

Outline:

- Jordan blocks of discrete-time systems
- zero order hold and discretization of continuous-time systems

1 Diagonal and Jordan forms of discrete-time systems

J	J^k
$\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$	$\begin{bmatrix} \lambda_1^k & \\ & \lambda_2^k \end{bmatrix}$
$\begin{bmatrix} \lambda & 1 \\ & \lambda \end{bmatrix}$	$\begin{bmatrix} \lambda^k & k\lambda^{k-1} \\ & \lambda^k \end{bmatrix}$
$\begin{bmatrix} \lambda & 1 & \\ & \lambda & 1 \\ & & \lambda \end{bmatrix}$	$\begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{1}{2!}k(k-1)\lambda^{k-2} \\ & \lambda^k & k\lambda^{k-1} \\ & & \lambda^k \end{bmatrix}$
$\begin{bmatrix} \lambda & 1 & & \\ & \lambda & 1 & \\ & & \lambda & 1 \\ & & & \lambda \end{bmatrix}$	$\begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{1}{2!}k(k-1)\lambda^{k-2} & \frac{1}{3!}k(k-1)(k-2)\lambda^{k-3} \\ & \lambda^k & k\lambda^{k-1} & \frac{1}{2!}k(k-1)\lambda^{k-2} \\ & & \lambda^k & k\lambda^{k-1} \\ & & & \lambda^k \end{bmatrix}$

- empty entries are all zeros in the above table
- understanding the results in the last and the second-to-last rows: $k\lambda^{k-1} = \frac{d}{dk}\lambda^k$, $\frac{1}{2!}k(k-1)\lambda^{k-2} = \frac{1}{2!}\frac{d^2}{dk^2}\lambda^k$, $\frac{1}{3!}k(k-1)(k-2)\lambda^{k-3} = \frac{1}{3!}\frac{d^3}{dk^3}\lambda^k$. These modes are due to the presence of $*/(1-\lambda z^{-1})^n$, $n > 1$ in the transfer function (after partial fractional expansion).
- write down yourself the case for $J = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$

2 Zero order hold and discretization of continuous-time systems

Find the zero order hold equivalence of $G(s) = e^{-Ls}$, $L = 2.5\Delta T$. Repeat the process for $L = 2.1\Delta T$. Explain the results. Here, ΔT is the sampling time.

