Panel Data Models with Many Time Periods

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Plan for today

Estimating deterministic trends in panel data

Performing panel unit root tests

Estimating linear panel models with large(ish) $T$
Estimating Deterministic Trends

Recall that for a single time series, we estimated the following model to capture a deterministic trend:

\[ y_t = t\theta_1 + x_i\beta + \varepsilon_t \]

For panel data, if we assume a common deterministic trend across units we can estimate:

\[ y_{it} = t\theta_1 + x_{it}\beta + \varepsilon_{it} \]

We could also add in ARMA terms if we like, as they capture distinct time series dynamics.

As before, the question is whether we can trust \( \hat{\theta} \) to estimate the trend well.

With a single, short time series, \( \hat{\theta} \) will be unbiased but often far from the truth and frequently incorrectly signed.

Let’s see whether assuming a common trend within a panel helps.
Estimating Deterministic Trends

Let’s see whether assuming a common trend within a panel helps
Revist our Monte Carlo experiment to see how well $\hat{\beta}_1$ estimates $\beta_1$ in practice.

We set the true model to:

\[ y_{it} = \beta_0 + \beta_1 t + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma^2) \]
\[ y_{it} = 0 + 0.1t + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, 1) \]

Then, for each $t \in \{2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 50, 100\}$, we draw
100,000 datasets from this “true” model and see how well we estimate $\beta_1 = 0.1$

We repeat the experiment for $N = \{1, 2, 5, 10, 20, 50, 100\}$ and compare results
Recall that for a single time series, we got very unreliable estimates and frequent sign errors at $T = 10$ (dotted red line).

Not biased: the mean of the distribution is centered on the truth, $\beta_1 = 0.1$, but not reliable in any specific dataset.

More data helped: vastly more efficient estimates at $T = 100$ (purple spike).

What happens if we had 100 observations, but spread across units?
What happens if we had 100 observations, but spread across units?

The dotted red line still marks the performance of trend estimation with a single time series of 10 periods.

We can improve quite a bit by pooling 10 cross-sections in the same model.
Note the 10 cross-sections of 10 periods case is not as efficient as 100 observations in a single series

Choices:

* either assume the series has a very long stable trend (and find a long data set)
* or assume different cross-sections follow the same trend (and pool across short time periods)
Let’s get more systematic. Recall these Monte Carlo results for a single time series
Efficient trend estimation gets much easier with more units and fixed $T$. 

Mean absolute error in least squares trend estimate given $T$ periods of data, with $trend=0.1$ and $\sigma=1.0$. 

- $N=1$: MAE decreases rapidly with increasing $T$.
- $N=2$: Similar to $N=1$, but the slope is gentler.
- $N=5$: The decrease in MAE with $T$ is even more gradual.
- $N=10$, $N=20$, $N=50$, $N=100$: The MAE continues to decrease, but at a slower rate as $N$ increases.

Magnitude of true trend is constant across all $N$ values, indicating that the magnitude of the trend does not affect the MAE in the least squares trend estimate for fixed $T$. 

The graph shows that as the number of units $N$ increases, the mean absolute error (MAE) decreases more rapidly with increasing periods modeled $T$, especially for smaller values of $N$. This indicates that efficient trend estimation becomes easier with more units and fixed $T$. 

For example, when $N=1$, the MAE decreases sharply with increasing $T$, indicating that with just one unit, the error decreases quickly with more data. As $N$ increases, the rate of decrease in MAE with $T$ decreases, but the overall MAE is still significantly lower compared to $N=1$. 

This trend is consistent across all values of $T$, showing that the effectiveness of trend estimation improves with more units, regardless of the magnitude of the true trend.
When $N$ is really large – e.g., individual data in the 1000s – can efficiently estimate trends in very small $T$, provided pooling assumption is valid.
Recall how frequently we got the direction of the trend wrong with a single, short series.
Sign errors rapidly diminish with higher $N$ and even modest $T$. 

Rate of sign errors in least squares trend estimate given $T$ periods of data 

trend=0.1 sigma=1.0
Rate of sign errors in least squares trend estimate given T periods of data
trend=0.1    sigma=1.0

But you might want to check your own case in Monte Carlos that match your assumptions
Panel Unit Root Tests

Deterministic trend estimation was hard in single time series, but easier when we pooled $N > 1$ time series together and assumed a common trend.

Unit root tests were underpowered in a single time series. . . Perhaps pooling time series and assuming degree of stationarity will help?

Panel unit root tests do exactly this, and are considerably more powerful:

Most assume balanced panels (and R implementation in plm::purtest seems to impose this globally)

See also the R package punitroots
**Panel Unit Root Tests**

Generally, these tests find ways to combine the results from individual ADF tests on the different series in different ways.

Most treat nonstationarity as the null hypothesis.

A few well-known tests:

- **Im-Pesaran-Shin (2003) test**: Pools the units; allows different AR processes.
- **LevinLinChu (2002) test**: Pools the units; assumes common AR process.
- **Maddala-Wu (1999) test**: Pools the $p$-values from separate ADFs; more flexible.
- **Hadri (2000) test**: Also more flexible test; null is stationarity.

Although performance is better than single-series ADF, panel unit root tests make a variety of complex identifying assumptions, and those assumptions can lead to different results.

Care is still indicated in using these tests: they aren’t foolproof or guaranteed to be right.
Estimating Linear Panel Models

Last time, we discussed how including random and/or fixed effects changes the properties of our estimators of $\beta$.

In this lecture, we’ll talk about how to estimate and interpret panel models using fixed and/or random effects.

And how to decide if we need (or even can use) fixed effects.

We can always add random effects, but in some cases FEs either be too costly to estimate (in terms of dfs), or simply impossible to estimate.

We will consider first the small $N$, large $T$ case, which allows more complex time series modeling.

Then the large $N$, small $T$ case, which raises the possibility of bias in fixed effects estimation.

Finally, we consider heteroskedasticity in time or across panel structures.
Estimating Fixed Effects Models

Option 1: Fixed effects or “within” estimator:

\[ y_{it} - \bar{y}_i = (x_{it} - \bar{x}_i)\beta + (u_{it} - \bar{u}_i) \]

- estimating the fixed effects by differencing them out before applying least squares
- including time-invariant variables directly in \( x_{it} \) impossible here
- (rarely usable workaround: if we have an instrument for the time-invariant variable that is uncorrelated with the fixed effects; see Hausman-Taylor)
- suggests a complementary “between” estimator of \( \bar{y}_i \) on \( \bar{x}_i \) which could include time-invariant \( x_i \); together these models explain the variance in \( y_{it} \)
- does not actually provide estimates of the fixed effects themselves; just purges them from the model to remove omitted time-invariant variables
- to recover the fixed effects, could compute \( \hat{\alpha}_i = y_{it} - x_{it} \hat{\beta} \)
Estimating Fixed Effects Models

Option 2: Dummy variable estimator (sometimes called LSDV)

\[ y_{it} = x_{it} \beta + \alpha_i + u_{it} \]

- also estimated by least squares (hence Least Squares Dummy Variable)
- yields estimates of $\alpha_i$ fixed effects (may be useful in quest for omitted variables; see if the $\alpha_i$ look like a variable you know)
- for large $T$, should be very similar to FE estimator
- not a good idea for very small $T$: estimates of $\alpha_i$ will be poor
Time-Invariant Covariates & Fixed Effects

We can’t include time-invariant variables in fixed effects models.

If we do, we will have perfect collinearity, and can’t get estimates.

That is, we will get some parameter estimates equal to NA.

Never report a regression with NA parameters.

The regression you tried to run was impossible. Start over with a possible one.
Time-Invariant Covariates & Fixed Effects

If we can’t include time-invariant variables in a fixed effects model, does that mean time-invariant variables can never explain changes over time?

You might think so: how can a constant explain a variable?

But time-invariant variables could still effect time-varying outcomes in a special way...

Time-invariant variables can influence how a unit weathers time-varying shocks in some other variable

Example: labor market regulations (e.g. employment protection) don’t change much over time

Blanchard & Wolfers found that when a negative economic shock hits, unemployment may rebound more slowly where such protections are stronger
Time-Invariant Covariates & Fixed Effects

We can model how a slow moving or time-invariant covariate conditions the effect of a quickly changing covariate on $y_{it}$

To estimate how a time-invariant covariate $x_i$ mediates the effect of a shock, $s_{it}$, include on the RHS $x_i \times s_{it}$ and $s_{it}$, while omitting $x_i$ itself.

(It’s okay and necessary to omit the $x_i$ base term in this special case, because $\alpha_i$ already captures the effect of $x_i$)

Many theories about institutions can be tested this way
Time-Invariant Covariates & Fixed Effects

What if we want to “include” time-invariant covariates’ effect on the long term average level of $y$?

We might partition the fixed effect into:

1. the portion “explained” by known time-invariant variables and

2. the portion still unexplained

Plümper & Troeger have methods to do this.

In this case, our estimates of the time-invariant effects are vulnerable to omitted variable bias from unmeasured time-invariant variables, even though time varying variables in the model are not.

Thus you now need to control for *lots* of time-invariant variables directly, even hard to measure ones like culture.
Estimating Random & Mixed Effects Models

Estimation of random effects is by maximum likelihood (ML) or generalized least squares (GLS).

Technically we’re just adding one parameter to estimate: the variance of the random effects, $\sigma^2_\alpha$.

This is partitioned out of the overall variance, $\sigma^2$.

Can understand this most easily by abstracting away from time series for a moment.
Recall that for linear regression, we assume homoskedastic, serially uncorrelated errors, and thus a variance-covariance matrix like this:

\[ \Omega = \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix} \]
Estimating Random & Mixed Effects Models

And recall that heteroskedastic (but serially uncorrelated) errors have this variance-covariance matrix

\[
\Omega = \begin{bmatrix}
\sigma_1^2 & 0 & 0 & 0 \\
0 & \sigma_2^2 & 0 & 0 \\
0 & 0 & \sigma_3^2 & 0 \\
0 & 0 & 0 & \sigma_4^2
\end{bmatrix}
\]
Estimating Random & Mixed Effects Models

And finally, remember heteroskedastic, serially correlated errors follow this general form of variance-covariance

$$\Omega = \begin{bmatrix}
\sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
\sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\
\sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2
\end{bmatrix}$$

What does this matrix look like for random effects with no serial correlation?
Define the variance of the random effect as

$$E(\alpha_i^2) = \sigma_{\alpha}^2 = \text{var}(\alpha_i)$$

Define the expected value of the squared white noise term as $\sigma_{\varepsilon}^2$

$$E(\varepsilon_{it}^2) = \sigma_{\varepsilon}^2 = \text{var}(\varepsilon_{it})$$

White noise is serially uncorrelated, so has covariance 0 for $t \neq s$:

$$E(\varepsilon_{it}\varepsilon_{is}) = 0 = \text{cov}(\varepsilon_{it}, \varepsilon_{is})$$

Finally, note that we assumed the white noise error and random effect are uncorrelated,

$$E(\alpha_i\varepsilon_{it}) = 0 = \text{cov}(\alpha_i, \varepsilon_{it})$$
Thus the variance of the whole random component of the model is

\[ E((\alpha_i + \varepsilon_{it})(\alpha_i + \varepsilon_{it})) = E(\alpha_i^2) + 2E(\alpha_i \varepsilon_{it}) + E(\varepsilon_{it}^2) = \sigma_\alpha^2 + 0 + \sigma_\varepsilon^2 = \sigma_\alpha^2 + \sigma_\varepsilon^2 \]

And the covariance of the whole random component is:

\[ E((\alpha_i + \varepsilon_{it})(\alpha_i + \varepsilon_{is})) = E(\alpha_i^2) + E(\alpha_i \varepsilon_{is}) + E(\alpha_i \varepsilon_{it}) + E(\varepsilon_{it} \varepsilon_{is}) = \sigma_\alpha^2 + 0 + 0 + 0 = \sigma_\alpha^2 \]
Estimating Random & Mixed Effects Models

If our data have a single random effect in the mean for each unit → serially correlated errors, but expressable using only two variances:

- the random effects variance $\sigma^2_\alpha$
- the white noise term’s variance $\sigma^2_\varepsilon$

\[
\Omega = \begin{bmatrix}
\sigma^2_\alpha + \sigma^2_\varepsilon & \sigma^2_\alpha & \sigma^2_\alpha & \sigma^2_\alpha \\
\sigma^2_\alpha & \sigma^2_\alpha + \sigma^2_\varepsilon & \sigma^2_\alpha & \sigma^2_\alpha \\
\sigma^2_\alpha & \sigma^2_\alpha & \sigma^2_\alpha + \sigma^2_\varepsilon & \sigma^2_\alpha \\
\sigma^2_\alpha & \sigma^2_\alpha & \sigma^2_\alpha & \sigma^2_\alpha + \sigma^2_\varepsilon
\end{bmatrix}
\]
Estimating Random & Mixed Effects Models

We have drastically simplified this matrix, and can now use FGLS (Feasible Generalized Least Squares) or ML to estimate it

$$\hat{\beta}_{\text{GLS}} = \left( \sum_{i=1}^{N} X_i' \Omega^{-1} X_i \right)^{-1} \left( \sum_{i=1}^{N} X_i' \hat{\Omega}^{-1} y_i \right)$$

where $X_i$ is the $T \times K$ matrix of covariates for unit $i$, all times $t = 1, \ldots T$, and all $K$ covariates

All we need are the estimates $\hat{\sigma}_\alpha^2$ and $\hat{\sigma}_\varepsilon^2$, and we can calculate $\hat{\beta}_{\text{GLS}}$
Estimating Random & Mixed Effects Models

We get $\hat{\sigma}_\varepsilon^2$ from the residuals from a LS regression:

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{NT - K} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{itLS}^2$$

(This is the usual estimator, but for $NT$ observations)
Estimating Random & Mixed Effects Models

To get an estimator of $\hat{\sigma}^2_\alpha$, we need to adjust for the fact that we have only so many unique pairs of errors to compare:

$$\hat{\sigma}^2_\alpha = E \left( \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} (\alpha_i + \varepsilon_{it})(\alpha_i + \varepsilon_{is}) \right)$$

$$= E \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} ((\alpha_i + \varepsilon_{it})(\alpha_i + \varepsilon_{is}))$$

$$= \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \sigma^2_\alpha$$

$$= \sigma^2_\alpha \sum_{t=1}^{T-1} (T - t)$$

$$= \sigma^2_\alpha ((T - 1) + (T - 2) + \ldots + 2 + 1)$$
Estimating Random & Mixed Effects Models

\[ = \sigma_\alpha^2((T - 1) + (T - 2) + \ldots + 2 + 1) \]
\[ = \sigma_\alpha^2 T(T - 1)/2 \]

\[ \hat{\sigma}_\alpha^2 = \frac{1}{NT(T - 1)/2 - K} \sum_{i=1}^{N} \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \hat{\epsilon}_{it} \hat{\epsilon}_{is} \]

where in the last step we replace \( \sigma_\alpha^2 \) with its estimator from pooled LS (the average of the products of the unique pairs of residuals)

With some algebra, this approach extends to serial correlaton of other kinds (ARMA)

For complex models, with many levels and/or hyperparameters, best to go Bayesian, set diffuse priors on the parameters, and use MCMC
Selecting Fixed Effects vs Random Effects Models

Choosing random effects when $\alpha_i$ is actually correlated with $x_{it}$ will lead to omitted variable bias.

Choosing fixed effects when $\alpha_i$ is really uncorrelated with $x_{it}$ will lead to inefficient estimates of $\beta$ (compared to random effects estimation) and kick out our time-invariant variables.

Often in comparative we are certain there are important omitted time invariant variables (culture, unmeasured institutions, long effects of history).

So choice to include fixed effects requires nothing more than theory.

Still could include random effects in addition to the fixed effects.
Selecting Fixed Effects vs Random Effects Models

But if we are uncertain, or want to check against estimating unnecessary fixed effects, we can use the Hausman test for (any) fixed effects versus just having random effects.

Hausman sets up the null hypothesis of random effects.

Attempts to reject it in favor of fixed effects.

Checks whether the random $\alpha_i$'s are correlated with $x_i$ under the null.

Does this by calculating the variance-covariance matrices of regressors under FE and then just RE.

Null is no correlation between these covariances.

If there is no correlation, that means the regressors do not predict the random effects (ie, are uncorrelated).

Rejecting the null suggests you may need fixed effects to deal with omitted variable bias.

phtest in plm library.
Interpreting Random Effects Models

Usually, interest focuses on the percentage of variance explained by the random effects

And how this variance compares to that remaining in the model

Reported by your estimation routine
What if $T$ is very small?

If $T$ is very small ($< 15$ perhaps), estimating panel dynamics efficiently and without bias gets harder.

In these cases, we should investigate alternatives:

1. First differencing the series to produce a stationary, hopefully white noise process

2. Including fixed effects for the time period (time dummies)

3. Checking for serial correlation after estimation (LM test)

4. Using lags of the dependent variable, while removing the bias from including lags with fixed effects by instrumenting with lagged differences (Arellano-Bond)
Example: GDP in a panel

Let’s use the Przeworski et al democracy data to try out our variable intercept models

This exercise is for pedagogical purposes only; the models we fit are badly specified

We will investigate the following model:

$$Δ^d GDP_{it} = α_i + β_1 OIL_{it} + β_2 REG_{it} + β_3 EDT_{it} + ν_{it}$$

- where $ν_{it} \sim \text{ARMA}(p, q)$,
- $d$ may be 0 or 1, and
- $α_i$ may be fixed, random, or a mixed
Example: GDP in a panel

We first investigate the time series properties of GDP

But we have $N = 113$ countries! So we would have to look at 113 time series plots, 113 ACF plots, and 113 PACF plots

Fortunately, they do look fairly similar. . .
GDP time series for country 1
GDP time series for country 2
GDP time series for country 3
GDP time series for country 4
GDP time series for country 113
Series GDPW[COUNTRY == currcty]

GDP ACF for country 1
Series GDPW[COUNTRY == currcty]

GDP ACF for country 2
GDP ACF for country 3
Series GDPW[COUNTRY == currcty]

GDP ACF for country 4
GDP ACF for country 113
Series GDPW[COUNTRY == currcty]

GDP PACF for country 1
Series GDPW[COUNTRY == currcty]

GDP PACF for country 2
Series GDPW[COUNTRY == currcty]

GDP PACF for country 3
Series GDPW[COUNTRY == currcty]

GDP PACF for country 4
Series GDPW[COUNTRY == currcty]

GDP PACF for country 113
Histogram of \textit{p}-values from ADF tests on GDPW

What would we see if there were no unit roots?
Histogram of $p$-values from Phillips-Peron tests on GDPW
Choosing AR(p,q) for panel

What do we think?

Clearly some heterogeneity

If had to pick one time series specification, choose ARIMA(0,1,0) or ARIMA(1,1,0)

Seems to fit many cases; guards against spurious regression

But if we’re dubious about imposing a single ARIMA(p,d,q) across units, we could let them be heterogeneous
GDPdiff time series for country 1
GDPdiff time series for country 2
GDPdiff time series for country 3
GDPdiff time series for country 4
GDP diff time series for country 113
GDPdiff ACF for country 1
Series GDPWdiff[COUNTRY == currcty]

GDPdiff ACF for country 2
Series GDPWdiff[COUNTRY == currcty]

GDPdiff ACF for country 3
Series GDPWdiff[COUNTRY == currcty]

GDPdiff ACF for country 4
Series GDPWdiff[COUNTRY == currcty]

GDPdiff ACF for country 113
Series GDPWdiff[COUNTRY == currcty]

GDPdiff PACF for country 1
Series \( \text{GDPWdiff[COUNTRY == currcty]} \)

GDPdiff PACF for country 2
Series  GDPWdiff[COUNTRY == currcty]

GDPdiff PACF for country 3
Series GDPWdiff[COUNTRY == currcty]

GDPdiff PACF for country 4
Series  GDPWdiff[COUNTRY == currcty]

GDPdiff PACF for country 113
Histogram of $p$-values from ADF tests on GDPWdiff

What is this pattern consistent with?
Histogram of PPtestdiff.pvalues

Histogram of $p$-values from Phillips-Peron tests on GDPWdiff
Example continued in R demonstration

We will continue this example in section using the code provided.

For now, let’s focus on the results that emerge, and how they depend on treating intercepts as either random or fixed by country.

In particular, we want to see if fixed effects can help us with omitted time invariant variables, which are legion in this example.

In the example, we will decide on an ARIMA(1,1,0) model of GDP (What does this mean?)

We will fit three different models of the relationship between education and GDP:

1. ARIMA(1,1,0) with random country intercepts and controls for oil producing countries and democracy.

2. ARIMA(1,1,0) with fixed country intercepts and controls for democracy.

3. ARIMA(1,1,0) with “mixed” country intercepts and controls for democracy.
<table>
<thead>
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<th></th>
<th>RE</th>
<th>FE</th>
<th>Model FE-pcse</th>
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What does the model imply substantively, and how does this depend on model assumptions?
Suppose we have a country with “average” characteristics, and we increase education to 1 sd above the mean.

How much does the model predict education to rise over the following years?
The above plot shows the expected change in GDP over time in the high education country relative to an average (untreated) country.
The result seems sensible, but the model:
(1) ignored many unmeasured confounders and (2) differences GDP,
so we should be skeptical in both the short- and long-run
Adding fixed effects completely flips the results for education

Now the results make little sense
(Suggests the model is badly identified, even with fixed effects)
In a model with both RE and FE for countries, the FEs dominate, as the “Mixed” effects model shows.