{LW}{\pi r^2}\frac{1}{\sqrt{2g}}\, 2\Big[\sqrt{h}\Big]_0^{40\,\mathrm{cm}}$$

$$= \frac{2 \times 1}{\pi \times (0.02)^2} \times 2\sqrt{\frac{0.4}{2 \times 9.8}} = 455 \text{ sec!}$$

e. Show that the variation of pressure p with height h in the atmosphere is given by $p = p_0 \exp(-\rho_0/p_0)gh$, where $p_0 = 10^5$ Pa is the pressure at ground level and $\rho_0 = 1.3$ kg/m^3 is the density at ground level. Assume constant temperature (T_0) throughout the atmosphere, and that all gases are ideal. [8]

$$\frac{dp}{dh} = -\rho g \qquad p = \frac{n}{V} RT \propto \rho \text{ at const } T$$

$$\therefore \frac{p}{p_0} = \frac{\rho}{\rho_0} \qquad \therefore \rho = \frac{\rho_0}{p_0} p$$

$$\therefore \frac{dp}{dh} = -\frac{\rho_0 g}{p_0} p \qquad \therefore p = p_0 e^{-\left(\frac{\rho_0}{p_0} gh\right)}$$

f. A balloon is filled with helium to a volume $V_0 = 2$ liters at ground level. What is the buoyant force F_B on it? (Ignore the effects of tension in the rubber.) [6]

$$F_B = \rho_0 V_0 g = 1.3\,\text{kgm}^{-3} \times 0.002\,\text{m}^3 \times 9.8\,\text{ms}^{-2}$$

$$= 0.025\,\text{N}$$

g. The balloon is released and rises to a height of 1 km. Assuming that it rises *isothermally*, by how much does the buoyant force on the balloon change during its rise? [8] V = volume at height h

$$F_B = \rho V g. \qquad pV = p_0 V_0 \quad \text{so} \quad V = \frac{p_0 V_0}{p}$$

$$\text{and } \rho = \frac{\rho_0}{p_0} p$$

$$\therefore F_B = \frac{\rho_0}{p_0} p \cdot \frac{p_0 V_0}{p} g = \rho_0 V_0 g$$

$\therefore$ no change in F_B

h. Assuming *instead* that the balloon rises to the same height *adiabatically* (the rubber must be thermally insulating), by how much does the buoyant force change? [8]

For He $\frac{c_p}{c_v} = \gamma = \frac{5}{3}$

$$\text{Now} \quad pV^\gamma = p_0 V_0^\gamma \quad \text{so} \quad V = \left(\frac{p_0}{p}\right)^{1/\gamma} V_0$$

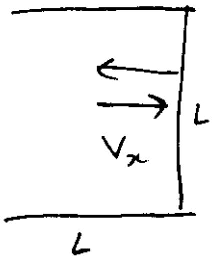
$$\therefore F_B = \left(\frac{\rho_0}{p_0} p\right)\left[\left(\frac{p_0}{p}\right)^{1/\gamma} V_0\right] g = \rho_0 V_0 g \cdot \left(\frac{p_0}{p}\right)^{1/\gamma - 1}$$

$$= \rho_0 V_0 g \left(\frac{1}{e^{-\left(\frac{\rho_0}{p_0} gh\right)}}\right)^{\frac{3}{5}-1} \qquad \exp\left(-\frac{2}{5} \cdot \frac{1.3\,\text{kgm}^{-3}}{10^5\,\text{Nm}^{-2}} \times 9.8\,\text{ms}^{-2} \times 10^3\,\text{m}\right)$$

$$= \rho_0 V_0 g \exp\left(-\frac{2}{5} \frac{\rho_0}{p_0} gh\right) \qquad = 0.95 \qquad \text{so } \Delta F_B = -5\%$$

Question 2. (Assume all gases are ideal in this question).

a. Show using kinetic theory that the pressure p exerted by a noninteracting gas of N molecules in a box of volume V is given by $p = (1/3)(N/V)mv_{rms}^2$, where m is the mass of a molecule and v_{rms} is the root mean squared speed. [10]



$$L^2 p = (\text{total no. molecules}) \times \overline{\begin{pmatrix}\text{collision}\\ \text{frequency}\end{pmatrix} \times \begin{pmatrix}\text{impulse transferred}\\ \text{per collision}\end{pmatrix}}^{\text{average}}$$

$$= N \times \overline{\frac{v_x}{2L} \times 2mv_x} = \frac{Nm\overline{v_x^2}}{L}$$

$$\therefore p = \frac{Nm\overline{v_x^2}}{L^3} \qquad v_{rms}^2 = \overline{v_x^2 + v_y^2 + v_z^2} = 3\overline{v_x^2}$$

$$\therefore p = \frac{N}{V} m . \frac{v_{rms}^2}{3}$$

b. A gas-tight, rigid box of volume $V = 10$ liters contains a mixture of $n = 0.01$ moles of hydrogen (H_2) and $n/2$ moles of oxygen (O_2) at room temperature, $T_0 = 27°$ C. What is the total pressure inside the box? [6]

$$p_{tot} = p_{H_2} + p_{O_2} = \frac{n}{V}RT + \frac{n}{2V}RT$$

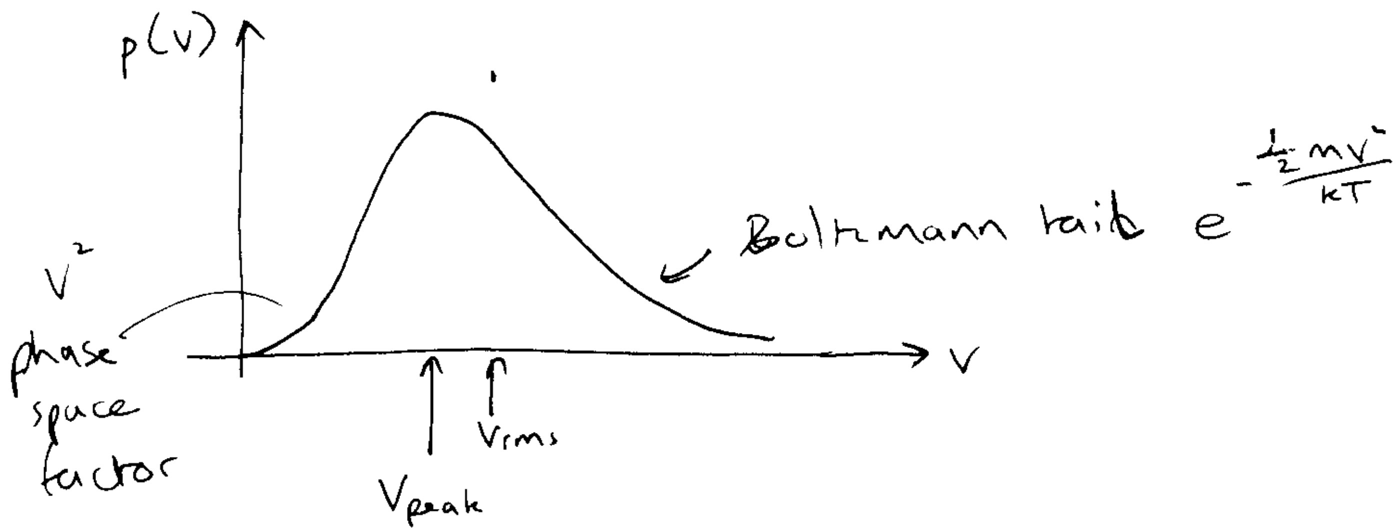
$$= \frac{3}{2}\frac{nRT}{V} = \frac{3}{2} \times \frac{0.01 \times 8.3\,\text{J/K} \times 300\,\text{K}}{10^{-2}\,\text{m}^3} \simeq 3700\,\text{Pa}$$

c. What is v_{rms} for a hydrogen molecule in the box? [6]

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT \quad \text{so} \quad v_{rms} = \sqrt{\frac{3k_BT}{m}} = \qquad m = \frac{2 \times 1\text{g}}{N_A} = \frac{2\text{g} \times k_B}{R}$$

$$v_{rms} = \sqrt{\frac{3RT}{2\text{g}}} = \left(\frac{3 \times 8.3\,\text{J/K} \times 300}{0.002\,\text{kg}}\right)^{1/2} = 1930\,\text{ms}^{-1}$$

d. Sketch the distribution function for the speed v of the hydrogen molecule [4]. Is v_{rms} its most likely speed? [2] Indicate the origins of the behavior of the function at $v << v_{rms}$ and $v >> v_{rms}$. [2]



e. Triggered by a spark, the gases react suddenly and completely to produce pure water vapor (H_2O) at temperature $T_f = 300°$ C. What is the pressure immediately after the reaction? [4]

$$p = \frac{n}{V} R T_f = \frac{0.01}{0.010\,m^3} \times 8.3\,J/K \times 573\,K$$

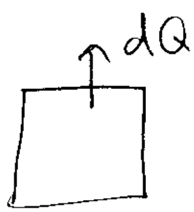
$$\doteq 4800\ Pa$$

f. The water vapor is then allowed to cool down to room temperature (there is not enough for it to condense). What is the final pressure? [4]

$$p = \frac{n}{V} R T_0 = \frac{0.01}{0.010\,m^3} \times 8.3\,J/K \times 300\,K$$

$$= 2490\ Pa$$

g. Assuming water behaves as an ideal gas with 6 unfrozen quadratic degrees of freedom per molecule, what is the change in entropy of the water vapor as it cools? [9]



573 → 300 K

$$\Delta S = \int_{573\,K}^{300\,K} \frac{-dQ}{T} = \int_{300\,K}^{573\,K} \frac{n c_v\,dT}{T}$$

c_v = molar spec. heat

$$c_v = \frac{q_{eff}}{2} R = 3R$$

$$\therefore \Delta S = n.3R \int_{300\,K}^{573\,K} \frac{dT}{T} = 3nR \ln \frac{573}{300}$$

$$= 3 \times 0.01 \times 8.3\,J/K \times 0.65$$

$$= 0.16\ J/K$$

h. The amount of heat released to the environment during this entire combustion process can be calculated from the difference in total thermodynamic potential between the hydrogen-oxygen mixture and the water vapor. Which thermodynamic potential – U, H, F or G – should be used *in this particular situation*, and why? [3]

U. Because $\Delta U = \Delta Q + W$ and here $W = 0$

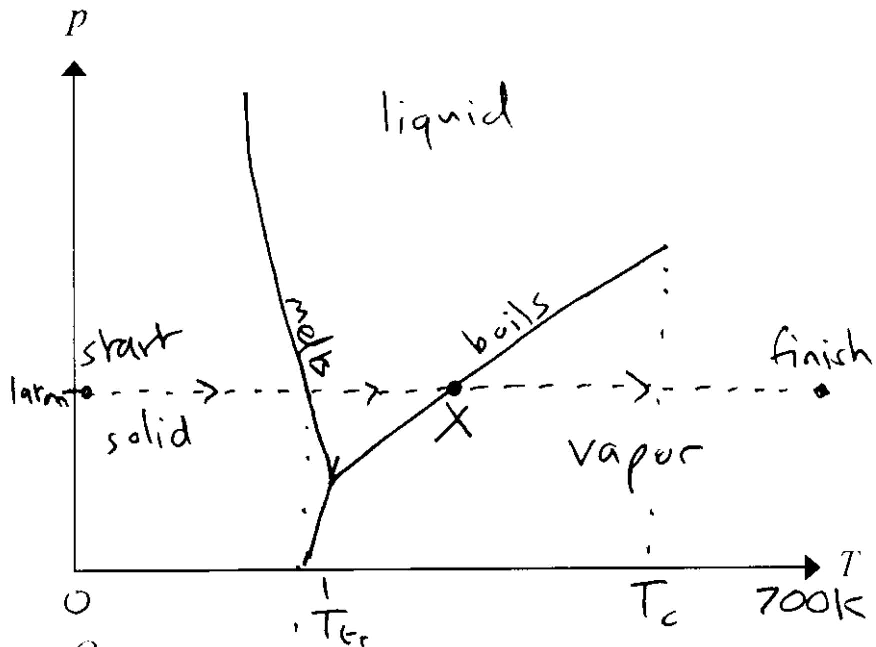
so $\Delta Q = \Delta U$.

$$= U_{H_2} + \tfrac{1}{2} U_{O_2} - U_{H_2O}$$

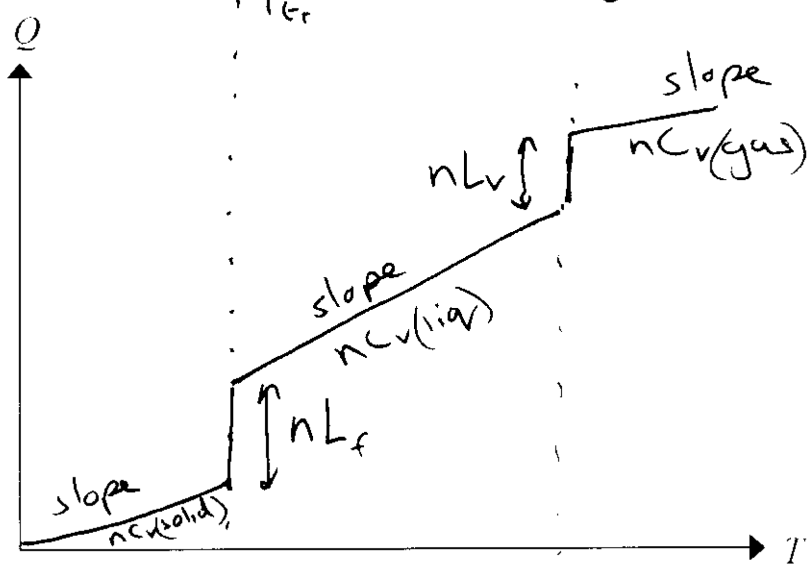
Question 3.
n moles of water is held at a constant pressure of 1 atm in a piston. The water is initially ice at close to $T = 0$ K. Heat is then gradually passed into it until it finally becomes gas at 700 K.

a. Sketch and label on the p-T axes to the right a phase diagram for water, with the T-axis ranging from 0 K to 700 K, which is above the critical point $T_c = 647$ K, and the p-axis from zero to about 3 atm.[10]

b. Sketch and annotate on the diagram the path (a dotted line) taken by the water during this process. Label the point where the liquid is in equilibrium with its vapor with an "X" [4]

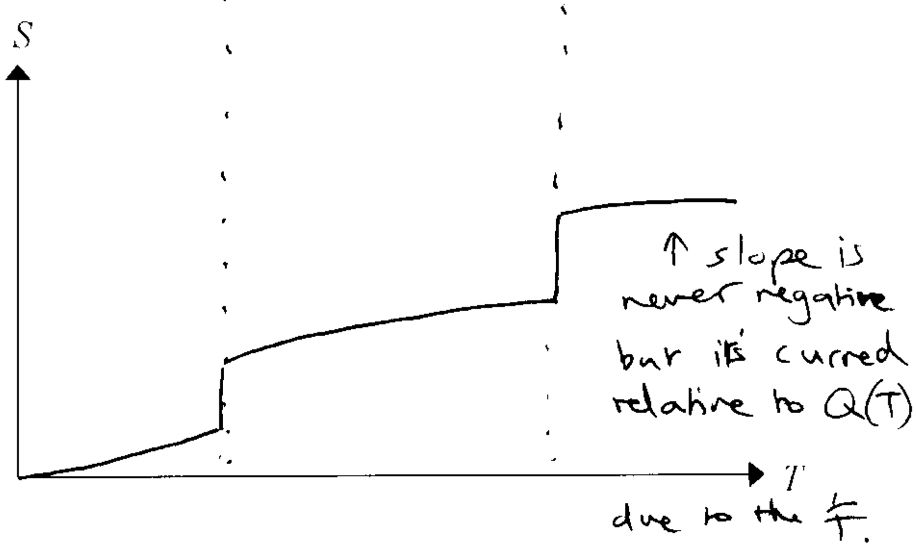


c. Sketch, on the Q-T axes to the right, the total heat absorbed as a function of T during this process. Label all quantities involved (heat capacities, latent heats). [10]

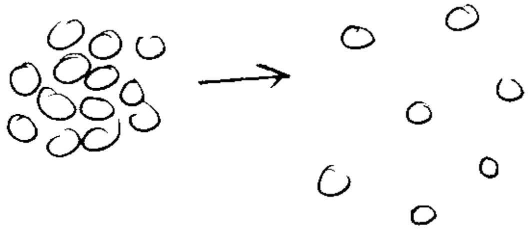


d. Sketch, on the S-T axes to the right, the entropy of the ice as a function of T. [6]

$$S = \int \frac{dQ}{T}$$

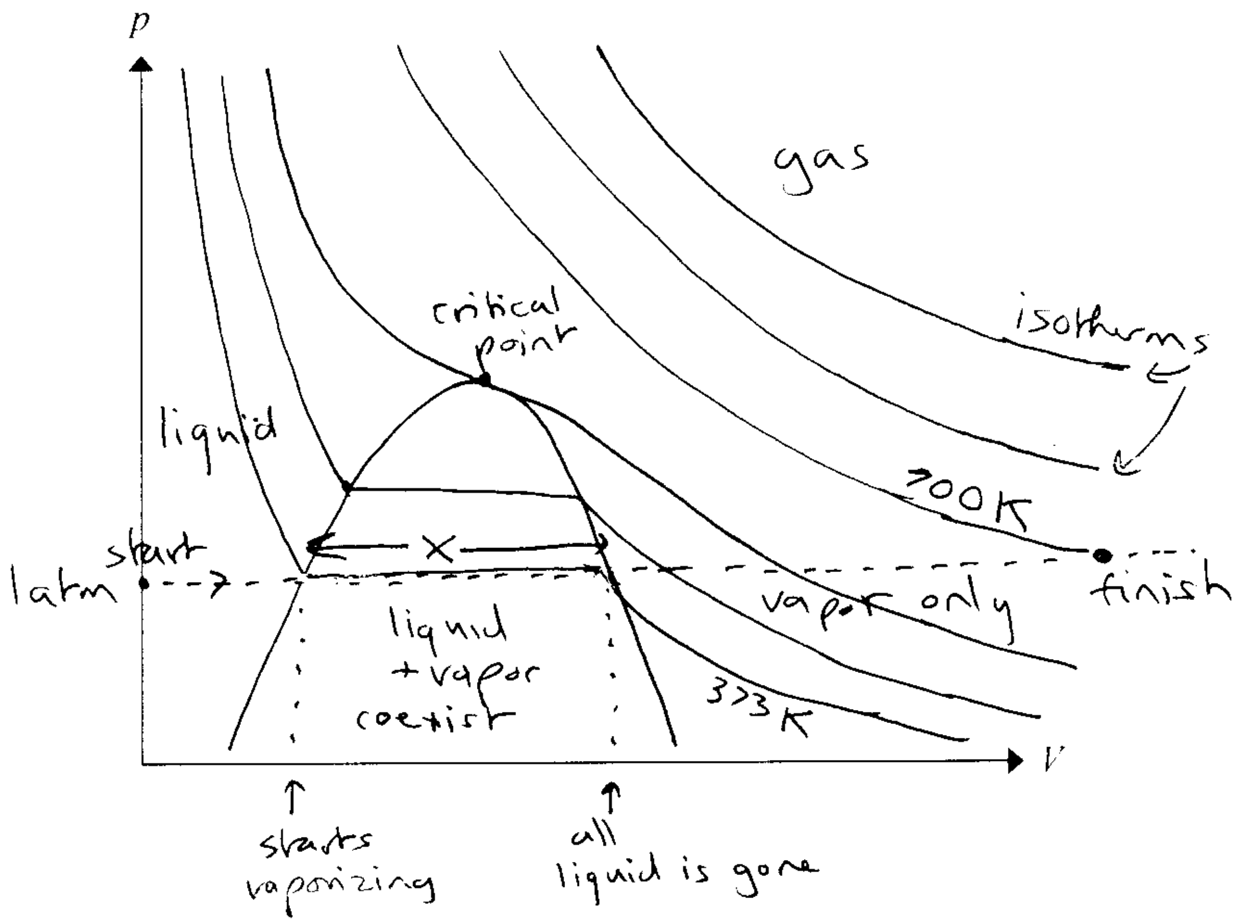


e. When energy is added as heat to the liquid at the boiling point, where microscopically does most of this energy go to? [4]



It goes to overcome short-range attractive forces, ie, "bonds". There's no increase in the speed.

f. Sketch and label, on the p-V axes below, isotherms of liquid and gaseous water, indicating the regions of different phase, the mixture, and their boundaries. [10]

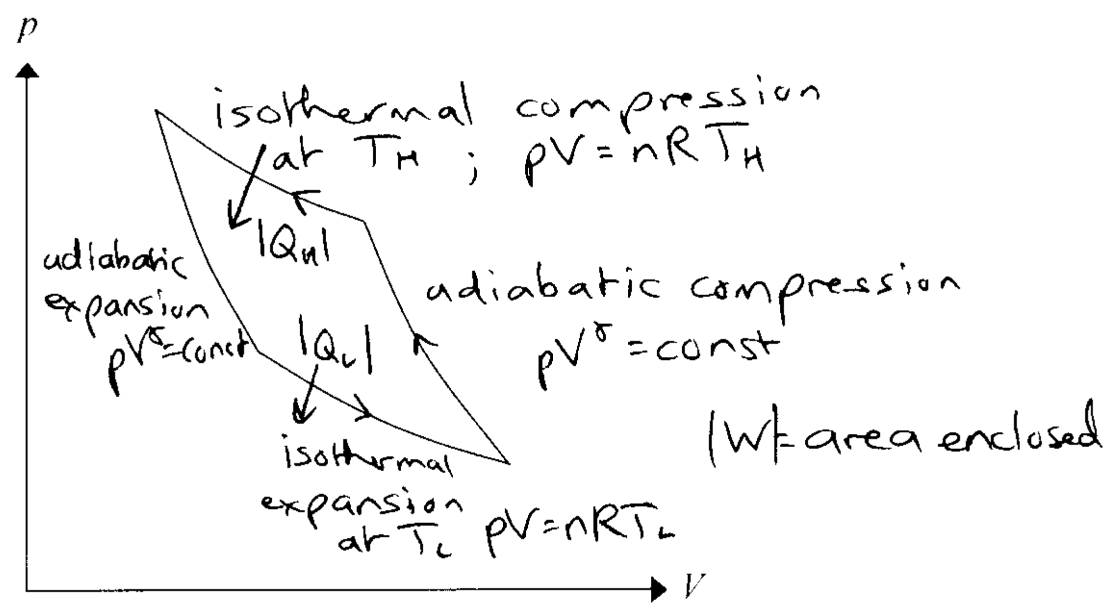


g. Sketch and annotate the path (draw a dotted line) taken on this p-V diagram by the water in part (b) as heat is added. You may neglect the solid phase. [4] What does point "X" from part (b) correspond to on this diagram? [2]

region marked X

Question 4.

a. A Carnot *refrigerator* takes its working substance, an ideal gas, reversibly around the cycle shown below between heat reservoirs at temperatures T_H and T_L. Annotate the diagram, using terms such as 'isothermal expansion', and including indications of (a) where heat flows in and out of the gas, (b) the functional form of each part of the cycle, and (c) what corresponds to the total work input during the cycle. [10]



b. The coefficient of performance of a refrigerator is defined by $C = |Q_L|/|W|$. Why do we choose this definition? [4]

You want to know how much heat it extracts, $|Q_L|$, for how much work you do, $|W|$

c. What is the coefficient of performance, C_C, for the Carnot refrigerator? [6]

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L} \quad \text{for Carnot} \qquad |W| = |Q_H| - |Q_L|$$

$$\therefore C_C = \frac{|Q_L|}{|W|} = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{T_L}{T_H - T_L}$$

d. What factors typically cause a real refrigerator to be less efficient than a Carnot fridge? [5]

Friction in the parts
Viscosity of the gas
Irreversible & nonequilibrium temperature, pressure variations in the system

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e. Show that for a real refrigerator, having coefficient of performance $C < C_C$, operating between the same two heat reservoirs, the entropy of the universe increases over each cycle. [10]

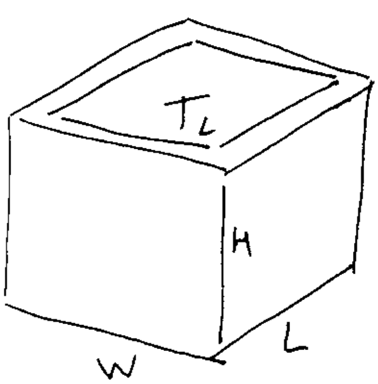
If $C < C_c$ then $\frac{|Q_L|}{|Q_H| - |Q_L|} < \frac{T_L}{T_H - T_L}$ $\therefore \frac{|Q_H|}{|Q_L|} > \frac{T_H}{T_L}$

$\therefore \frac{|Q_H|}{T_H} > \frac{|Q_L|}{T_L}$

$\therefore \Delta S_{univ} = + \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}$

> 0

f. An air conditioner with *half* the coefficient of performance of a Carnot refrigerator, $C = C_p/2$, is used to cool a mud hut in the desert. The hut has dimensions $L \times W \times H = 4$ m $\times$ 3 m $\times$ 2 m, and the thickness of the walls is $d = 10$ cm. The floor and the ceiling are superinsulated. The air temperature inside is to be maintained at $T_L = 20°$ C, while outside it is at $T_H = 40°$ C. Assume all the unwanted heat flow into the room is by conduction through the walls~~, whose thickness is 20 cm~~, and whose thermal conductivity is $K = 1.0$ W/m/°C. Estimate the electrical power needed to run the air conditioner. [15]



Heat flow in =

$\dot{Q}_{in} = K A_{total} \left(\frac{T_H - T_L}{d} \right)$ Fick's law

$= K \times 2H(W+L) \times \left(\frac{T_H - T_L}{d} \right)$

$C = \frac{|\dot{Q}_{out}|}{|\dot{W}_{ork}|} = \frac{1}{2} C_c = \frac{\frac{1}{2} T_L}{T_H - T_L}$

$\therefore$ power reqd $= |\dot{W}_{ork}| = \frac{|\dot{Q}_{out}|}{C} = \frac{|\dot{Q}_{in}|}{C} = \frac{K \times 2H(W+L)\left(\frac{T_H - T_L}{d}\right)}{\frac{1}{2}\left(\frac{T_L}{T_H - T_L}\right)}$

$= \frac{4KH(W+L)(T_H - T_L)^2}{d T_L}$ $= \frac{4 \times 1 \times 2 \times (4+3) \times (20)^2}{0.1 \times 293}$ $= 765$ W