Review of heteroskedasticity

Recall that in cross-sectional LS, heteroskedasticity

- is assumed away
- if present, biases our standard errors

We noted two approaches

- Model the heteroskedasticity directly with an appropriate ML model, or
- Less optimally, continue to use the wrong method (LS), but try to correct the se’s; these are known as Huber-White, sandwich, or robust standard errors

How do these approaches transfer to the time series context? to panel data?
Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly.

One possibility is to let heteroskedasticity evolve dynamically.
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We can let heteroskedasticity be (sort-of) “ARMA”, under the name “GARCH”.

Generalized Autoregressive Conditional Heteroskedasticity:

\[ y_t = \mu_t + \varepsilon_t \]
\[ \varepsilon_t \sim f_{\mathcal{N}} (0, \sigma_t^2) \]
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\]

In words, \( y_t \) is an ARMA\((P, Q)\)-GARCH\((C, D)\) distributed time-series.

(Of course, we could leave out \( x \) and/or \( z \) if we wanted.)
Dynamic heteroskedasticity

\[ y_t = \mu_t + \varepsilon_t \]
\[ \varepsilon_t \sim f_{\mathcal{N}} \left( 0, \sigma_t^2 \right) \]

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Models like the above are workhorses of financial forecasting

Can estimated by ML as usual

In R, `garch()` in the `tseries` package does GARCH

May have to look around a bit for ARMA-GARCH
Dynamic and panel heteroskedasticity

Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example, . . .

- Serial correlation: $\mathbb{E}(\varepsilon_{is}\varepsilon_{it}) = \sigma_{st} \neq 0$

(Reduced/eliminated by appropriate ARMA specification)
Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example...

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- Contemporaneous correlation: \( E(\varepsilon_it \varepsilon_{jt}) = \sigma_{ij} \neq 0 \)
  
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- Dynamic heteroskedasticity: $\sigma_{it}^2 = f(\sigma_{i,t-k}^2)$
  (How to fix?)
Consider a Panel ARMA-GARCH model:

\[ y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim fN \left( 0, \sigma_{it}^2 \right) \]
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Suppose we estimated \( y_{it} \) as a function of \( \mu_{it} \), but ignored the structure of the error term.

That is, we estimate a panel ARMA or GMM, but assume \( \varepsilon_{it} \) is homoskedastic and serially uncorrelated, conditional on the covariates and lags in \( \mu_{it} \).
Dynamic and panel heteroskedasticity

If we ignore this:

$$\sigma_{i,t}^2 = \exp(\eta_i + \zeta_t + z_{it}\gamma) + \sum_{c=1}^{C} \sigma_{i,t-c}^2\lambda_c + \sum_{d=1}^{D} \varepsilon_{i,t-d}^2\xi_d$$

We would miss three non-standard features of the error variance-covariance:

- Panel heteroskedasticity, from $\eta_i$, the unit random effect in the variance function
- Contemporaneous correlation, from $\zeta_t$, the time random effect in the variance function
- Conditional heteroskedasticity: $\lambda_i$ and $\xi_i$ make the variance time dependent

Thankfully, few reasonable models are this complex. . .
Suppose we think this AR(1) with panel heteroskedasticity is appropriate:

\[ y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_N \left(0, \sigma^2_i \right) \]

\[ \mu_{it} = \alpha_i + x_{it} \beta + y_{i,t-p} \phi \]

\[ \sigma^2_i = \exp(\eta_i) \]

Only source of heteroskedasticity is now \( \eta_i \):
panel heteroskedasticity, not dynamic heteroskedasticity

We could switch this to contemporaneous correlation, by swapping \( \zeta_t \) for \( \eta_i \)

Roughly the model Beck & Katz advocate as a baseline for comparative politics

Suggest estimating by LS then correcting se’s for omission of \( \eta_i \) & contemp. corr.

This procedure yields “panel-corrected standard errors”, PCSEs

What are they, and how do we compute them?
What would we do if we had a plain-vanilla cross-sectional regression and suspected or detected heteroskedasticity?

Recall the standard errors from LS are the square roots of the diagonal elements of

$$\hat{V}(\hat{\beta}) = \hat{\sigma}^2 (X'X)^{-1}$$

So if $\sigma^2$ varies by $i$, these will be badly estimated.
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$$\hat{\sigma}_i^2 = \hat{\epsilon}_i^2$$

A “heteroskedasticity robust” formula for the Var-Cov matrix follows:

$$\hat{V}(\hat{\beta}) = (X'X)^{-1} \left( \sum_i \hat{\epsilon}_i x_i' x_i \right) (X'X)^{-1}$$

The standard errors of our parameters ($\beta$’s) are the square roots of the diagonal of this matrix.
Review: Adjusting standard errors for heteroskedasticity

\[ \hat{V}(\hat{\beta}) = (X'X)^{-1} \left( \sum_i \hat{\varepsilon}_i x_i' x_i \right) (X'X)^{-1} \]

SE’s calculated from this equation are known by many names:

- Huber-White standard errors
- Robust standard errors
- Sandwich standard errors
- Heteroskedasticity consistent standard errors

If you have a single time series, Newey-West standard errors generalize this concept to include robustness to serial correlation.

For panel data there are many further options, leading to a vast literature exploring refinements to this basic concept.
Panel-corrected standard errors

To calculate panel-corrected standard errors, we need to estimate the correct variance-covariance matrix.

Why not just use Huber-White? That would ignore panel structure, which is inefficient if we know how that structure affects heteroskedasticity:

\[
\hat{\sigma}^2_{it} = \hat{\epsilon}^2_{it}
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$$\hat{\sigma}^2_{it} = \hat{\epsilon}^2_{it}$$

Beck and Katz’s panel correction produces sharper estimates of $\hat{\sigma}^2_{it}$ by borrowing strength across the observations from a single unit:

$$\hat{\sigma}^2_{it} = \hat{\sigma}^2_{i} = \frac{1}{T}(\hat{\epsilon}^2_{i,1} + \hat{\epsilon}^2_{i,2} + \cdots + \hat{\epsilon}^2_{i,T})$$
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Beck and Katz’s panel correction also accounts for contemporaneous correlations across units:

\[ \hat{\sigma}_{i,j} = \frac{1}{T}(\hat{\varepsilon}_{i,1}\hat{\varepsilon}_{j,1} + \hat{\varepsilon}_{i,2}\hat{\varepsilon}_{j,2} + \cdots + \hat{\varepsilon}_{i,T}\hat{\varepsilon}_{j,T}) \]

Note the above will work better if \( T \) is large relative to \( N \)
Panel-corrected standard errors

Building this intuition out into a variance-covariance matrix involves a bit of algebra.

To make PCSEs, suppose the variance-covariance matrix $\Omega$ is $NT \times NT$ block-diagonal with an $N \times N$ matrix $\Sigma$ of contemporaneous covariances on diagonal.
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In other words, allow for unit or contemporaneous heteroskedasticity that stays the same over time

Visualizing this large matrix is tricky

Note that “$NT \times NT$ block-diagonal” means we are ordering the observations first by time, then by unit (reverse of our usual practice)
Panel-corrected standard errors

\[
\Omega_{NT \times NT} =
\]
\[
\begin{bmatrix}
\sigma^2_{\varepsilon_1} & \ldots & \sigma_{\varepsilon_1, \varepsilon_i} & \ldots & \sigma_{\varepsilon_1, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\sigma_{\varepsilon_i, \varepsilon_1} & \ldots & \sigma^2_{\varepsilon_i} & \ldots & \sigma_{\varepsilon_i, \varepsilon_i} & \ldots & \sigma_{\varepsilon_i, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\sigma_{\varepsilon N, \varepsilon 1} & \ldots & \sigma_{\varepsilon N, \varepsilon i} & \ldots & \sigma^2_{\varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma^2_{\varepsilon_1} & \ldots & \sigma_{\varepsilon_1, \varepsilon_i} & \ldots & \sigma_{\varepsilon_1, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma_{\varepsilon i, \varepsilon 1} & \ldots & \sigma^2_{\varepsilon i} & \ldots & \sigma_{\varepsilon i, \varepsilon i} & \ldots & \sigma_{\varepsilon i, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma_{\varepsilon N, \varepsilon 1} & \ldots & \sigma_{\varepsilon N, \varepsilon i} & \ldots & \sigma^2_{\varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma^2_{\varepsilon_1} & \ldots & \sigma_{\varepsilon_1, \varepsilon_i} & \ldots & \sigma_{\varepsilon_1, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma_{\varepsilon i, \varepsilon 1} & \ldots & \sigma^2_{\varepsilon i} & \ldots & \sigma_{\varepsilon i, \varepsilon i} & \ldots & \sigma_{\varepsilon i, \varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ldots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & \sigma_{\varepsilon N, \varepsilon 1} & \ldots & \sigma_{\varepsilon N, \varepsilon i} & \ldots & \sigma^2_{\varepsilon N} & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ 
\end{bmatrix}
Panel-corrected standard errors

Instead, suppose $\Omega$ is $NT \times NT$ block-diagonal
with an $N \times N$ matrix $\Sigma$ of contemporaneous covariances on diagonal
Panel-corrected standard errors

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In other words, allow for unit or contemporaneous heteroskedasticity that stays the same over time.

Beck and Katz (1995) estimate $\Sigma$ using LS residuals $e_{i,t}$:

$$\hat{\Sigma}_{i,j} = \sum_{t=1}^{T} \frac{e_{i,t} e_{j,t}}{T}$$

And then use $\hat{\Sigma}$ to construct the covariance matrix.
Panel-corrected standard errors

Monte Carlo experiments show panel-corrected standard errors are “correct” unless contemporaneous correlation is very high or $T$ is small relative to $N$ (Note: alternative is to estimate random effects in variance by ML.)

Beck and Katz suggest using LS with PCSEs and lagged DVs as a baseline model

Most practitioners think fixed effects should also be used

Most important: getting the right lag structure & including FE\text{s} where appropriate

PCSE\text{s} (or other var-cov correction) is a second-order concern

In R, package pcse will calculate PCSE\text{s} for a linear regression

Even easier: In the plm package, vcovBK() will produce a panel corrected var-cov matrix from a plm object

If $N$ is large relative to $T$, consider the Driscoll and Kraay alternative, vcovSCC()
Panel-corrected standard errors: Application

Let’s apply Beck-Katz PCSEs to our panel ARIMA/plm example: we’ll replace the usual variance-covariance matrix with the panel corrected variance covariance matrix.

We must make this substitution manually after estimation to get corrected standard errors, confidence intervals, and var-cov matrices:

1. to print the `summary()` of a `plm` model

   *Example:* `summary(plm.res, .vcov=vcovBK(plm.res))`

2. to use the `coeftest()` function on a `plm` model

   *Example:* `coeftest(plm.res, .vcov=vcovBK(plm.res))`

3. to simulate parameters with `mvrnorm()` for computing counterfactuals

   *Example:* `mvrnorm(10000, coef(plm.res), vcovBK(plm.res))`
<table>
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<tr>
<th></th>
<th>RE</th>
<th>FE</th>
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</table>
Recall the fixed effects results.

These are uncorrected for panel heteroskedasticity or contemporaneous correlation.
Panel correction usually makes little difference in long $T$ small $N$ contexts.

But in short $T$, robust standard errors can be quite important...
Heteroskedastic and serial correlation consistent Var-Cov

In the PCSEs approach, the focus is on panel heteroskedasticity.

It is assumed that serial correlation has been adequately modeled and purged.

A reasonable check when we have a few dozen periods of data, though similar in most cases to either ordinary SEs or White SEs.

But what if we have a low $T$? We might be more worried about residual serial correlation (and don’t have practical access to ARMA diagnostics or fitting).

Now there is more need for a correction to the variance covariance that corrects for observed error correlation across units and across periods.

Arellano (1987) provides a heteroskastic and autocorrelation consistent variance-covariance matrix: in plm, `vcovHC()`

Use the same commands as above, but with `vcovHC()` instead of `vcovBK()`.

Particularly important to correct with panel GMM estimators.
Our prior results for cigarette taxes used the Arellano heteroskedastic and serial correlation consistent var-cov matrix.

What would happen if we had used the ordinary, homoskedastic var-cov matrix to compute CIs?
The effects sizes are mostly unchanged: adjustments to standard errors affect CIs, not point estimates.

But the CIs are radically different under the traditional var-cov estimator.

Far too small (invisible even!) for the misspecified linear models.

And too large for the more correctly specified log-log models!
Just as panel GMM point estimates are sensitive to assumptions, so are the standard errors.

Use caution, and prefer `vcovHC()` to `vcov()` in PGMM models.

Be sure to check which var-cov matrix your functions are using: the default may be wrong!