Position And Orientation

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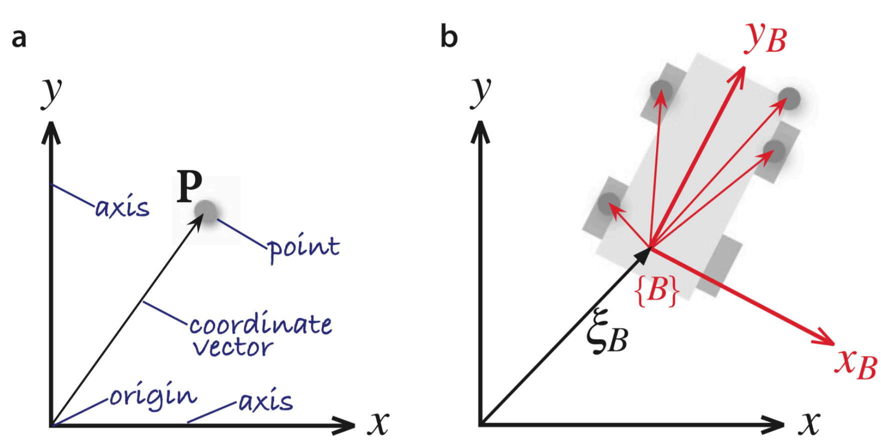
Contents

1 Basic concepts

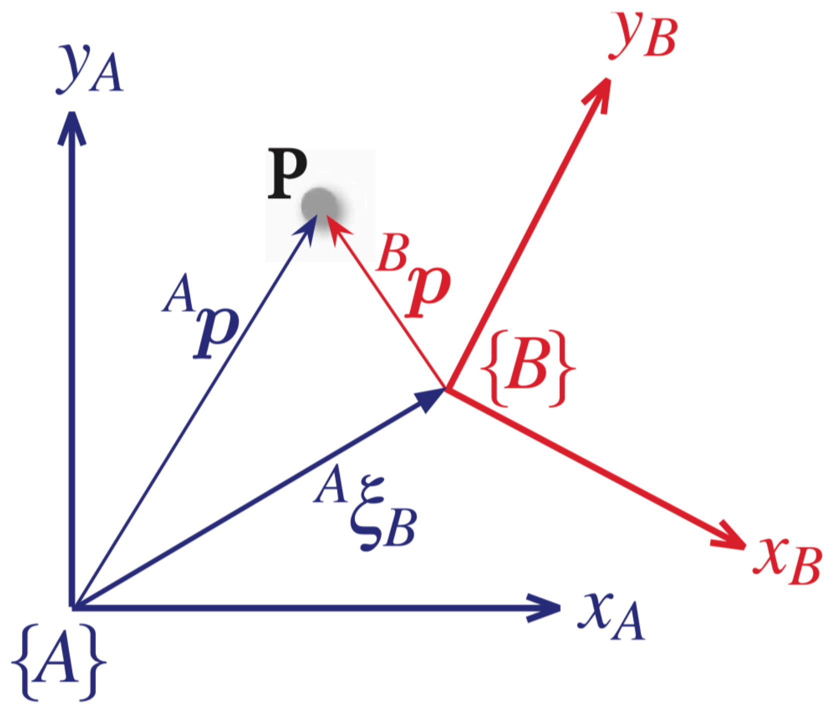
- location, e.g., the object is 2m due north: represented as a vector containing a denominate number plus a direction
- orientation, e.g., the door is facing west
- pose: the combination of position and orientation, e.g., the car is 2m due north and facing west



- A point in space is a familiar concept from mathematics and can be described by a coordinate vector.
- A coordinate frame, or Cartesian coordinate system, is a set of orthogonal axes which intersect at a point known as the origin.
- An object:
  - comprises infinitely many points
  - unlike a point, also has an orientation.
  - If we attach a coordinate frame to an object, as shown in Fig. 2.1b, we can describe every point within the object as a constant vector with respect to that frame.

1.1 Pose of the coordinate frame

- denoted by ξ - pronounced ksi.
  - given two frames $\{A\}$ and $\{V\}$, ${}^{A}\xi_{B}$ describes the relative pose of $\{B\}$ w.r.t. $\{A\}$
    - leading superscript: the reference coordinate frame
    - subscript: the frame being described
    - if the initial superscript is missing, we assume that the change in pose is relative to the world coordinate frame $\{O\}$
    - imagine picking up $\{A\}$ and applying a displacement and a rotation so that it is transformed to $\{B\}$
- example



${}^A p = {}^A\xi_B \bullet {}^B p$

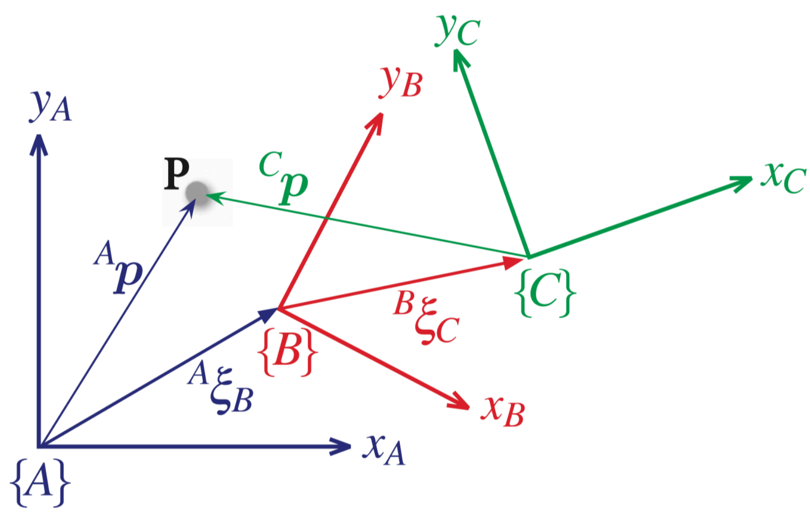
here, the operator $\bullet$ transforms the vector, resulting in a new vector that describes the same point but w.r.t. a different coordinate frame.

- important characteristic of relative poses: they can be composed or compounded

  ${}^A\xi_C = {}^A\xi_B \bigoplus {}^B\xi_C$: the pose of $\{C\}$ relative to $\{A\}$ can be obtained by compounding the relative poses from $\{A\}$ to $\{B\}$ and $\{B\}$ to $\{C\}$.

the $\bigoplus$ operator: indicates composition of relative poses.

- example

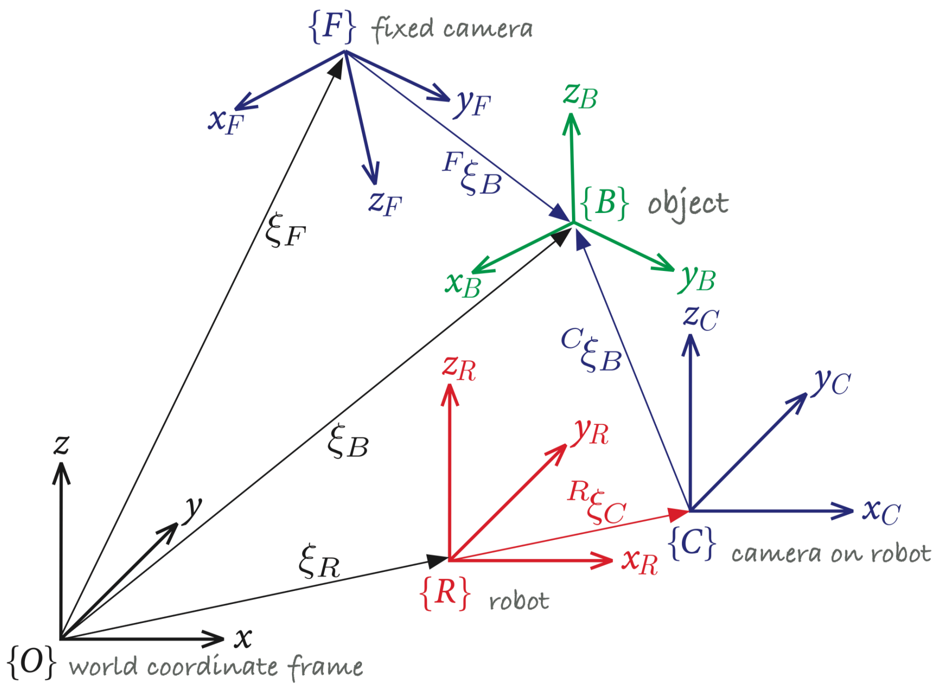


the point p can be described by

$$^A p = \left({}^A\xi_B \bigoplus {}^B\xi_C \right) {}^C p$$

1.2 Coordinate frames: from 2d to 3d

- 2d coordinate frames: appropriate for e.g., mobile robots that operate in a planar world
- 3d coordinate frames: needed by e.g., the pose of a flying or underwater robot, or the end of a tool carried by a robot arm



$$\xi_F \oplus {}^F\xi_B = \xi_R \oplus {}^R\xi_C \oplus {}^C\xi_B$$
$$\xi_F \oplus {}^F\xi_R = \xi_R$$