Network Analysis and Applications in Biology: Introduction

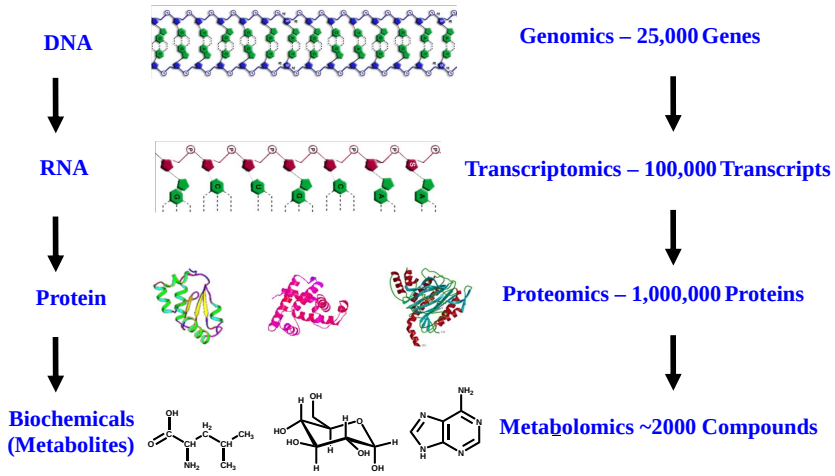
Ali Shojaie & George Michailidis

ENAR 2020

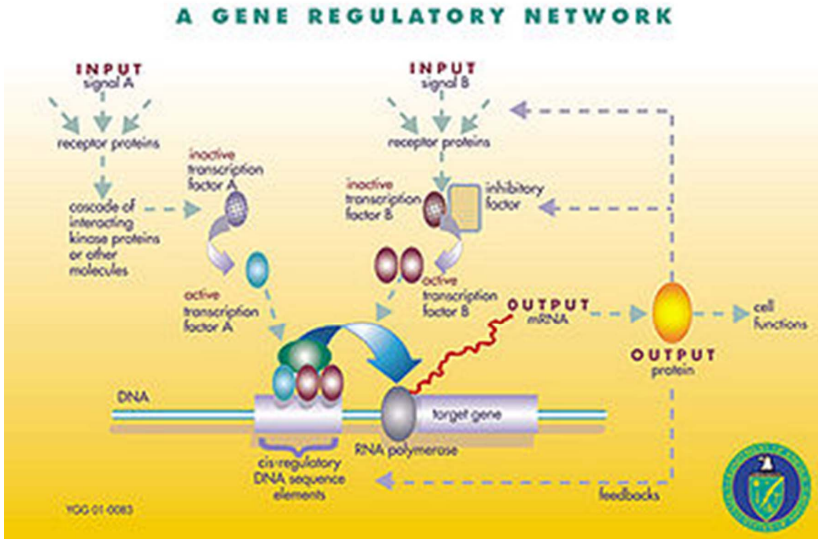
Why Study Networks?

- Components of biological systems (genes, proteins etc) interact with each other to carry out cell functions.
- Examples of such interactions include signaling, regulation and interactions between proteins.
- We cannot understand the function and behavior of biological systems by studying individual components ($2 + 2 \neq 4$!).
- Networks provide an efficient representation of complex interactions in cells, and a basis for mathematical/statistical models to study these systems.

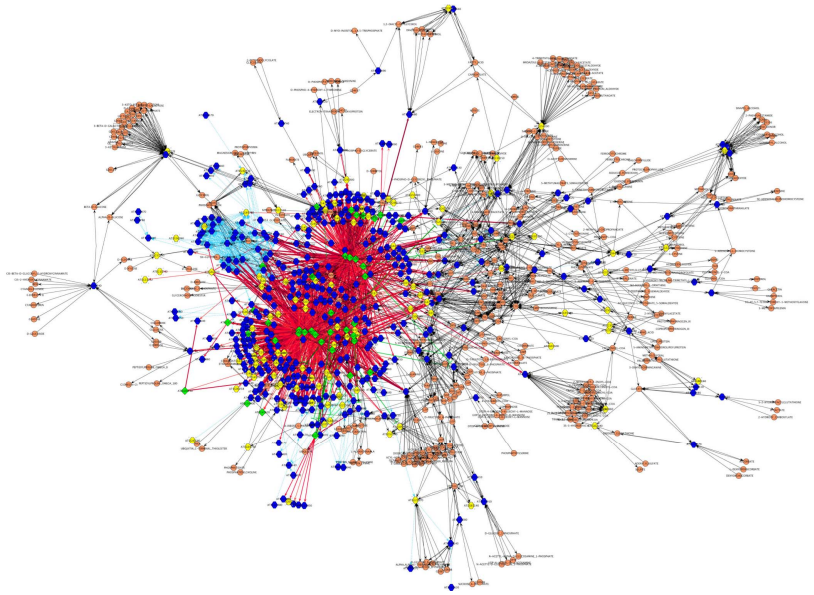
Central Dogma of Molecular Biology (Extended)



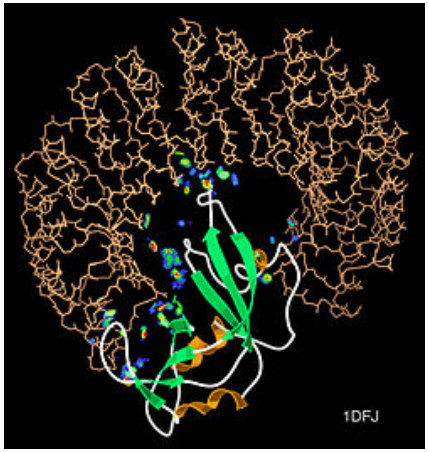
Networks in Biology: Gene Regulatory Interactions



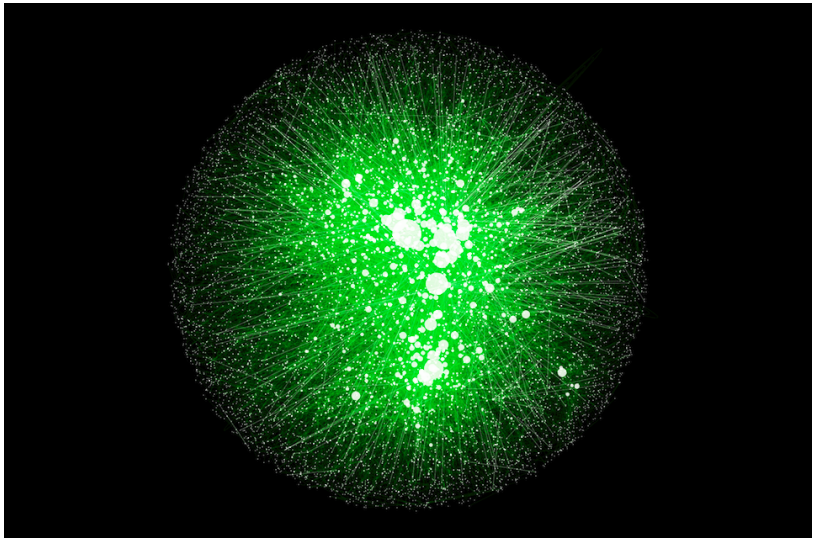
Networks in Biology: Gene Regulatory Networks



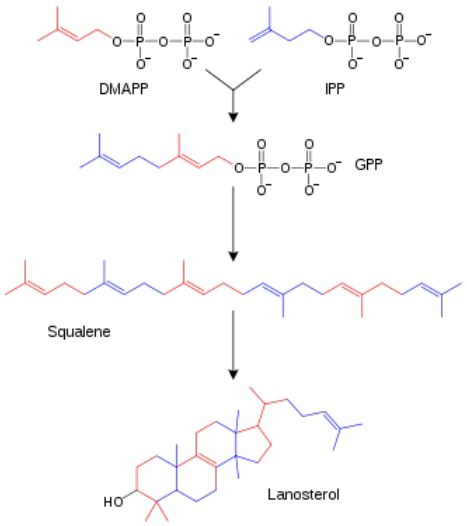
Networks in Biology: Protein-Protein Interaction



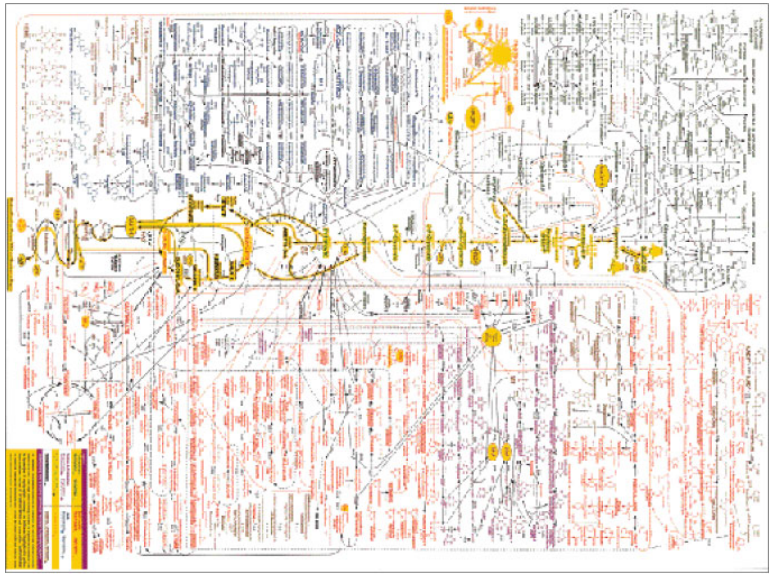
Networks in Biology: Protein-Protein Interactions (PPI)



Networks in Biology: Metabolic Reactions



Networks in Biology: Metabolic Pathways



But Do Networks Matter?

- They Do!
- Recent studies have linked changes in gene/protein networks with many human diseases.

Systems Biology and Emerging Technologies

Gene Networks and microRNAs Implicated in Aggressive Prostate Cancer

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But Do Networks Matter?

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Estrogen-Regulated Gene Networks in Human Breast Cancer Cells: Involvement of E2F1 in the Regulation of Cell Proliferation

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But Do Networks Matter?

Cell PRESS

Cancer Cell
Article

A Transcriptional Signature and Common Gene Networks Link Cancer with Lipid Metabolism and Diverse Human Diseases

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But Do Networks Matter?

And, incorporating the knowledge of networks improves our ability to find causes of complex diseases.

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www.molecularsystemsbiology.com

molecular systems biology

REPORT

Network-based classification of breast cancer metastasis

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Networks: A Short Primer

- A network is a collection of nodes V and edges E.
- We assume there are p nodes in the network, and that the nodes correspond to random variables $X_1, \ldots X_p$.
- Edges can be undirected $X - Y$ or directed $X \to Y$.

- Consider the node set $V = \{1, 2, 3\}$.
- Then edges can be:

$$\text{undirected: } E_1 = \{1 - 2, 2 - 3\}$$
$$\text{directed: } E_2 = \{1 \to 3, 3 \to 2\}$$

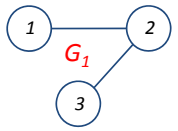
★ We focus primarily on *undirected* networks.

Networks: A Short Primer

- A convenient way to represent the edges of the network is to use an adjacency **matrix** A
- Adjacency matrix is a square matrix, with a **nonzero entry in** (i, j) **and** (j, i) **if there is an edge** between nodes i and j

$$A = \begin{bmatrix} . & \mathbf{x} & . \\ \mathbf{x} & . & . \\ . & . & . \end{bmatrix} \to \mathbf{x} \text{ shows an an edge between 1 and 2}$$

Example:



$$A = \begin{bmatrix} 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & \mathbf{1} \\ 0 & \mathbf{1} & 0 \end{bmatrix}$$

What Do Edges in Biological Networks Mean?

- In gene regulatory networks, an edge from gene i to gene j often means that i controls the expression of j: as i's expression changes, j's expression also increases/decreases.
- In protein-protein interaction networks, an edge between proteins i and j often means that *the two proteins bind together and form a protein complex*. Therefore, we expect that these proteins are generated at similar rates.
- In metabolic networks, an edge between compound i and j often means that *the two compounds are involved in the same reaction*, meaning that they are generated at relative rates.
- Thus, edges represent some type of association among genes, proteins or metabolites, defined generally to include *linear or nonlinear* associations; more later....

Statistical Models for Biological Networks

- We use the framework of graphical models
- In this setting, nodes correspond to "random variables"
- In other words, each node of the network represents one of the variables in the study
  - In gene regulatory networks, nodes $\equiv$ genes
  - In PPI networks, nodes $\equiv$ proteins
  - In metabolic networks, nodes $\equiv$ metabolites
- In practice, we observe n measurements of each of the variables (genes/proteins/ metabolites) for say different individuals, and want to determine which variables are connected, or use their connection for statistical analysis

Our Plan

We will cover the following topics

- Methods for detecting signal on known networks
  - Network analysis based on centrality and clustering
  - Topology-based pathway enrichment analysis
- Methods for learning undirected networks
  - Co-expression networks
  - ARACNE
  - Conditional independence graphs
    - Gaussian observations (`glasso`, etc)
    - Non-Gaussian and non-linear data (`nonparanormal`, etc)
- [Will not discuss methods for **learning directed networks**]

Network Analysis and Applications in Biology: Analysis of Network-Structured Data

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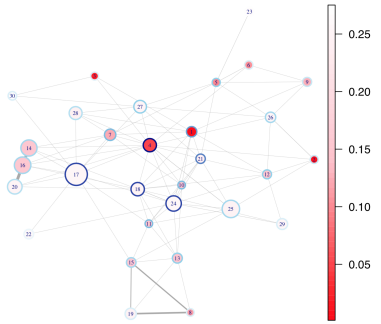
Introduction

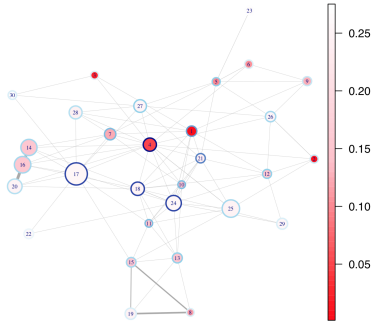
Suppose we observe activities of individual nodes (genes, proteins, brain regions, etc) on a network (gene regulatory network, structural connectivity network, etc)



How can we identify the important nodes?

and what does this even mean?

Identifying Important Nodes



How can we identify the important nodes?

- We can select the significant nodes based on p-values, after adjusting for multiple comparisons (FDR, etc)
- But the signal is often weak for lots of tests
- If we believe the network is informative, it may make sense to use the network to guide our selection

Identifying Important Nodes

Possible strategies:

- Identify individual nodes associated with the outcome by incorporating the network (signal detection on network)
- Test if (pre-specified) subnetworks are associated with the outcome (topology-based pathway enrichment analysis)
- Identify collections of (connected) nodes that are associated with the outcome (*de-novo identification of enriched modules*)

Signal Detection on Networks

Signal Detection on Networks

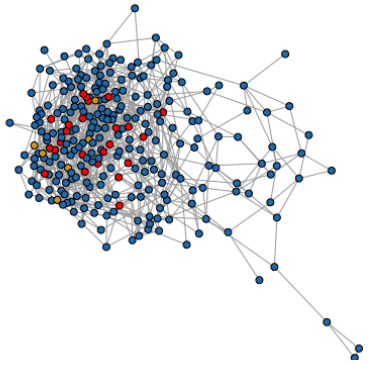
How can we identify the important nodes in a network?

The simplest option is to limit our search/testing to the central nodes in the network:

- Nodes connected to many other nodes, aka hub nodes
- Nodes that are close to many other nodes (closeness)
- Nodes that are on many network paths (betweenness)

Example: Functional Relevance of Hub Nodes

- Inferred genetic interaction network of cancer-related pathway in prostate cancer (data from TCGA)
- Hubs defined as nodes whose degrees are at the 75th percentile of the degree distribution



Other Measures of Centrality

- Closeness: Total distance of each node to other nodes:

$$\text{cl}_j = \left(\sum_{k \in V} d(j,k) \right)^{-1}$$

where $d(j,k)$ is the (shortest path) distance between j and k.

- Betweenness: The number of *paths* that go through a node:

$$\text{bw}_j = \sum_{i \neq j \neq k} \frac{\pi_{ik}(j)}{\pi_{ik}}$$

where $\pi_{ik}(j)$ is the number of paths between i and k that go through j, and π_{ik} is the total number of paths between them.

Identifying "Central" Nodes

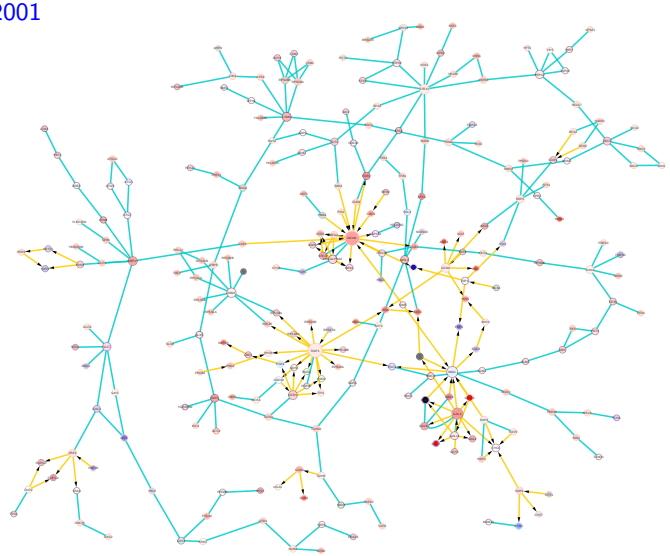
Calculating centrality measures using `igraph`:

- Hub nodes: `hub_score(graph)`
- Closeness: `closeness(graph, vids)`
  - use `estimate_closeness()` for larger networks)
- Betweenness: `betweenness(graph, vids)`
  - use `estimate_betweenness()` for larger networks

Topology-Based Pathway Enrichment Analysis

Yeast GAL Pathway

Ideker et al, 2001



Topology-Based Pathway Enrichment Analysis

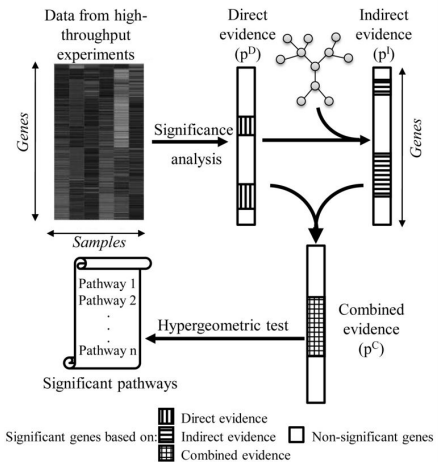
Test for changes in activities of node (genes, brain ROIs, etc) in pre-specified subnetworks, while incorporating network information

Two possible null hypotheses:

- Competitive null hypothesis: activity of each pathway is compared with other pathways, often using a permutation test
  - Assume few genes are differentially connected, and may be sensitive to the choice of gene sets
- Self-contained null hypothesis: activity of each pathway is compared against the null distribution
  - More rigorous, but may be sensitive to modeling assumptions (*Goemen & Buhlmann* (07), *Ackermann & Strimmer* (09))

PathNet[1]

A simple topology-based pathway enrichment method:



PathNet: Details

- Each gene's p-value from differential expression is combined with p-values of its neighbors using Fisher's methods

$$\mathrm{SI}_j = \sum_{k \in \mathrm{ne}(j)} \left\{ -\log_{10}\left(p_k^D\right) \right\}.$$

  - The indirect p-value, p^I is calculated from SI_j by permutation
- Direct (p_j^D) and indirect (p_j^I) p-values are then combined (p_j^C)
- The significance of p_j^C for genes in each pathway is assessed using a hypergeometric test
- Implemented in Bioconductor package `PathNet`

topologyGSA[2]

- topologyGSA (Gene Set Analysis Exploiting Pathway Topology) assumes that data are normally distributed:

$$X^1 \sim N(\mu^1, \Sigma^1), \quad X^2 \sim N(\mu^2, \Sigma^2)$$

- It obtains estimates of Σ^1 and Σ^2 based on the networks (think graphical lasso, but with known nonzero entries)
- It then performs two tests:
  - equality of covariance matrices: $H_0^c : \Sigma^1 = \Sigma^2$
  - equality of means $H_0^m : \mu^1 = \mu^2$ — it uses different methods depending on the result of H_0^c
- Implemented in R-package `topologyGSA` (also in `graphite`)

[2]Massa et al (2010)

Signaling Pathway Impact Analysis (SPIA)[3]

- Combines overrepresentation analysis (ORA) with measure of perturbation of a given pathway under a given condition
- A bootstrap procedure is used to assess the significance of the observed pathway perturbation (difficult to extend to comparison of > 2 conditions)
- Currently not applicable to all pathways (more later)
- Analyzes each pathway separately (ignores connections between pathways)
- Implemented in the Bioconductor package `SPIA`

[3]Tarca et al (2009)

The SPIA Methodology

SPIA combines two types of evidence

(i) the overrepresentation of DE genes in a given pathway

- measured by the p-value for the given number of DE genes $P_{NDE} = P(X \geq N_{DE} \mid H_0)$

The SPIA Methodology

SPIA combines two types of evidence

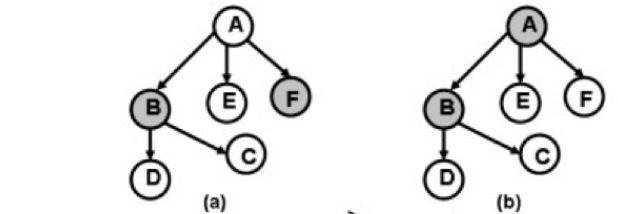
(ii) the abnormal perturbation of the pathway

- the perturbation for each gene in the pathway is defined as $PF(g_i) = \Delta E(g_i) + \sum_{j=1}^{p} \beta_{ij} \frac{PF(g_j)}{N_{DS}(g_j)}$
  - $PF(g_i)$ is the perturbation factor of gene i (not known)
  - β_{ij} is the magnitude of effect of gene j on gene i; currently, $beta_{ij} = 1$ if $j \rightarrow i$
  - $\Delta E(g_i)$ is the fold change in expression of gene i
  - $N_{DS}(g_j)$ is the number of downstream genes from gene j

The SPIA Methodology

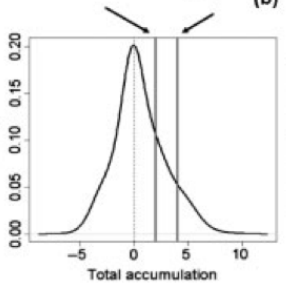
- The accumulated activity of each gene can then be calculated as $ACC(g_i) = B \cdot (I - B)^{-1} \Delta E$
  - B is the normalized matrix of β's: $B_{ij} = \beta_{ij} / N_{DS}(g_j)$
  - ΔE is the vector of fold changes
  - Requires B to be invertible; would not work otherwise
- The total accumulated perturbation of the pathway is then given by $t_A = \sum_i ACC(g_i)$
- The p-value for pathway perturbation is given by $P_{PERT} = P(T_A \geq t_A \mid H_0)$, which is calculated using a bootstrap approach

The SPIA Methodology



Gene	ΔE	PF	Acc
A	0	0	0
B	2	2	0
C	0	1	1
D	0	1	1
E	0	0	0
F	4	4	0
Total			2.0

P_{PERT}=0.57



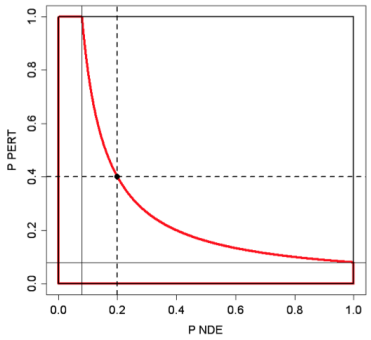
Gene	ΔE	PF	Acc
A	1.5	1.5	0
B	2	2.5	0.5
C	0	1.25	1.25
D	0	1.25	1.25
E	0	0.5	0.5
F	0	0.5	0.5
Total			4.0

P_{PERT}=0.24

The SPIA Methodology

SPIA combines two types of evidence

- The final p-value for each pathway is calculated based on the p-values from parts (i) and (ii):
  - $P_G(i) = c_i - c_i \ln(c_i)$
  - $c_i = P_{NDE}(i) P_{PERT}(i)$



An Example in R: Data on Colorectal Cancer

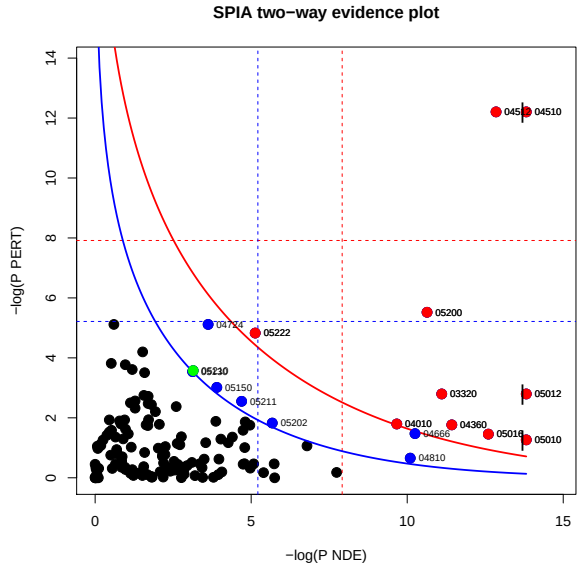
```
data(colorectalcancer)

#pathway analysis using SPIA
#use nB=2000 or higher for more accurate results
#uses older version of KEGG signaling pathways graphs
res <- spia(de=DE_Colorectal, all=ALL_Colorectal, organism="hsa", beta=NULL,
    nB=2000, plots=FALSE, verbose=TRUE, combine="fisher")

#now combine pNDE and pPERT using the normal inversion method without
#running spia function again
res$pG=combfunc(res$pNDE,res$pPERT,combine="norminv")
res$pGFdr=p.adjust(res$pG,"fdr")
res$pGFWER=p.adjust(res$pG,"bonferroni")
plotP(res,threshold=0.05)

#highlight the colorectal cancer pathway in green
points(I(-log(pPERT))~I(-log(pNDE)),data=res[res$ID=="05210",],col="green",
    pch=19,cex=1.5)
```


The SPIA Methodology



Network-Based Gene Set Analysis (NetGSA)[4]

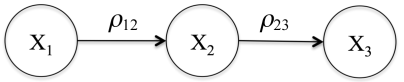
- Generalizes SPIA, to allow for more complex experiments & incorporate interactions among pathways
- Assesses the overall behavior of arbitrary subnetworks (pathways): changes in gene expression & network structure
- Uses latent variables to model the interaction between genes defined by the network
- Uses mixed linear models for inference in complex data
- Computationally challenging for large networks, unless pathways separately analyzed (similar to SPIA)

[4] S & M (2009, 2010); Ma, S & M (2016)

Problem Setup

- Gene (protein/metabolite) expression data for K experimental conditions and J_k time points
- Network information (partially) available in the form of a directed weighted graph $G = (V, E)$, with vertex set V corresponding to the genes/proteins/metabolites and edge set E capturing their associations
- Network edges can be directed $j \to k$ or undirected $j \leftrightarrow k$
- Edges defines the effect of nodes on their immediate neighbors; the weight associated with each edge corresponds to the value of partial correlation
- Represent the network by its adjacency matrix A: $A_{jk} \neq 0$ iff $k \to j$ & for undirected edges, $A_{jk} = A_{kj}$

The Latent Variable Model: Main Idea



$$\begin{aligned} X_1 &= \gamma_1 \\ X_2 &= \rho_{12} X_1 + \gamma_2 = \rho_{12}\gamma_1 + \gamma_2 \\ X_3 &= \rho_{23} X_2 + \gamma_3 = \rho_{23}\rho_{12}\gamma_1 + \rho_{23}\gamma_2 + \gamma_3 \end{aligned}$$

Thus $X = \Lambda\gamma$ where

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{12} & 1 & 0 \\ \rho_{12}\rho_{23} & \rho_{23} & 1 \end{pmatrix}$$

The Latent Variable Model

- Let Y be the ith sample in the expression data
- Let $Y = X + \varepsilon$, with signal X and noise $\varepsilon \sim N_p(0, \sigma^2_\varepsilon I_p)$
- The influence matrix Λ measures the propagated effect of genes on each other through the network, and can be calculated based on the adjacency matrix A
- Using $X = \Lambda\gamma$, we get

$$Y = \Lambda\gamma + \varepsilon, \quad \Rightarrow \quad Y \sim N_p(\Lambda\mu, \sigma^2_\gamma \Lambda\Lambda' + \sigma^2_\varepsilon I_p)$$

where $\gamma \sim N_p(\mu, \sigma^2_\gamma I_p)$ are latent variables

Mixed Linear Model Representation

Rearranging the expression matrix into np-vector $\mathbf{Y}$, we can write

$$\mathbf{Y} = \mathbf{\Psi}\beta + \mathbf{\Pi}\gamma + \varepsilon$$

where β and γ are fixed and random effect parameters and

$$\varepsilon \sim N_{np}(\mathbf{0}, R(\theta_\varepsilon)), \qquad \gamma \sim N_{np}(\mathbf{0}, \sigma^2_\gamma \mathbf{I_{np}})$$

- Temporal Correlation incorporated through R

In general, the design matrices, $\mathbf{\Psi}$ and $\mathbf{\Pi}$ depend on the experimental settings (similar to ANOVA), and are functions of Λ

Estimation of MLM Parameters

MLE for β:

$$\hat{\beta} = (\Psi' \hat{W}^{-1} \Psi)^{-1} \Psi' \hat{W}^{-1} \mathbf{Y}$$

where $W = \sigma_\gamma^2 \Pi\Pi' + R$.

$\hat{\beta}$ depends on estimates of σ_γ^2 and θ_ε^2 (estimated using restricted maximum likelihood (REML)).

Inference using MLM

- Let ℓ be a contrast vector (a linear combination of fixed effects), and consider the test:

$$H_0 : \ell\beta = 0 \quad vs. \quad H_1 : \ell\beta \neq 0$$

- Use t-test to test the significance of each hypothesis separately

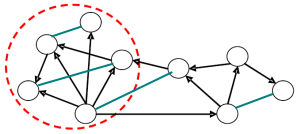
$$T = \frac{\ell\hat{\beta}}{\sqrt{\ell \hat{C} \ell'}}$$

where $C = (\Psi' W^{-1} \Psi)^{-1}$

- Under the null hypothesis, T is approximately t-distributed with degrees of freedom that needs to be estimated

"Optimal" Choice of Contrast Vector

- An intuitive choice is the indicator (membership) vector for the pathway, **b**, but this only captures changes in mean
- Need to *de-couple the effect of subnetwork* from other nodes



"Optimal" Choice of Contrast Vector

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{12} & 1 & 0 \\ \rho_{12}\rho_{23} & \rho_{23} & 1 \end{pmatrix}$$

Consider the set, $\mathbf{b} = (0, 1, 1)$; then

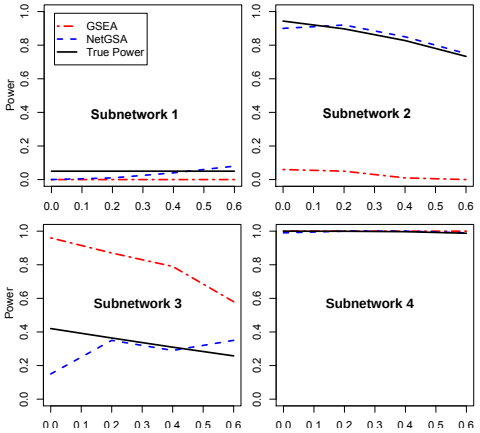
$$(\mathbf{b}\Lambda) = (\rho_{12} + \rho_{12}\rho_{23}, 1 + \rho_{23}, 1)$$

On the other hand,

$$(\mathbf{b}\Lambda \cdot \mathbf{b}) = (0, 1 + \rho_{23}, 1)$$

Comparison in Simulated Data

Subnetwork	Mean	Network Influence
1	$\mu_1 = \mu_2 = 1$	$\rho_1 = \rho_2 = 0.2$
2	$\mu_1 = 1, \mu_2 = 2$	$\rho_1 = \rho_2 = 0.2$
3	$\mu_1 = \mu_2 = 1$	$\rho_1 = 0.2, \rho_2 = 0.7$
4	$\mu_1 = 1, \mu_2 = 2$	$\rho_1 = 0.2, \rho_2 = 0.7$



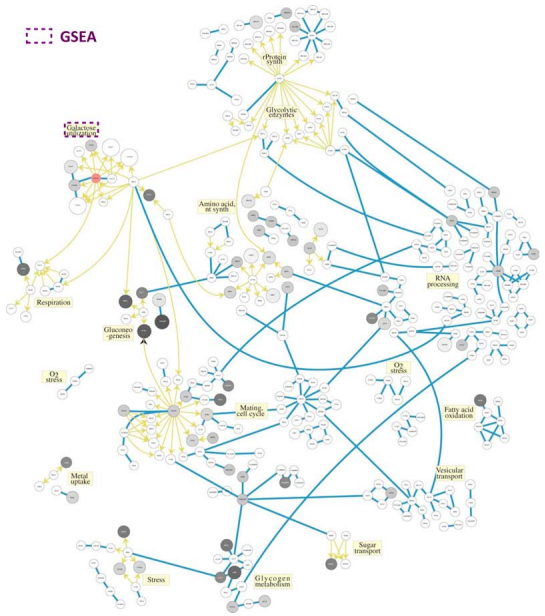
Yeast Galactose Utilization Pathway

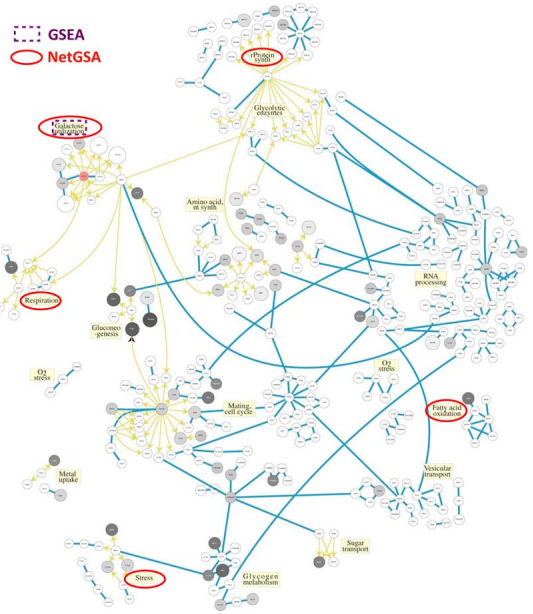
Ideker et al (2001) data on yeast Galactose Utilization Pathway

- Gene expression data for 2 experimental conditions: (gal+) and (gal–)
- Gene-gene and protein-gene interactions as well as association weights found from previous studies
- Q: which pathways respond to the change in growth medium?

Analysis of Yeast GAL Data

- Data:
  - gene expression data for 343 genes
  - 419 interactions found from previous studies and integration with protein expression (association among genes also available)
- Results:
  - GSEA finds *Galactose Utilization Pathway* significant
  - NetGSA finds several other pathways with biologically meaningful functions related to survival of yeast cells in gal–





Environmental Stress Response in Yeast

Gene expression data on Yeast Environmental Stress Response (ESR) (*Gasch et al.*, 2000)

- 3 combinations of experimental factor, heat shock and osmotic changes (sorbitol), over 3 time points
- Temporal correlation
- Network correlation
- Q: Which pathways indicate response to environmental stress
  - in different experimental conditions
  - over time

Yeast ESR Data

Gasch et al (2000)

- Gene Expression Data

Experiment	Obs. Time (after 33C)
Mild heat shock (*29C to 33C*), no sorbitol	5, 15, 30 min
Mild Heat Shock, 1M sorbitol at 29C & 33C	5, 15, 30 min
Mild Heat Shock, 1M sorbitol at 29C	5, 15, 30 min

- Network Data
  - Use YeastNet (*Lee et al.*, 2007) for gene-gene interactions (102,000 interactions among 5,900 yeast genes)
  - Use independent experiments of *Gasch et al.* to estimate weights
  - Pathways are defined using GO functions

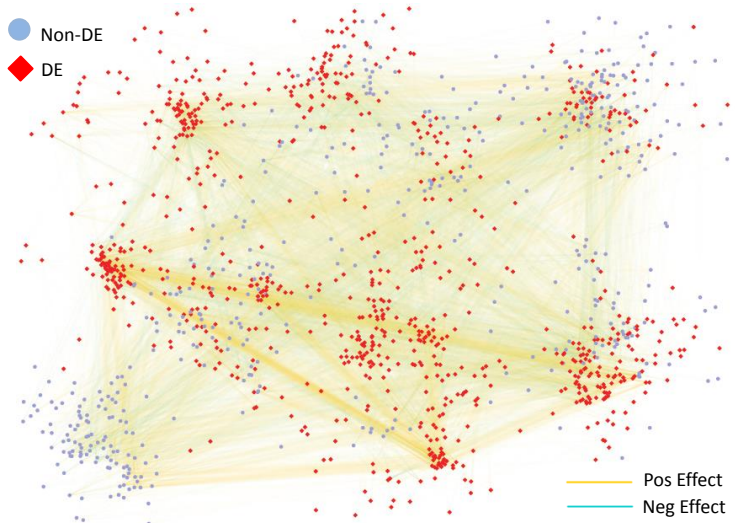
Model and Results

- Model: Let j and k be indices for time and levels of sorbitol

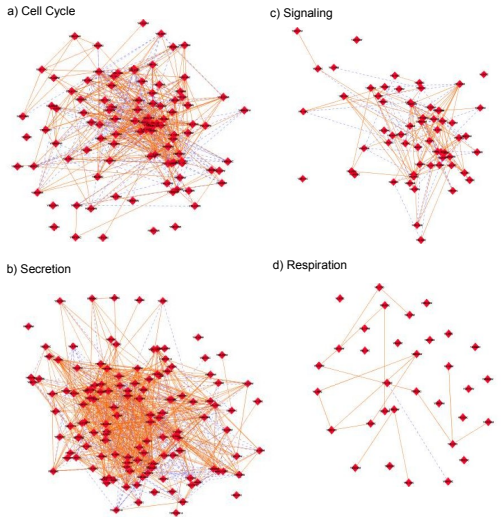
$$\mathbb{E}Y_{11} = \Lambda\mu, \qquad \mathbb{E}Y_{jk} = \Lambda(\mu + \alpha_j + \delta_k) \quad j,k = 2,3$$

- Temporal correlation is modeled directly via R (as $AR(1)$ process)
- Results:
  - $\sim$ 3000 genes,
  - 47 pathways showed significant changes of expression
  - 24 pathways showed changes over time
  - 29 pathways showed changes in response to different sorbitol levels
  - 12 pathways showed both types of changes
  - Significant pathways overlap with the gene functions recognized by *Gasch et al.*

Yeast ESR Network

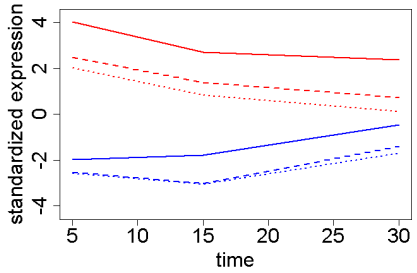


Significant subnetworks



Expression Profiles

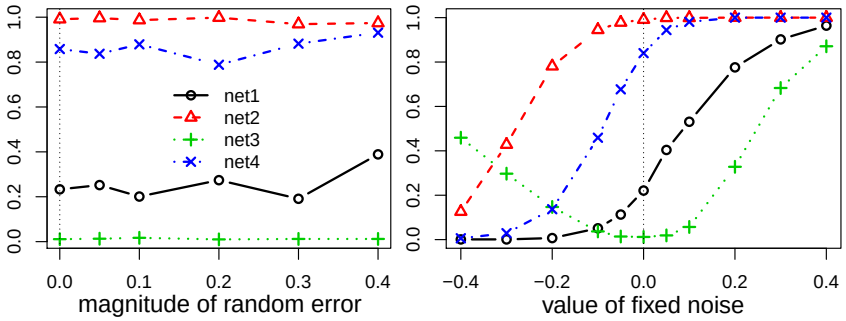
Average Standardized Expression Levels of Pathways



- Induced and Suppressed Pathways
- Can observe the transient patterns of expressions as predicted by *Gasch et al.*

Effect of Noise In Network Information

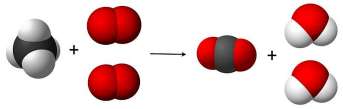
- Let $\tilde{A}$ be observed network information, and A be the truth.
- It can be shown that, if $\|\tilde{A} - A\|$ is small then, NetGSA still works (is *asymptotically most powerful unbiased test*)



Metabolic Profiling in Bladder Cancer

Targeted metabolic profiling of bladder cancer (BCa) (*Putluri et al.*, 2012)

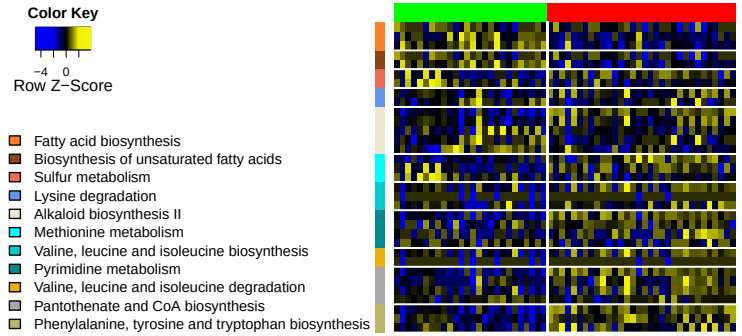
- 58 bladder cancer and adjacent benign samples
- Pathways information obtained from KEGG



- Varying number of identified metabolites per pathway (3-15)
- Q: Which pathways show differential activity in BCa?

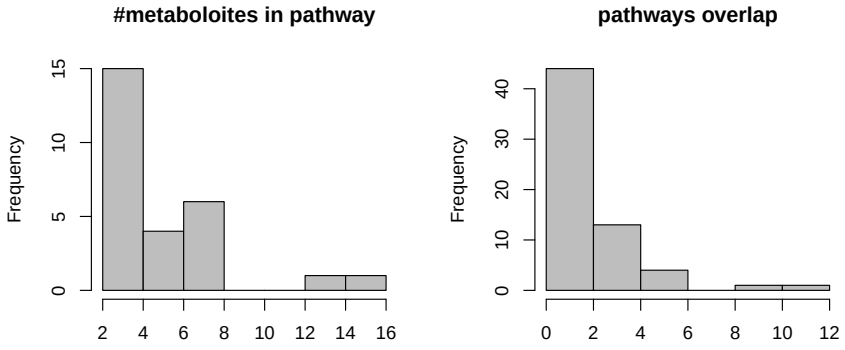
Metabolic Profiling in BCa

- 63 metabolites identified, mapped to 70 pathways
- 27 pathways with at least 3 members



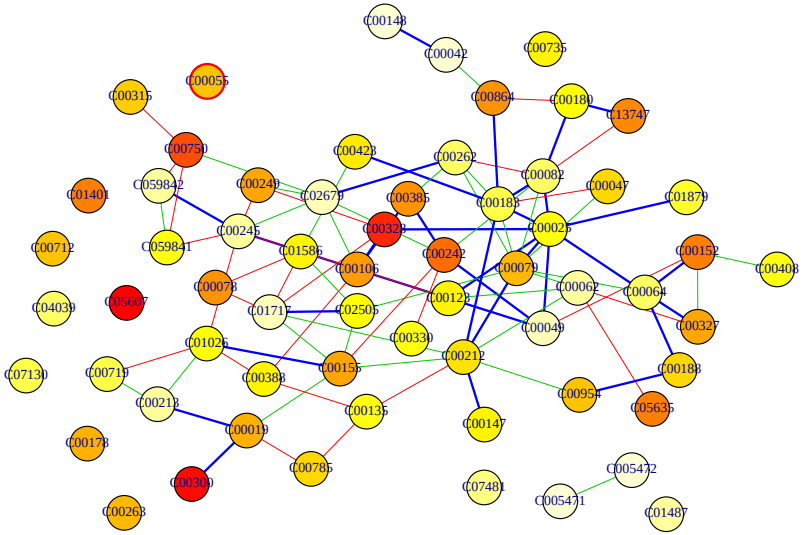
Metabolic Profiling in BCa

- Small pathway sizes & significant overlap among pathways



- Existing methods may not work well

Metabolic Interaction Network

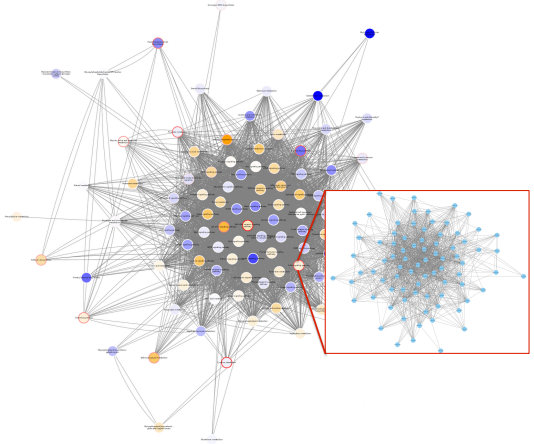


Significant Pathways

- GSEA does not identify any pathway as differential
- GSA identifies Fatty Acid Biosynthesis as differential
- NetGSA identifies another 7 pathways corresponding to role of Amino Acid Metabolism in BCa, similar to *Putluri et al* (2012)

R-Package `netgsa`

```
adjmats <- prepareAdjMat(data, groups, edges, TRUE)
res  <- NetGSA(adjmats$Adj, data, groups, pathways, "REHE")
plot(res)  #interactive plotting in Cytoscape
```



Selected Topology Based Pathway Enrichment Methods

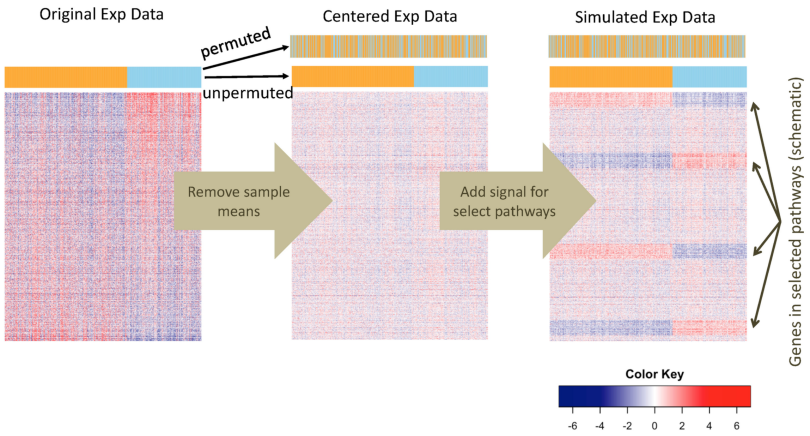
Method	Null hypothesis	Input
Pathway-Express	competitive	DE genes & p-values; sample labels; pathway topology
NetGSA	self-contained	expression matrix; sample labels; pathway membership; network information
SPIA	competitive	DE genes with p-values; sample labels; pathway topology
topologyGSA	self-contained	Gene expression matrix; sample labels; pathway topology
CAMERA	competitiv	Gene expression matrix; sample labels; pathway membership
DEGraph	self-contained	Gene expression matrix; sample labels; pathway topology
PathNet	competitive	DE genes with p-values; sample labels; pathway topology

Overview of tested pathway enrichment methods. All methods return the p-values before and/or after correcting for multiple comparisons.

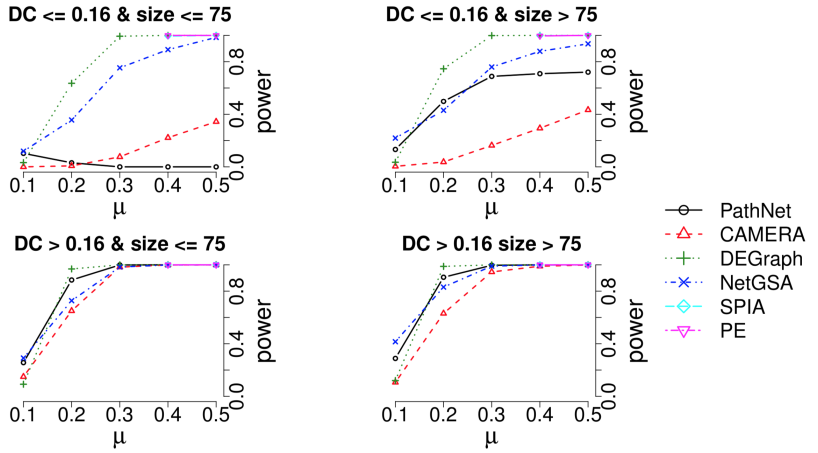
Comparison of these Methods Using Synthetic Data (Ma, Shojaie, Michailidis, 2019)

- Comparison of topology-based pathway enrichment methods using two synthetic data sets
  - Gene expression data $p \approx 3000$
  - Metabolomics data $p \approx 100$
- *In silico* data sets with known signal:
  1. Remove the original signal, but keep the correlation structure
  2. Perturb means in one condition (differential expression) for nodes in selected pathways
  3. Also use sample permutation to create data with equal correlation structure

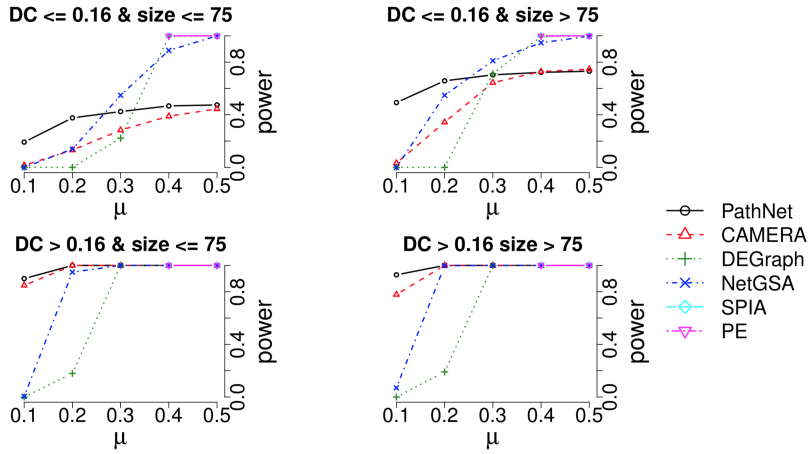
Comparison Using Synthetic Data



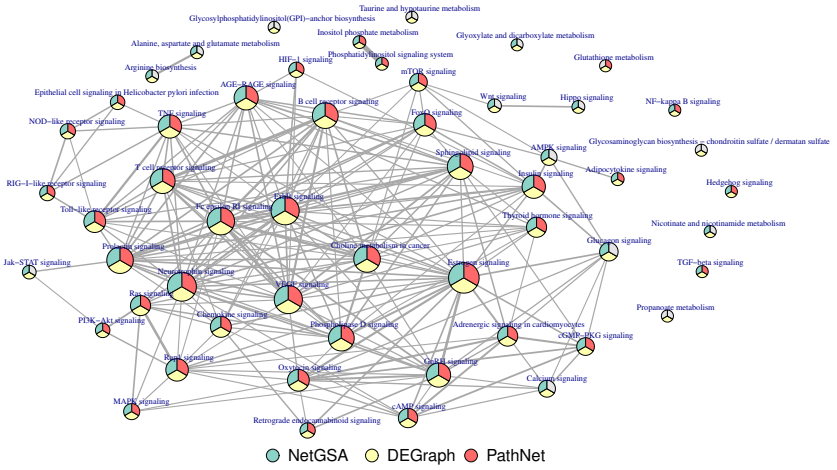
Results for Gene Expression Data — Equal Covariance



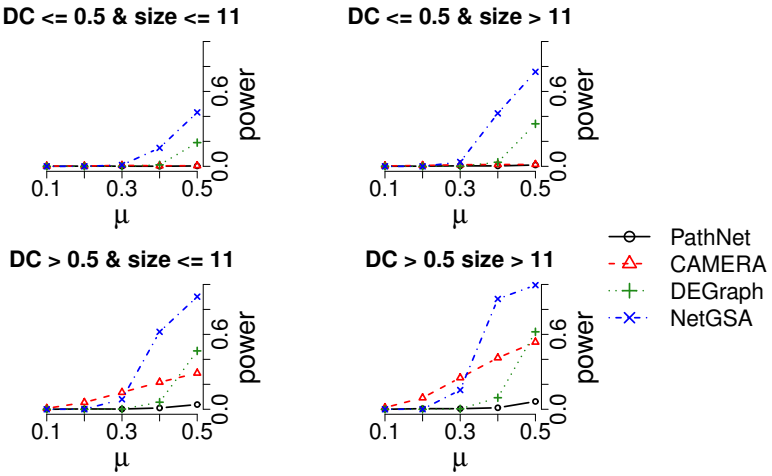
Results for Gene Expression Data — Diff Covariance



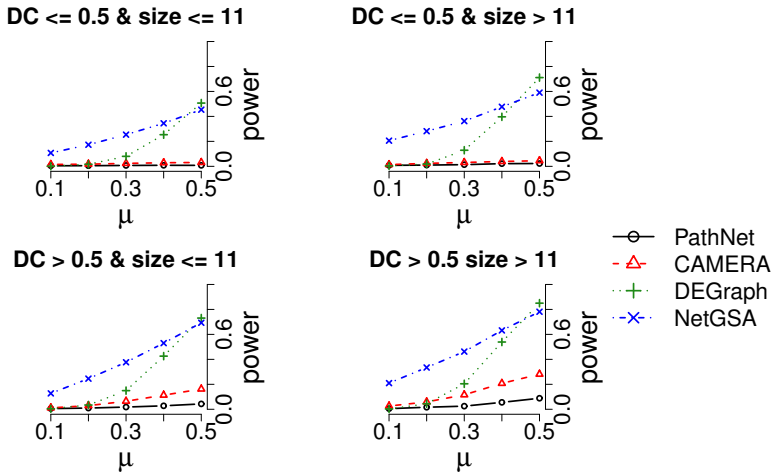
Results for Gene Expression Data



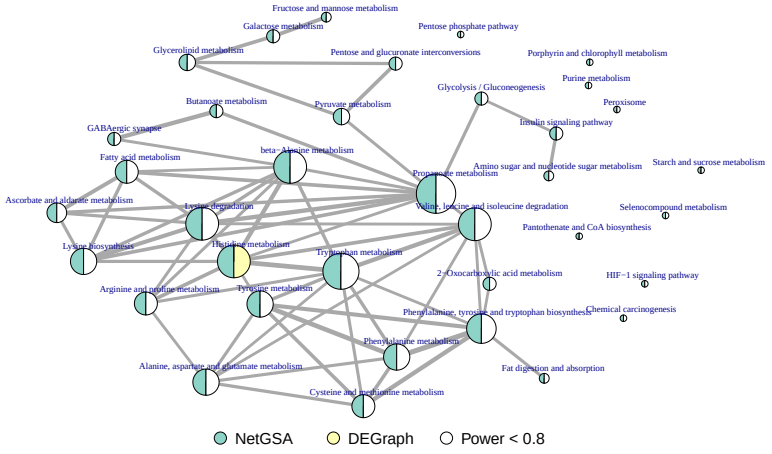
Results for Metabolomics Data — Equal Covariance



Results for Metabolomics Data — Diff Covariance

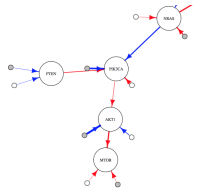


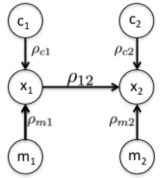
Results for Metabolomics Data

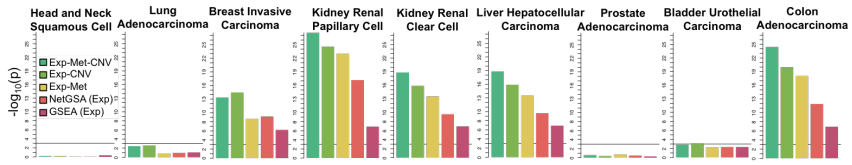


Multi-Omics NetGSA

Pan-cancer integration of expression, methylation and CNV in BRAF (TCGA data)[5]







[5] Zhang et al (2018)

Identifying Enriched Modules in Networks

Identifying Enriched Modules in Networks

Two general strategies:

- Assess the significance of data-driven modules (WGCNA):
  1. Identify modules (network clustering, etc)
  2. Assess the significance of modules
- Search for enriched (connected) subnetworks (often using greedy search methods)

- Advantage: No need to rely on known pathways — especially useful when known pathways are not complete, etc
- Disadvantage: Interpretation may become challenging...

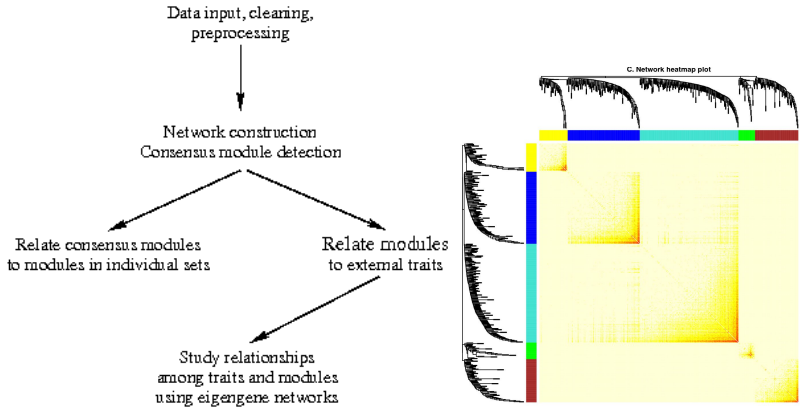
WGCNA[6]

- We previously talked about weighted gene co-expression (WGCNA), but for estimating networks
- However, WGCNA is also used for topology-based enrichment analysis, although in a different way than many other topology-based methods
- Here's how it works:
  1. Estimate the co-expression network (more in the next lecture)
  2. Find modules by clustering the nodes in the estimated network
  3. Summarize the expressions of genes in each module using PCA (eigen-genes)
  4. Test if the eigen-genes are associated with the outcome

[6] Horvath & Zhang (2005); Langfelder et al (2008)

WGCNA

- Here's how it works:



Let's look at an example in R...

Walktrap[7]

- Searches for connected modules containing significant genes
  - Weights each edges based on the significance of its corresponding nodes

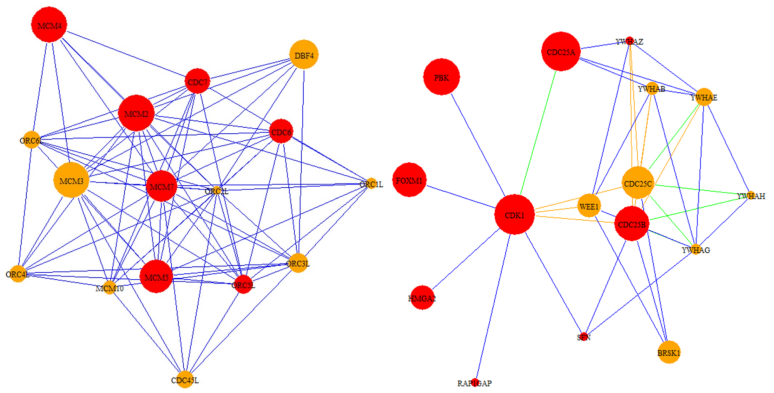
$$w_{ij} = \left(|\mathrm{FC}_i| + |\mathrm{FC}_j|\right)/2$$

  - Connected significant modules are found through community detection using a random walk with transition probability

$$P_{ij} = \frac{w_{ij}}{\sum_j w_{ij}}$$

[7] Petrochilos et al (2013)

Identifying Cancer-Related Modules



Summary

- Network-based methods (centrality-based, pathway topology, etc) rely on network information — helpful if correct network information avail
- What if network information is not available?
- What about differences in network structures — differential network biology[8]?

[8] Ideker & Krogan (2012)

Network Analysis and Applications in Biology: Learning Undirected Networks

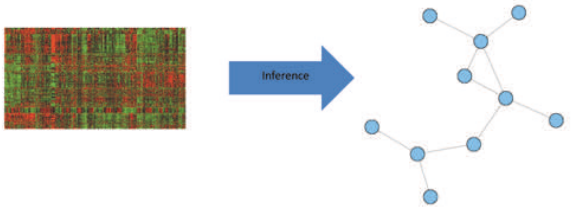
Ali Shojaie & George Michailidis

ENAR 2020

Learning Undirected Networks

Learn network from data (structure learning):

- Data matrix: $X_{n\times p}$.
- Features correspond to the p nodes in the network.
- Goal: Learn edges between nodes $\equiv$ learn the statistical relationships between features.



Why Do We Need Network Inference?

- Despite progress, our knowledge of interactions is limited.
- The entire genome is a vast landscape, and experiments for discovering networks are very expensive.
- From a statistical point of view, network estimation is related to estimation of covariance matrices, which has many independent applications in statistical inference and prediction (*more about this later*).
- Finally, and perhaps most importantly, gene and protein networks are dynamic and changes in these networks have been attributed to complex diseases.

Network Inference — An Overview

Two general classes of network inference methods:

- Methods based on marginal measures of association:
  - Co-expression Networks (based on linear measures of association)
  - Methods based on mutual information (can accommodate non-linear associations)
- Methods based on conditional measures of association:
  - Methods assuming (multivariate) normality (`glasso`, etc)
  - Generalizations to allow for nonlinear dependencies (`nonparanormal`, etc)

Graphical Models

Probabilistic Graphical Models[1]

Joint multivariate probability distribution where dependencies can be represented as a network.

Advantages:

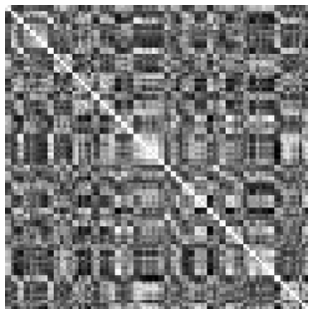
- Graphical models offer efficient factorized forms for joint distributions with easily interpretable dependencies.
  - **Conditional dependencies** denoted via an edge in network.
- Convenient visual representation.

[1] For a detailed technical introduction, see *Graphical Models, Exponential Families, and Variational Inference* by Wainwright & Jordan (2008)

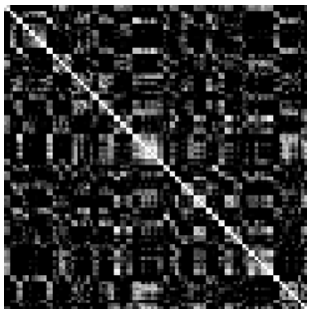
Marginal Association Networks

Correlation Networks (Association Networks)

- Simplest (and most-widely used!) method for estimating networks — key assumption:
  large correlation $\equiv$ presence of an edge
- Let $r(i,j)$ be correlation between X_i and X_j; we claim an edge between i and j if $|r(i,j)| > \tau$.
  - τ: a user-specified threshold (tuning parameter).



Correlation matrix



Thresholded correlation matrix

Limitations of Correlation Networks

1. The estimation is highly dependent on the choice of τ.
2. Correlations capture **linear** associations, but many real-world relationships are nonlinear.
3. Large correlations can occur due to confounding.

Limitations of Correlation Networks

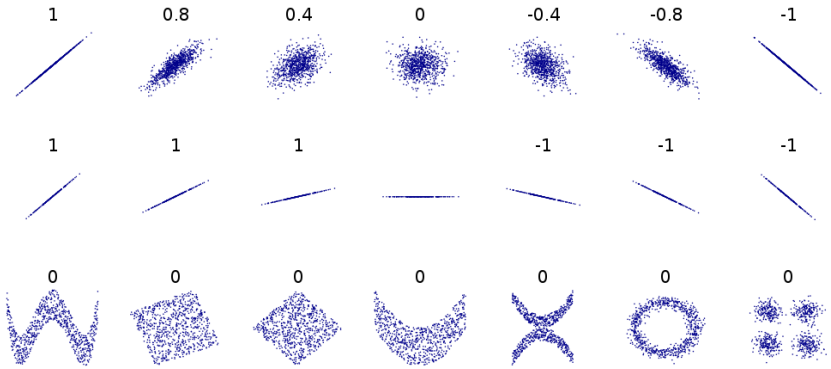
The estimation is highly dependent on the choice of τ.

- We can work with weighted co-expression networks (WGCNA)
- We can instead test $H_0 : r_{xy} = 0$
  - A commonly used test is based on the Fisher transformation

$$Z = \frac{1}{2} \ln \left(\frac{1+r}{1-r} \right) = \text{artanh}(r) \sim_{H_0} N \left(0, \frac{1}{\sqrt{n-3}} \right)$$

Limitations of Correlation Networks

Correlations capture **linear** associations, but many real-world relationships are nonlinear.



Limitations of Correlation Networks

Correlations capture **linear** associations, but many real-world relationships are nonlinear.

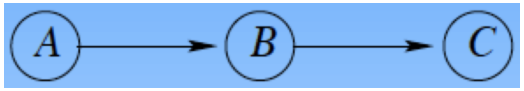
- We can use other measures of association, for instance, Spearman correlation or Kendal's τ.
  - These methods define the correlation between two variables, based on the ranking of observations, and not their exact values.
  - They can better capture non-linear associations.
- We can instead use mutual information; this has been used in many algorithms, e.g. ARACNE.

ARACNE: Algorithm for the Reconstruction of Accurate Cellular NEtworks[2]

1. Identifies statistically significant gene-gene co-regulation based on mutual information
2. It then eliminates indirect relationships in which two genes are co-regulated through one or more intermediates

[2] Margolin et al (2006)

Key Idea: Data Processing Inequality (DPI)



$$I(A, C) \leq min[I(A, B), I(B, C)]$$

where

$$I(g_i, g_j) = \log P(g_i, g_j)/P(g_i)P(g_j)$$

- Look at every triplet and remove the weakest link
- Need to estimate marginal and joint (pairwise) probabilities (using Gaussian Kernel)

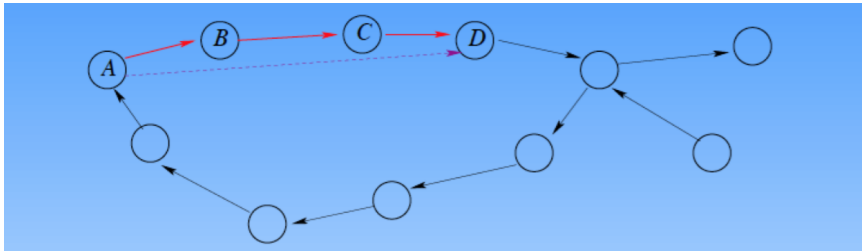
Algorithm Details

- The algorithm examines each gene triplet for which all pairwise MIs are greater than a cut-off and removes the edge with the smallest value based on DPI.
  - Each triplet is analyzed even if its edges have been selected for removal by prior DPI applications to other triplets.
  - The least of the three MIs can come from indirect interactions only, and checking against the DPI may identify gene pairs that are not independent, but still do not interact.

Rationale and Guarantees

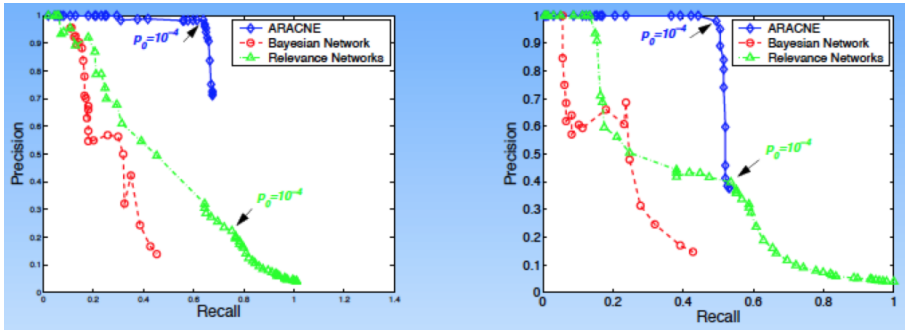
- If MIs are estimated with no errors, then ARACNE reconstructs the underlying interaction network exactly, if the network is a tree and has only pairwise interactions.
- The maximum MI spanning tree is a subnetwork of the network built by ARACNE.

Rationale and Guarantees

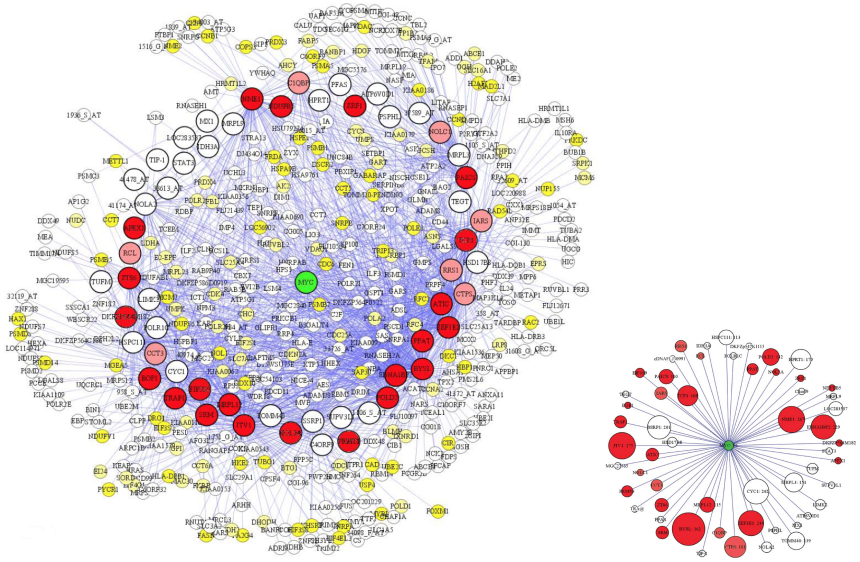


Theorem. Let π_{ik} be the set of nodes forming the shortest path in the network between nodes i and k. Then, if MIs can be estimated without errors, ARACNE reconstructs an interaction network without false positives edges, provided: (a) the network consists only of pairwise interactions, (b) for each $j \in \pi_{ik}$, $I_{ij} \geq I_{ik}$. Further, ARACNE does not produce any false negatives, and the network reconstruction is exact iff (c) for each directly connected pair ij and for any other node k, we have $I_{ij} > \min[I_{ik}, I_{jk}]$.

Performance on Synthetic Data



Application: B-lymphocytes Expression Data



Application: B-lymphocytes Expression Data

- MYC (proto-oncogene) subnetwork (2063 genes)
- 29 of the 56 (51.8%) predicted first neighbors biochemically validated as targets of the MYC transcription factor.
- New candidate targets were identified, 12 experimentally validated.
  - 11 proved to be true targets.
- The candidate targets that have not been validated are possibly also correct.

Software

- Implemented in the R-package `minet`:

```
source("http://bioconductor.org/biocLite.R")
biocLite("minet")
```

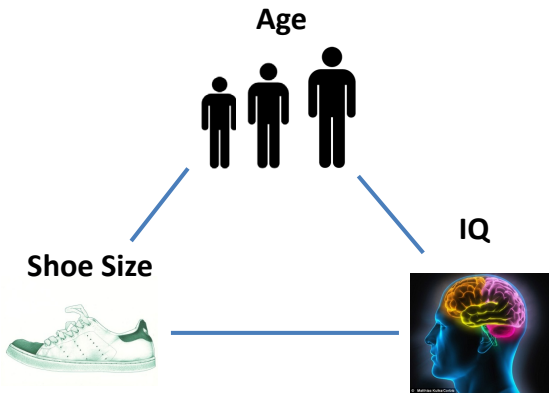
- Main estimation function `aracne(mim, eps=0)`
  - mim: mutual information matrix

```
mim <- build.mim(syn.data, estimator="spearman")
```

  - eps: threshold for setting an edge to zero, prior to searching over triplets

Limitations of Correlation Networks

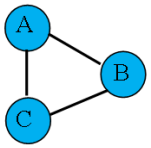
Large correlations can occur due to confounding.



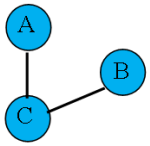
Markov Networks

Markov Network
An *undirected graphical model* that characterizes conditional dependence (≡ direct relationships).

- *Edge*: Two nodes are **conditionally dependent**.
- *No edge*: Two nodes are **conditionally independent**.
- Conditions on all other nodes.



$$A \perp B \mid C$$

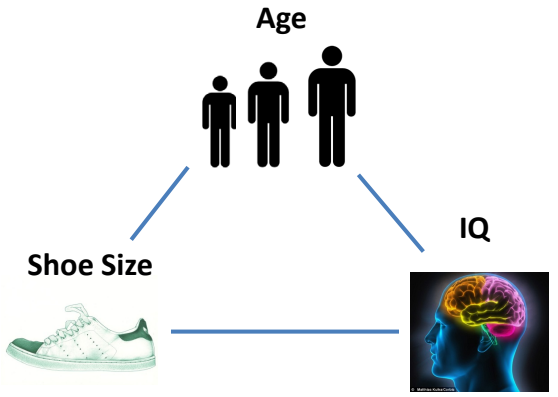


Markov Networks — Conditional Dependence

Regression Interpretation:

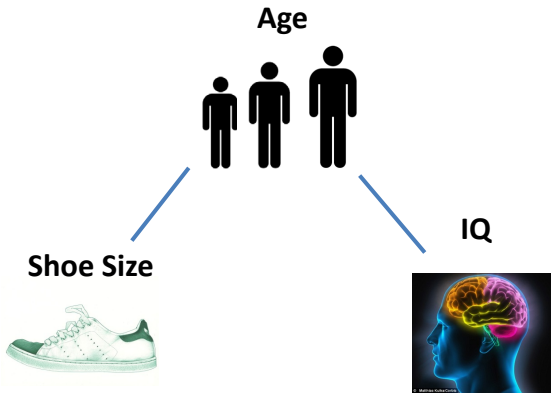
- Imagine trying to predict the observations in Node A (response) by the observations of all other nodes (predictors).
- Node B predictive of Node A (with all other nodes in model).
  - A is conditionally dependent on B.
  - Edge.
- Because of other nodes in model, Node B does not add any predictive value for Node A.
  - A is conditionally independent of B.
  - No Edge.

Markov Networks — Conditional Dependence



Correlation.

Markov Networks — Conditional Dependence



Conditional Dependence.

Markov Networks — Conditional Dependence

How can we learn conditional dependencies?

- A and B are conditionally independent given C if

$$P(A, B \mid C) = P(A \mid C)P(B \mid C)$$

  - Generally difficult (need to estimate multivariate densities).
- Alternatively, can use nonparametric approaches, e.g. conditional mutual information, but not easy in high dimensions.
- Often resort to models, or simple measures, such as partial correlations...

Partial Correlation

- Partial correlation measures the correlation between A and B after the effect of the other variables are removed.
  - In our example, this means correlation between shoe size and IQ, after adjusting for age.
- The partial correlation between A and B given C is given by:

$$\rho_{AB \cdot C} \equiv \rho(A, B|C) = \frac{\rho_{AB} - \rho_{AC}\rho_{BC}}{\sqrt{1-\rho_{AC}^2}\sqrt{1-\rho_{BC}^2}}.$$

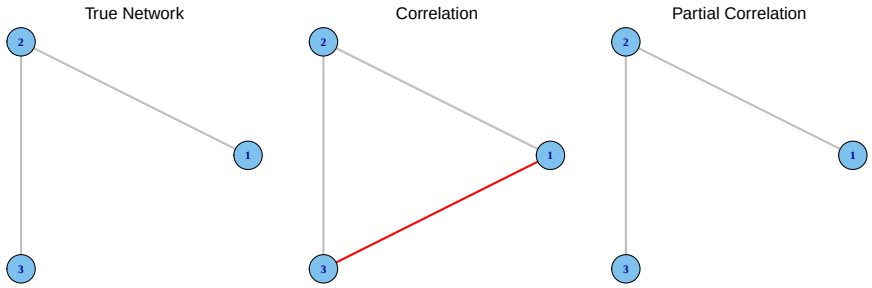
- Alternatively, regress A on C and get the residual, r_A; do the same for B to get r_B. The partial correlation between A and B give C is $\mathrm{Cor}(r_A, r_B)$.

Partial Correlation

- Partial correlation is symmetric $\Rightarrow$ undirected network
- Partial correlation takes values between -1 and 1
- In partial correlation networks, we draw an edge between A and B, if the partial correlation between them is large
- Calculation of partial correlation is more involved

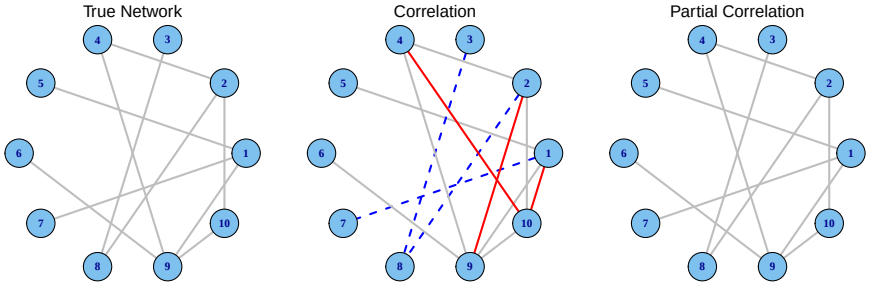
A Simple Example

$$Correlation = \begin{bmatrix} 1 & .8 & .7 \\ .8 & 1 & .8 \\ .7 & .8 & 1 \end{bmatrix} \quad PartialCorr = \begin{bmatrix} 1 & .6 & 0 \\ .6 & 1 & .6 \\ 0 & .6 & 1 \end{bmatrix}$$



A Larger Example

- A network with 10 nodes and 20 edges
- $n = 100$ observations
- Estimation using correlation & partial correlation (20 edges)

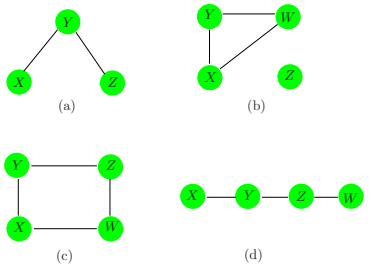


Gaussian Graphical Models (GGMs)

Partial Correlation for Gaussian Random Variables

- For Gaussian (multivariate normal) random variables, partial correlation between X_i and X_j given all other variables is given by the inverse of the (standardized) covariance matrix Σ.
  - The (i, j) entry in Σ^{-1} gives the partial correlation between X_i and X_j given all other variables $X_{\setminus i,j}$.
  - Multivariate normal: $X \sim N(0, \Sigma)$
  - $\Theta \equiv \Sigma^{-1}$ = inverse covariance/precision/concentration matrix.
  - Zeros in $\Theta \Longrightarrow$ conditional independence!
  - Edges correspond to non-zeros in Θ.

Partial Correlation for Gaussian Random Variables



$$\begin{pmatrix} - & \times & 0 \\ \times & - & \times \\ 0 & \times & - \end{pmatrix} \quad \begin{pmatrix} - & \times & \times & 0 \\ \times & - & \times & 0 \\ \times & \times & - & 0 \\ 0 & 0 & 0 & - \end{pmatrix}$$

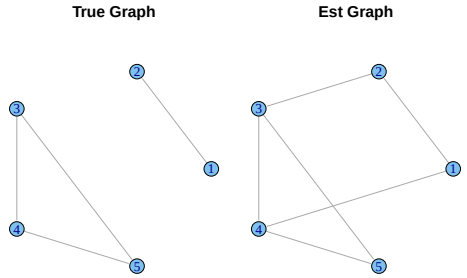
$$\begin{pmatrix} - & \times & 0 & \times \\ \times & - & \times & 0 \\ 0 & \times & - & \times \\ \times & 0 & \times & - \end{pmatrix} \quad \begin{pmatrix} - & 0 & 0 & \times \\ 0 & - & \times & 0 \\ 0 & \times & - & \times \\ \times & 0 & \times & - \end{pmatrix}$$

Estimating GGMs

From the discussion so far, to estimate the network, we can

1. Calculate the empirical covariance matrix: for (centered) $n \times p$ data matrix X, $S = (n-1)^{-1} X^\top X$.
2. Get the inverse of S. Non-zero values of S^{-1} give the edges.

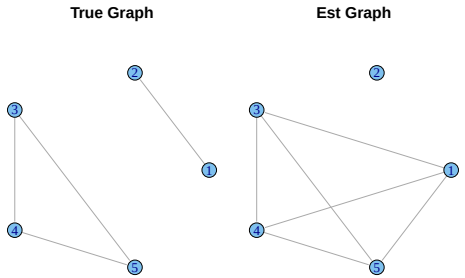
While simple, this may not work well in practice, even with large samples!



Estimating GGMs in High Dimensions

Many problems arise in high-dimensional settings, when $p \gg n$.

- First, S is not invertible if $p > n$!
- Even if $p < n$, but n is not very large, we may still get poor estimates, and many false positives/negatives.



Estimating GGMs in High Dimensions

- A number of methods have been recently proposed for estimating GGMs in high dimensions.
- The main idea in most of these methods is to use a regularization penalty, like the lasso.
- We discuss two approaches:
  - neighborhood selection
  - graphical lasso

Estimating GGMs in High Dimensions – Method 1

The idea behind neighborhood selection, is to estimate the graph by fitting a penalized regression of each variable on all other variables.

- Find neighbors of each node X_j by l_1-penalized regression or lasso:

$$\underset{\beta^j}{\text{minimize}} \quad \|X_j - X_{\neq j}\beta^j\|_2^2 + \lambda \sum_{k \neq j} |\beta_k^j|$$

- The final estimate is found by combining all of the edges from these individual regression problems.
  - Symmetry — β_k^j not always same as β_j^k.
  - Use min or max rule.

Estimating GGMs in High Dimensions – Method 2

Estimate a sparse Θ via penalized maximum likelihood estimation (MLE).

Graphical Lasso (`glasso`)

$$\underset{\Theta}{\text{maximize}} \quad \log\det(\Theta) - \text{tr}(S\Theta) - \lambda\|\Theta\|_1$$

- Blue: Log-likelihood; $\log\det$ denotes the logarithm of the determinant of Θ and tr the trace (sum of diagonal elements) $S\Theta$.
- Red: Penalty term encourages zeros on the off-diagonal elements of Θ.

Comparing the Two Approaches

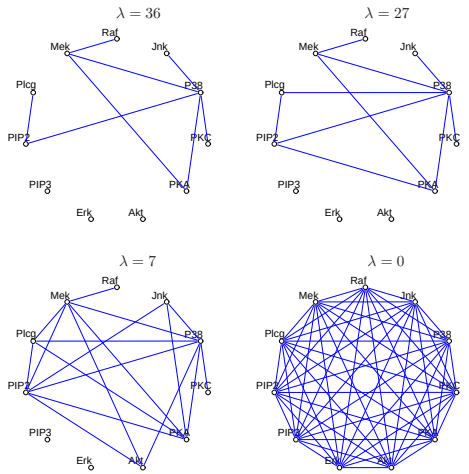
- Neighborhood selection is an approximation for graphical lasso:
  - Consider regression of X_j on $X_k, j \neq k$
  - Then, the regression coefficient for neighborhood selection is related to the j, k element of Θ:

$$\beta_k^j = -\frac{\Theta_{jk}}{\Theta_{jj}}$$

- Neighborhood selection is computationally more efficient, and may gives better estimates, but doesn't give an estimate of Θ!

A Real Example

- Flow cytometry proteomics in single cells (Sachs et al, 2003).
- $p = 11$ proteins measured in $n = 7466$ cells



How to Choose λ?

- λ modulates trade-off between model fit and network sparsity:
  - $\lambda = 0$ gives a dense network (no sparsity).
  - As λ increases, network becomes more sparse.
- A number of approaches proposed in the literature and used in practice
  1. Cross-Validation — tends to yield overly dense networks.
  2. Extended BIC — adjusted BIC for high dimensions.
  3. Controlling the probability of falsely connecting disconnected components at level α (Banerjee et al, 2008):

$$\lambda(\alpha) = \frac{t_{n-2}(\alpha/2p^2)}{\sqrt{n-2+t_{n-2}(\alpha/2p^2)}},$$

  ($t_{n-2}(\alpha)$ is the $(100-\alpha)$% quantile of t-dist with $n-2$ d.f.)

  4. Stability selection — Choose λ that gives the most **stable network** (R: huge package)

Other Types of Graphical Models

Nonparanormal (Gaussian Copula) Models

- Suppose $X \nsim N(0, \Sigma)$, but there exist monotone functions $f_j, j = 1, \ldots p$ such that $[f_1(X_1), \ldots f_p(X_p)] \sim N(0, \Sigma)$
  - X has a nonparanormal distribution $X \sim NPN_p(f, \Sigma)$.
  - f and Σ are parameters of the distribution, and estimated from data.
  - For continuous distributions, the nonparanormal family is the same as the Gaussian copula family
- To estimate the nonparanomal network:
  i) transform the data: $[f_1(X_1), \ldots f_p(X_p)]$
  ii) estimate the network of the transformed data (e.g. calculate the empirical covariance matrix of the transformed data, and apply glasso or neighborhood selection)

A Related Procedure

- Liu et al (2012) and Xue & Zou (2012) proposed a closely related idea using rank-based correlation
  - Let r_j^i be the rank of x_j^i among $x_j^1, \ldots, x_j^n$ and $\bar{r}_j = (n+1)/2$ be the average rank
  - Calculate Spearman's ρ or Kendall's τ

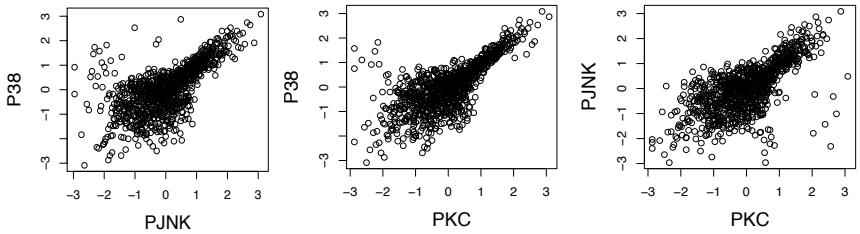
$$\hat{\rho}_{jk} = \frac{\sum_{i=1}^n (r_j^i - \bar{r}_j)(r_k^i - \bar{r}_k)}{\sqrt{\sum_{i=1}^n (r_j^i - \bar{r}_j)^2 \sum_{i=1}^n (r_k^i - \bar{r}_k)^2}}$$

$$\hat{\tau}_{jk} = \frac{2}{n(n-1)} \sum_{1 \le i < i' \le n} \text{sign}\left((x_j^i - x_j^{i'})(x_k^i - x_k^{i'})\right)$$

- If $X \sim NPN_p(f, \Sigma)$, then $\Sigma_{jk} = 2\sin(\rho_{jk}\pi/6) = \sin(\tau_{jk}\pi/2)$
- Therefore, we can estimate Σ^{-1} by plugging in rank-based correlations into graphical lasso (R-package `huge`)

A Real Data Example

- Protein cytometry data for cell signaling (Sachs et al, 2005)
- Transform the data using a Gaussian copula (Liu et al, 2009), giving marginal normality
- Pairwise relationships still seem non-linear



- Shapiro-Wilk test rejects multivariate normality: $p < 2 \times 10^{-16}$

Graphical Models for Discrete Random Variables

- In many cases, biological data are not Gaussian: SNPs, RNAseq, etc
- Need to estimate CIG for other distributions: binomial, poisson, etc
- In this case, the estimators do not have a closed-form!
- A special case, which is computationally more tractable, is the class of pairwise MRFs

Pairwise Markov Random Fields

- The idea of pairwise MRFs is to "assume" that only two-way interactions among variables exist
  - The pairwise MRF associated with graph G over the random vector X is the family of probability distributions $P(X)$ that can be written as

$$P(X) \propto \exp \sum_{(j,k)\in E} \phi_{jk}(x_j, x_k)$$

  - For each edge $(j,k) \in E$, ϕ_{jk} is called the edge potential function
- For discrete random variables, any MRF can be transformed to an MRF with pairwise interactions by introducing additional variables[3]

[3] Wainwright & Jordan, 2008

Graphical Models for Binary Random Variables

- Suppose $X_1, \ldots, X_p$ are binary random variables, corresponding to, e.g. SNPs, or DNA methylation
- A special case of discrete graphical models is the Ising model for binary random variables

$$P_\theta(x) = \frac{1}{Z(\theta)} \exp\left\{ \sum_{(j,k)\in E} \theta_{jk} x_j x_k \right\}$$

  - A pairwise MRF for binary data, with $\phi_{jk}(x_j, x_k) = \theta_{jk} x_j x_k$
  - $x^i \in \{-1, +1\}^p$
  - The partition function $Z(\theta)$ ensures that the distribution sums to 1
  - $(j,k) \in E$ iff $\theta_{jk} \neq 0$!

Graphical Models for Binary Random Variables

- We can consider a neighborhood selection[4] approach with an ℓ_1 (lasso) penalty to find the neighborhood of each node $N(j) = \{k \in V : (j,k) \in E\}$
- For $j = 1, \ldots, p$, need to solve (after some algebra)

$$\min_\theta \left\{ n^{-1} \sum_{i=1}^{n} \left[f(\theta; x^i) - \sum_{k \neq j} \theta_{jk} x_j^i x_k^i + \lambda \|\theta_{-j}\|_1 \right] \right\}$$

  - $f(\theta; x) = \log\left\{\exp\left(\sum_{k\neq j} \theta_{jk} x_k\right) + \exp\left(-\sum_{k \in -j} \theta_{jk} x_k\right)\right\}$
- This is equivalent to solving p penalized logistic regression problems, which is straightforward (R-package `glmnet`)

[4] Ravikumar et al (2010)

Other Non-Gaussian Distributions

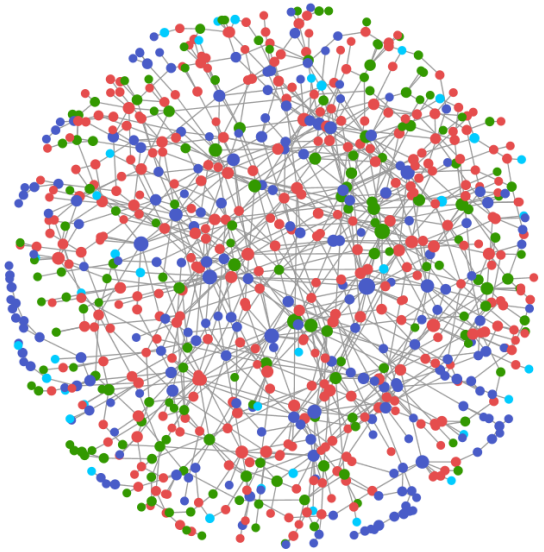
- Assume a pairwise graphical model

$$P(X) \propto \exp\left\{ \sum_{j \in V} \theta_j \phi_j(X_j) + \sum_{(j,k) \in E} \theta_{jk} \phi_{jk}(X_j, X_k) \right\}$$

- Then, similar to the Ising model, graphical models can be learned for other members of the exponential family
  - Poisson graphical models (for e.g. RNAseq), Multinomial graphical models, etc
  - All of these can be learned using a neighborhood selection approach, using the `glmnet` package[5]
  - We can even learn networks with multiple types of nodes (gene expression, SNPs, and CNVs)[6]

[5] Yang et al (2012)
[6] Yang et al (2014), Chen et al (2015)

Mixed Graphical Models



A General Approach for Estimation of Graphical Models

- Consider n iid observations from a p-dimensional random vector $x = (X_1, \ldots, X_p) \sim \mathcal{P}$
- Consider the (undirected) graph $G = (V, E)$ with vertices $V = \{1, \ldots, p\}$
- Want to estimate edges $E \subset V \times V$ that satisfy $\forall j \in V$, $\exists N(j)$ such that:

$$p_j(X_j | \{X_k, k \neq j\}) = p_j(X_j | \{X_k : k \in N(j)\}) = p_j(X_j | \{X_k : (k,j) \in E\})$$

- $N(j)$ is the minimal set of variables on which the conditional densities depend

Estimating Conditional Independencies

Question: how to condition?

- Approach 1: Estimate the joint density $f(X_1, \ldots, X_p)$; then get the conditionals $f_j(X_j \mid X_{-j})$
  - Efficient, coherent
  - Computationally challenging
  - Restrictive: how many joint distributions do you know?
  - Hard to check if assumptions hold!
- Approach 2: Estimate the conditionals directly $f_j(X_j \mid X_{-j})$
  - Computationally easy
  - Leads to easy & flexible models (regression)!
  - May not be efficient or coherent

A Semi-parametric Approach

- Consider additive non-linear relationships (additive model):

$$X_j \mid X_{-j} = \sum_{k \neq j} f_{jk}(X_k) + \varepsilon$$

- Then if $f_{jk}(X_k) = f_{kj}(X_j) = 0$, we conclude that X_j and X_k are conditionally independent, given the other variables
- In other words, we assume that conditional distributions and conditional means depend on the same set of variables
- We then use a semi-parametric approach for estimating the conditional dependencies

SpaCE JAM[7]

- Sparse Conditional Estimation with Jointly Additive Models (SpaCE JAM)

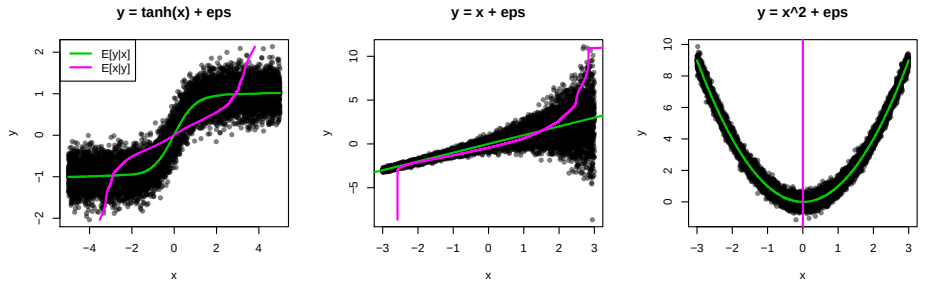
$$\underset{f_{jk} \in \mathcal{F}}{\text{minimize}} \frac{1}{2n} \sum_{j=1}^{p} \|x_j - \sum_{k \neq j} f_{jk}(x_k)\|_2^2 + \lambda \sum_{k>j} \left(\|f_{jk}(x_k)\|_2^2 + \|f_{kj}(x_j)\|_2^2 \right)^{1/2}$$

  - $f_{jk}(x_k) = \Psi_{jk} \beta_{jk}$
  - Ψ_{jk} is a $n \times r$ matrix of basis functions for f_{jk}
  - β_{jk} is an r-vector of coefficients
  - The standardized group lasso penalty for functions $\|f_{jk}\|_2$
- This is a convex problem, and block coordinate descent converges to the global minimum

[7] Voorman et al (2014), R-package `spacejam`

SpaCE JAM

Estimating f_{jk} and f_{kj} seems redundant...



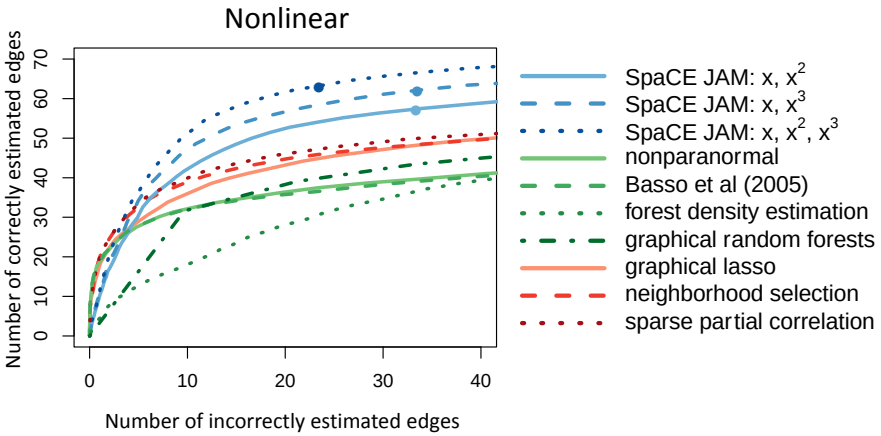
but necessary for non-linear functions

Other Flexible Procedures

- Forest density estimation (Liu et al, 2011) assumes that underlying graph is a forest, and estimates the bivariate densities non-parametrically.
- Graphical random forests (Fellinghauer et al, 2013) uses random forests to flexibly model conditional means
  - They consider conditional dependencies through conditional mean
  - They allow for general random variables, discrete or continuous
  - Use a random forest to estimate $E[X_j \mid X_{\setminus j}]$ non-parametrically
  - Theoretical properties have not yet been justified

Comparison on Simulated Data

non-linear relationships ($p = 100$, $n = 50$)



Comparison on Simulated Data

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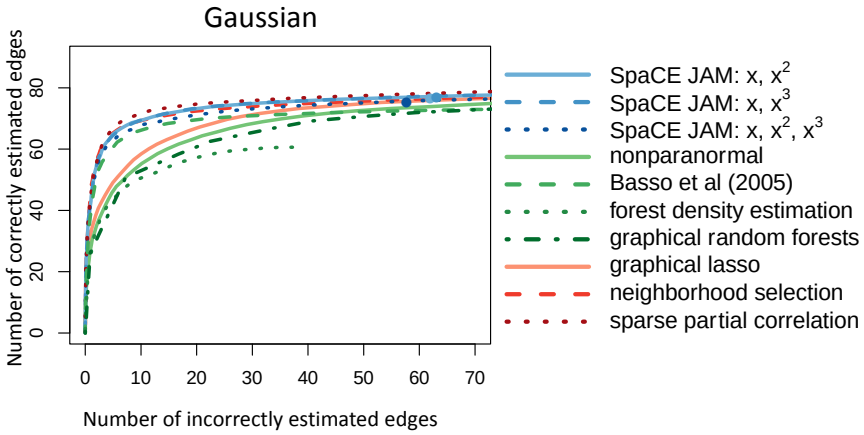
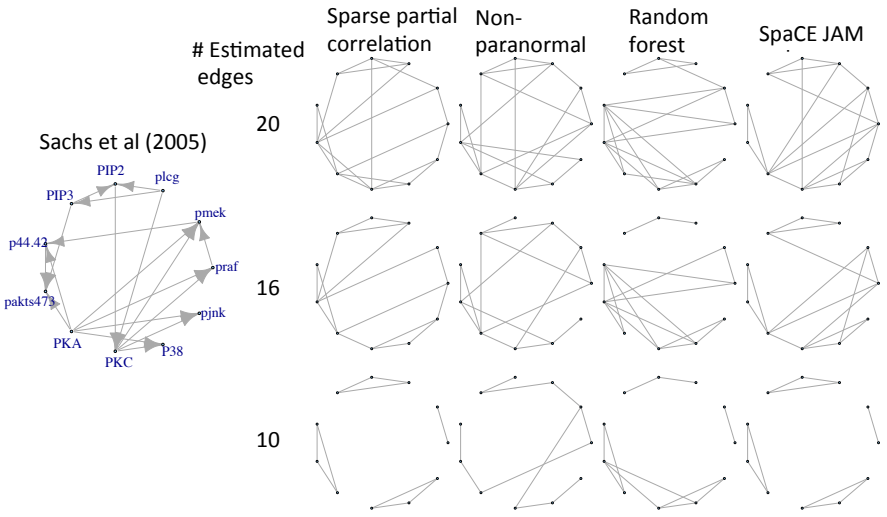


Estimation of Cell Signaling Network



Other Extensions of GGMs

- Multiple Graphical Models
  - For groups of observations, estimate graphical models with shared structure across groups and individual structure within groups.
- Time Varying Graphical Models
  - Smoothly varying graph over time estimated via local kernel smoothers.
  - Change points in graph structure over time estimated via fusion penalties.
- Latent Variable Graphical Models
  - Assume observed features are dependent on latent variables which exhibit a low-rank effect. Estimate a sparse (graph structure) plus low-rank inverse covariance matrix.

Joint Estimation of Multiple Graphical Models

A Brief Introduction

Key idea:

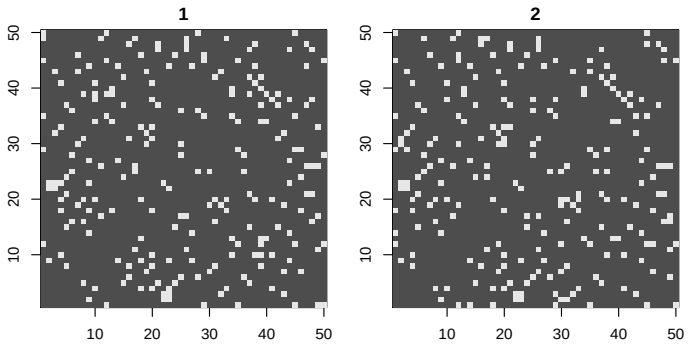
- We observe data from different a priori known sub-populations
  - Sub-populations may correspond to sub-types of a disease (e.g. neural, proneural, mesenchymal and classical in glioblastoma)
- For each sub-population, there exists an undirected graphical model
- The underlying graphical models share common structure

Main issue: What type of common structure is assumed, and how to enforce it?

Illustrative framework: Gaussian case

- $X_k \sim N(0, \Sigma_k), \quad k = 1, \ldots, K.$
- $\Theta_k \equiv \Sigma_k^{-1}, k = 1, \ldots, K$ so that $\Theta_k \sim \Theta_\ell$, for all $k \neq \ell$
- Different approaches build either on maximum likelihood estimation or on neighborhood selection

Pictorial Motivation - I



Selected Approaches - I

Rich literature on the topic with many variants appearing in statistics, machine learning and bioinformatics literature

- Hierarchical penalty[8]:
  - Let $\Theta_k(i,j) = \alpha(i,j)\gamma_k(i,j), \quad k = 1, \ldots, K$
  - For identifiability, assume $\alpha(i,j) \geq 0$ for all variable pairs (i,j)
  - $P(\Theta) = \lambda_\alpha |\alpha|_1 + \lambda_\gamma \sum_{k=1}^K |\gamma_k|_1$ — combine two lasso penalties

[8] Guo et al (2011)

Selected Approaches - II

- Fusing penalties[9]:
  - Variant 1:

$$P(\Theta) = \sum_{k=1}^K \lambda_k |\Theta_k|_1 + \sum_{k \neq \ell} \lambda_{k,\ell} |\Theta_k - \Theta_\ell|_1$$

element-wise fused lasso penalty, encourages similarities between all elements of the K graphical models

  - Variant 2:

$$P(\Theta) = \sum_{k=1}^K \lambda_k |\Theta_k|_1 + \sum_{i,j} \sqrt{\sum_{k=1}^K (\Theta_k(i,j))^2}$$

encourages strong fusing towards a common graphical model

[9] Danaher et al (2014)

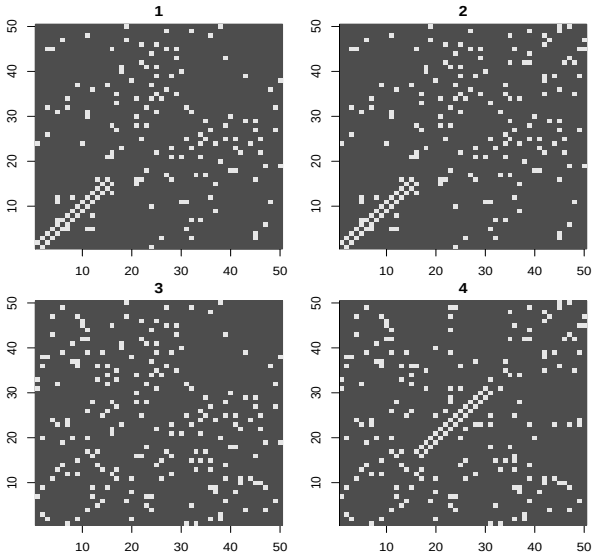
Selected Approaches - III

- Cai et al (2016) propose a mixed ℓ_∞/ℓ_1 norm:

$$\min_{\{\Theta\}_{k=1}^K} \left(\max_{1 \leq k \leq K} |\Theta_k|_1\right)$$
$$\text{s.t. } \max_{i,j} \left(\sum_{k=1}^K \frac{n_k}{n} |S_k \Theta_k - I|^2_{(i,j)} \right)^{1/2} \leq t_n$$

- The objective function encourages sparsity across all K models. The constraint is imposed on the maximum of the element-wise group ℓ_2 norm to encourage the groups to share a common graphical structure.

Pictorial Motivation - II



Selected Approaches - IV

- Saegusa & S (2016) encode similarity between different sets of edges for pairs of models (k, ℓ) through a Laplacian penalty
- Ma & M (2016) use group lasso penalties across different subsets of the edges

Both approaches require external information through prior knowledge (e.g. functional pathways, literature, etc.)

Application: Lipid Interaction Networks in CKD

- Chronic Kidney Disease (CKD) is strongly linked to cardiovascular morbidity and mortality.
- Despite the diversity of human plasma lipidome, studies of CKD have been traditionally limited to measuring total cholesterol, triglycerides, and lipoproteins
- New technologies allow researchers to profile a large number of lipid species ($p \sim 450$) from various lipid classes

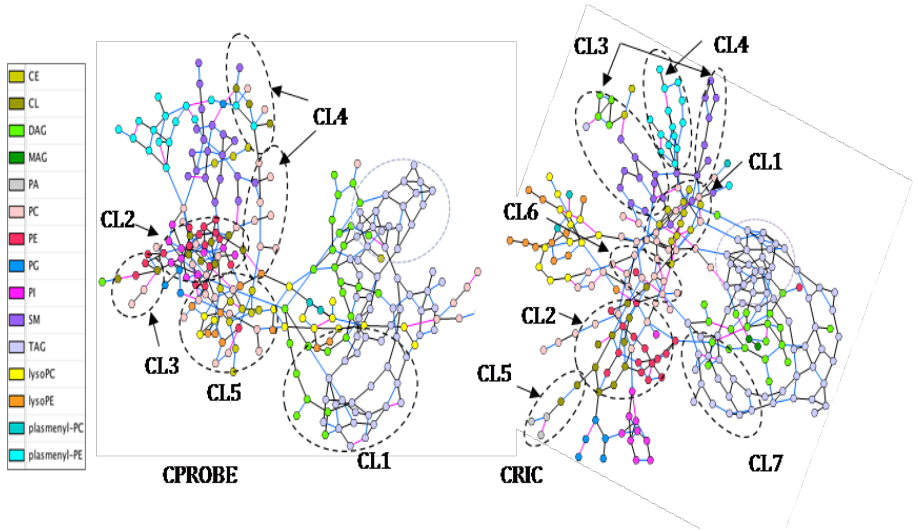
Lipid Interaction Networks in CKD

Objective: Understand lipid interactions from two related study cohorts[10]

- Clinical Phenotyping Resource and Biobank Core (CPROBE) — progressors vs non-progressors patients
- Chronic Renal Insufficiency Cohort (CRIC) — early stage CKD vs late stage CKD patients

[10] Analysis pipeline and results in Ma et al (2019)

CPROBE/CRIC Modules



CPROBE/CRIC Differential Sub-Networks[11]

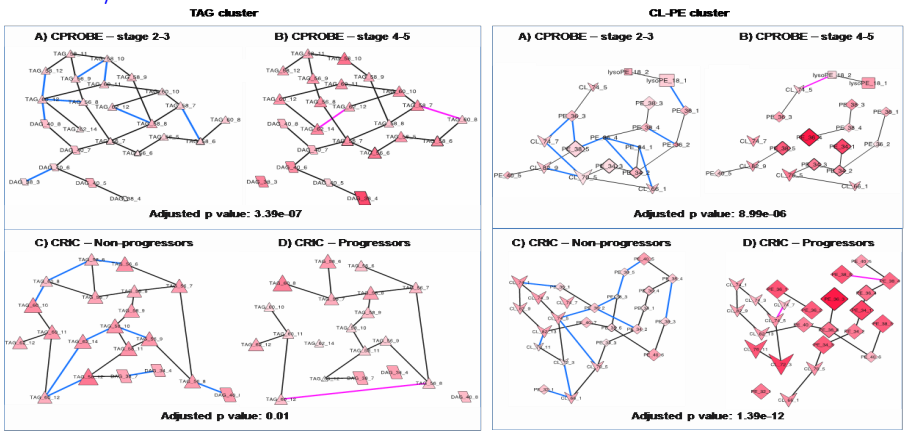


Figure: Black edges: common backbone; blue edges: present in early stage/NP; red edges: present in late stage/P

[11] Based on NetGSA enrichment analysis — see Lecture 2

Biological Relevance of Discovered Modules

- Of interest is the "disregulation" of a module comprising of triacylglycerols (TAGs) and another one of cardiolipins with phosphatidylethanolamines (CL-PE) in CRIC/CPROBE
- Of particular interest is the second (CL-PE) module, that points to role of cellular lipid metabolism and specifically the activity of the mitochondrial respiratory chain after checking the module for enrichment; thus, the loss of lipids may lead to decreased mitochondrial fusion and fragmented mitochondria
- Concordant with recent findings in the literature that mitochondrial damage and dysfunction might be a highly prevalent abnormality in early CKD (eGFR>60)