

1 Observers

- With full-state observers, the order of the system is doubled.
- The enlarged system is not fully observable. Half of the states are observable.

1.1 Continuous-time case

$$\begin{aligned}\dot{x} &= Ax + Bu \\ \dot{\hat{x}} &= A\hat{x} + Bu + L(y - C\hat{x}) \\ y &= Cx\end{aligned}$$

Error dynamics: $e = x - \hat{x}$, $\dot{e} = \dot{x} - \dot{\hat{x}} = Ae - LCe$

Enlarged system:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

The error dynamics can be arbitrarily assigned if the system is observable.

1.2 Discrete-time case

Standard discrete-time observer

$$\begin{aligned}x(k+1) &= Ax(k) + Bu(k) \\ \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k)) \\ y(k) &= Cx(k)\end{aligned}$$

Error dynamics: $e(k) = x(k) - \hat{x}(k)$,

$$\begin{aligned}e(k+1) &= (Ax(k) + Bu(k)) - (A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))) \\ &= Ae(k) - LCe(k)\end{aligned}$$

Enlarged system:

$$\begin{aligned}\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) \\ y(k+1) &= [C, 0] \begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix}\end{aligned}$$

Through a similarity transformation, the above is equivalent to the system with $\begin{bmatrix} x(k)^T, \hat{x}(k)^T \end{bmatrix}^T$ as the state vector.

The error dynamics can be arbitrarily assigned if the system is observable.

Discrete-time observer with predictor:

$$\begin{aligned}\hat{x}(k+1|k) &= A\hat{x}(k|k) + Bu(k) \\ \hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + L(y(k+1) - C\hat{x}(k+1|k)) \\ &= (I - LC)[A\hat{x}(k|k) + Bu(k)] + Ly(k+1) \\ &= (I - LC)A\hat{x}(k|k) + (I - LC)Bu(k) + Ly(k+1) \\ e(k+1) &= x(k+1) - \hat{x}(k+1|k+1) = (I - LC)Ae(k)\end{aligned}$$

The error dynamics can be arbitrarily assigned if (A, CA) is observable, which is guaranteed if A is invertible and (A, C) is observable.