Delta Robot Kinematics

3D printing–building by learning

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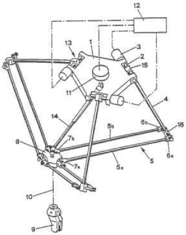
History

- Originated from delta robots (invented in 1980s, Switzerland)



Device for the movement and positioning of an element in space

Page bookmark	US4976582 (A) - Device for the movement and positioning of an element in space
Inventor(s):	CLAVEL REYMOND [CH] +
Applicant(s):	SOGEVA SA [CH] +
Classification:	**- international:** ***B25J11/00; B25J17/00; B25J17/02; B25J9/06; B25J9/10;*** (IPC1-7):
	- cooperative: **B25J17/0266; B25J9/0051; B25J9/1065;** Y10T74/20207
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Today

Widely used in pick-n-place operations of relatively light objects.

Fundamental Principles

- Actuators are all located in the workspace on the base
- Arm made of light materials

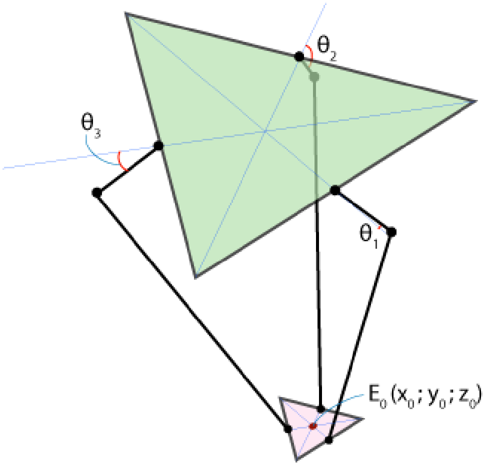
Hence the moving parts of the printer have a small inertia, allowing for very high speed and high accelerations.

Core Advantage

demo1 | demo2

Problem Definition

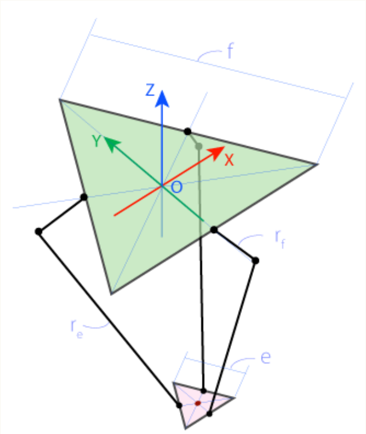
- **forward kinematics**: joint angles to position of the end effector
- **inverse kinematics**: (desired) position of the end effector to required joint angles



Inverse Kinematics

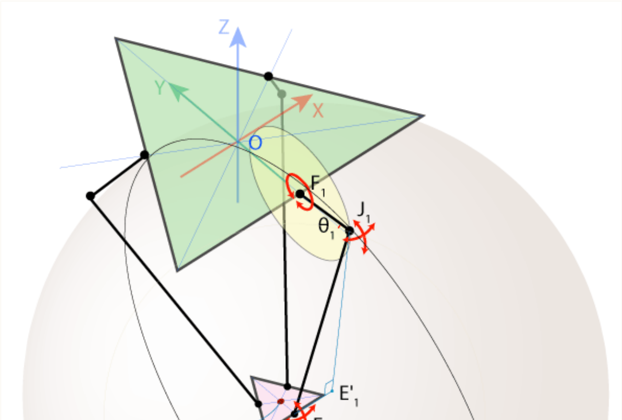
Dimensions:

- f: side of the fixed triangle (green in picture)
- e: side of the end effector triangle (pink in picture)
- r_f: length of upper joint
- r_e: length of lower joint (parallelogram joint)



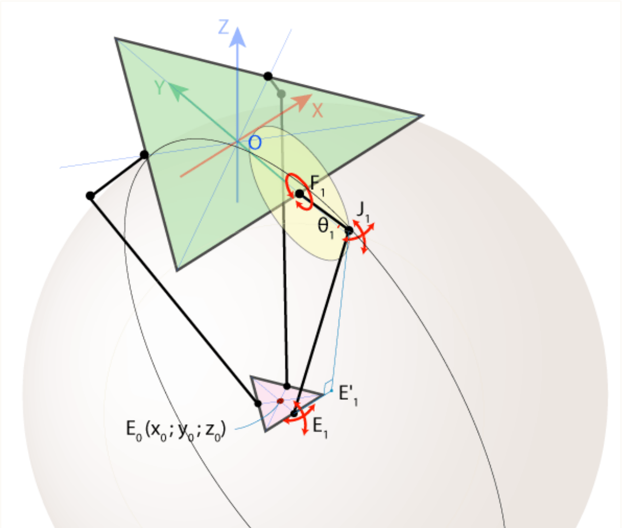
Inverse Kinematics » Geometry

- joint F_1J_1 only rotates in YZ plane (F_1J_1 forms a circle of radius r_f)
- J*1 and E*1 are called universal joints: E_1J_1 rotates freely relative to E_1, forming a sphere of radius r_e
- the fixed triangle and the end effector triangle are always parallel (no rotational motion for the end effector triangle)



Inverse Kinematics » Geometry

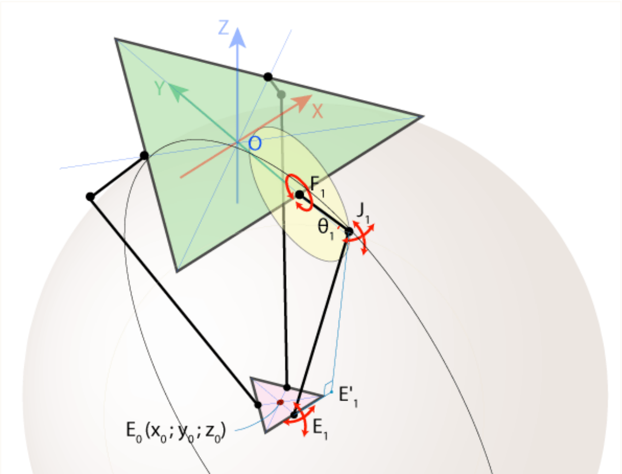
- define: the position of the center of the end effector as $E_0(x_0, y_0, z_0)$
- goal: given $E_0(x_0, y_0, z_0)$, find θ_i; $i = 1, 2, 3$



Inverse Kinematics » θ_1

- The sphere intersects with the YZ plane, forming a circle with center E_1' and radius $E_1'J_1$:

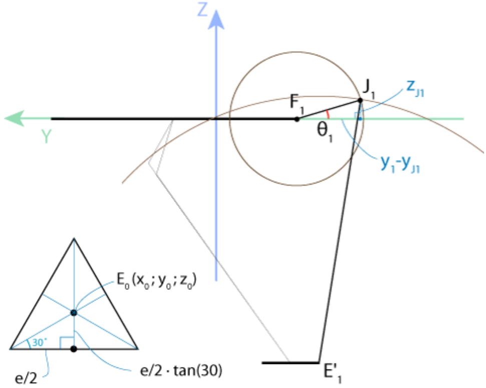
$$|E_1'J_1|^2 + |E_1E_1'|^2 = |E_1J_1|^2 = r_e^2$$



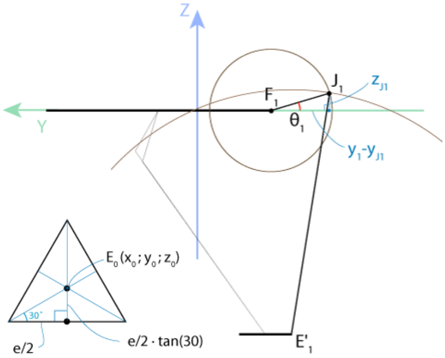
Inverse Kinematics » θ_1

Let's focus on the geometry in the YZ plane to find θ_1.

Big picture: decide E_1' and $|E_1'J_1|$ $\Rightarrow$Find the intersection of the two circles$\Rightarrow$Find J_1 $\Rightarrow$ $\theta_1 = \arcsin \frac{z_{J_1}}{r_f}$



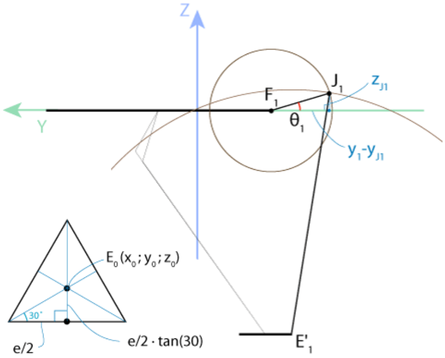
Inverse Kinematics » θ_1



- E_1 is the projection of E_0 to the bottom side of the end effector triangle on the XZ plane:

$$|EE_1| = \frac{e}{2}\tan 30^o = \frac{e}{2\sqrt{3}} \Longrightarrow E_1(x_0, y_0 - \frac{e}{2\sqrt{3}}, z_0)$$

Inverse Kinematics » θ_1



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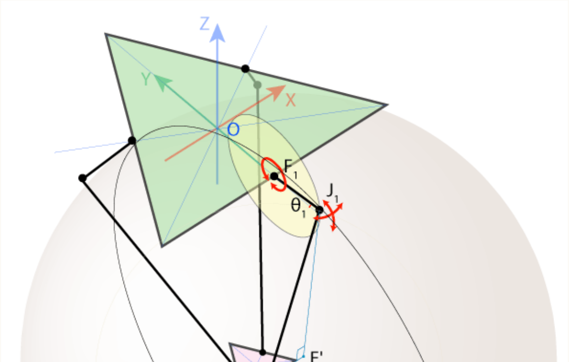
- E'_1 is the projection of E_1 onto the YZ plane:

$$|E_1 E'_1| = x_0$$

Inverse Kinematics » θ_1

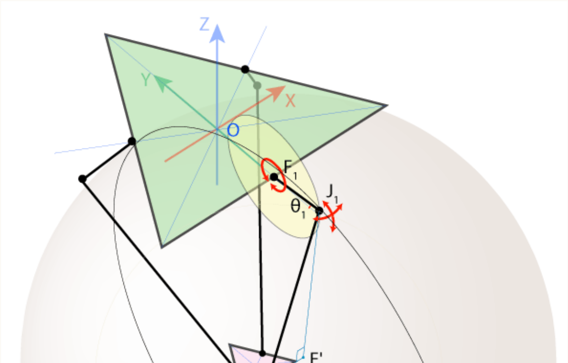
- We have

$$|E_1' J_1|^2 + |E_1 E_1'|^2 = |E_1 J_1|^2 = r_e^2$$



Inverse Kinematics » θ_1

- We have
$$|E_1' J_1|^2 + |E_1 E_1'|^2 = |E_1 J_1|^2 = r_e^2$$
- and
$$|E_1 E_1'| = x_0$$



Inverse Kinematics » θ_1

- We have

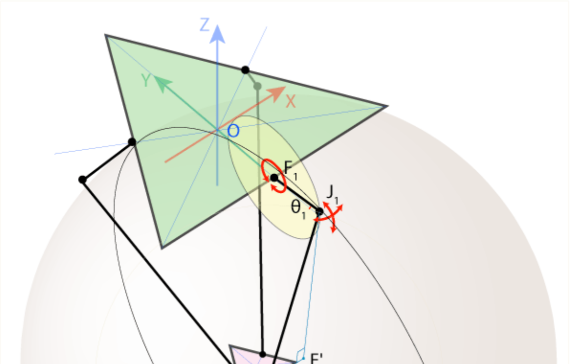
$$|E_1'J_1|^2 + |E_1E_1'|^2 = |E_1J_1|^2 = r_e^2$$

- and

$$|E_1E_1'| = x_0$$

- Hence

$$|E_1'J_1|^2 + |E_1E_1'|^2 = |E_1J_1|^2 = r_e^2$$



Inverse Kinematics » θ_1

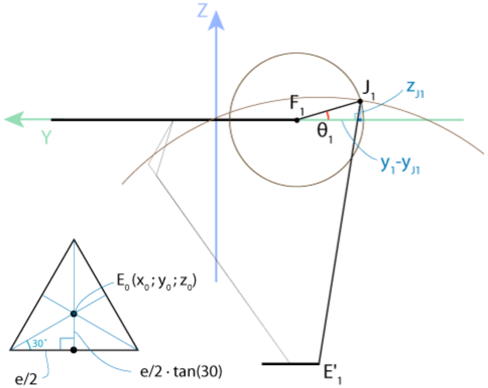
- The intersection of the two circles are defined by

$$(y_{J_1} - y_{F_1})^2 + (z_{J_1} - z_{F_1}^2)^2 = r_f^2$$

and

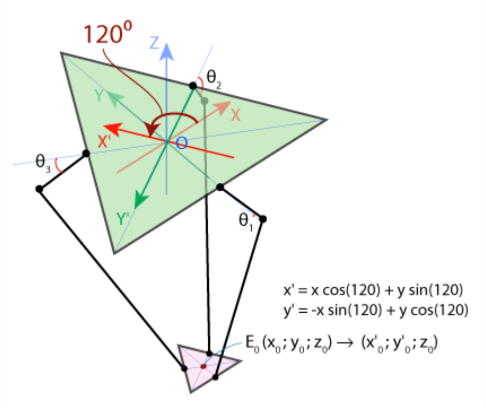
$$(y_{J_1} - y_{E_1'})^2 + (z_{J_1} - z_{E_1'}^2)^2 = r_e^2 - x_0^2$$

- solve for z_{J_1} and y_{J_1} to get θ_1



Inverse Kinematics » θ_2 and θ_3

- θ_2 and θ_3 can be similarly derived.
- but there is a shortcut: rotating the axis, we can use the exact same formula on the new coordinates

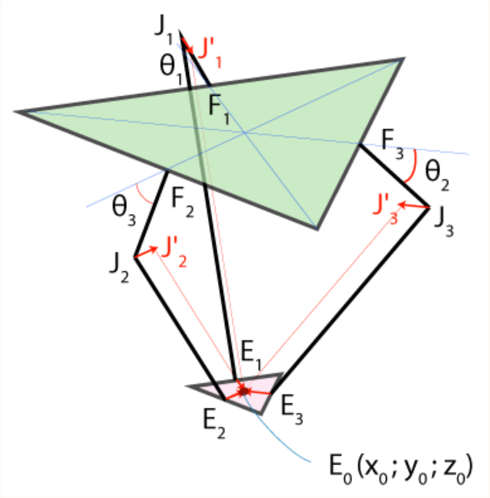


Forward Kinematics

Forward Kinematics

Goal:

- given θ_1, θ_2, θ_3
- find $E_0(x_0, y_0, z_0)$



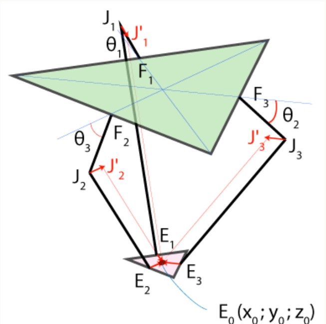
Forward Kinematics

Solution concept:

- given θ_1, θ_2, θ_3
- compute the coordinates of J_1, J_2, J_3
- move J_1, J_2, J_3 to J_1', J_2', J_3' using transition
- compute the intersection of the three spheres centered at J_1', J_2', J_3'

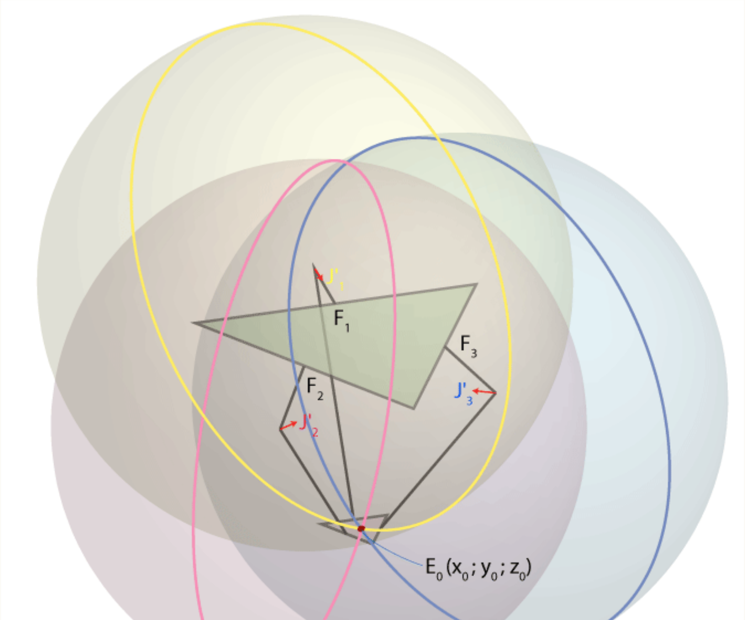
$$(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2 = r_e^2$$

- the intersection is E_0

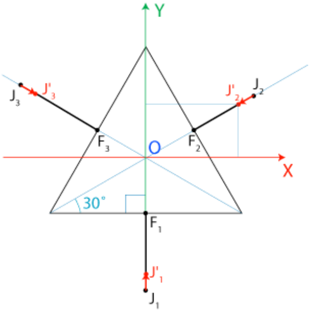


Forward Kinematics

Illustration of the intersection of the three spheres



Forward Kinematics » J_1', J_2', J_3'



$OF_1 = OF_2 = OF_3 = f/2\ \tan(30) = f/2\sqrt{3}$
$J_1J'_1 = J_2J'_2 = J_3J'_3 = e/2\ \tan(30) = e/2\sqrt{3}$
$F_1J_1 = r_f\cos(\theta_1)$, $F_2J_2 = r_f\cos(\theta_1)$, $F_3J_3 = r_f\cos(\theta_3)$

$J'_1(0; -(f-e)/2\sqrt{3} - r_f\cos(\theta_1); -r_f\sin(\theta_1))$
$J'_2([(f-e)/2\sqrt{3} + r_f\cos(\theta_2)]\cos(30); [(f-e)/2\sqrt{3} + r_f\cos(\theta_2)]\sin(30); -r_f\sin(\theta_2))$
$J'_3([(f-e)/2\sqrt{3} + r_f\cos(\theta_3)]\cos(30); [(f-e)/2\sqrt{3} + r_f\cos(\theta_3)]\sin(30); -r_f\sin(\theta_3))$

Forward Kinematics » equation for the intersection point

$$\begin{cases} x^2+(y-y_1)^2+(z-z_1)^2=r_e^2 \\ (x-x_2)^2+(y-y_2)^2+(z-z_2)^2=r_e^2 \\ (x-x_3)^2+(y-y_3)^2+(z-z_3)^2=r_e^2 \end{cases} \Rightarrow \begin{cases} x^2+y^2+z^2-2y_1y-2z_1z=r_e^2-y_1^2-z_1^2 & (1) \\ x^2+y^2+z^2-2x_2x-2y_2y-2z_2z=r_e^2-x_2^2-y_2^2-z_2^2 & (2) \\ x^2+y^2+z^2-2x_3x-2y_3y-2z_3z=r_e^2-x_3^2-y_3^2-z_3^2 & (3) \end{cases}$$

$$w_i = x_i^2+y_i^2+z_i^2$$

$$\begin{cases} x_2x+(y_1-y_2)y+(z_1-z_2)z=(w_1-w_2)/2 & (4)=(1)-(2) \\ x_3x+(y_1-y_3)y+(z_1-z_3)z=(w_1-w_3)/2 & (5)=(1)-(3) \\ (x_2-x_3)x+(y_2-y_3)y+(z_2-z_3)z=(w_2-w_3)/2 & (6)=(2)-(3) \end{cases}$$

From (4)-(5):

$$x = a_1z+b_1 \quad (7) \qquad y = a_2z+b_2 \quad (8)$$

$$a_1 = \frac{1}{d}\left[(z_2-z_1)(y_3-y_1)-(z_3-z_1)(y_2-y_1)\right] \qquad a_2 = -\frac{1}{d}\left[(z_2-z_1)x_3-(z_3-z_1)x_2\right]$$

$$b_1 = -\frac{1}{2d}\left[(w_2-w_1)(y_3-y_1)-(w_3-w_1)(y_2-y_1)\right] \qquad b_2 = \frac{1}{2d}\left[(w_2-w_1)x_3-(w_3-w_1)x_2\right]$$

$$d = (y_2-y_1)x_3-(y_3-y_1)x_2$$

Now we can substitute (7) and (8) in (1):

$$(a_1^2+a_2^2+1)z^2+2(a_1+a_2(b_2-y1)-z_1)z+(b_1^2+(b_2-y_1)^2+z_1^2-r_e^{\,2})=0$$

Solve the last equation and calculate x_0 and y_0 from equations (7) and (8).

References

- Paul Zsombor-Murray, Descriptive Geometric Kinematic Analysis of Clavel's "Delta" Robot, 2004
- http://reprap.org/wiki/Delta_geometry