B Raising the penalty Z

Consider Example 1, with evidence production functions specified by

$$\begin{aligned} y_D(x,G) &= y_P(x,B) = c^{-1}(x+c_0) \\ y_D(x,B) &= y_P(x,G) = c^{-1}(x) \end{aligned} \tag{20}$$

for some convex function $c(\cdot)$ and $c_0 > 0$.

Deterrence provided by a fully rewarded plaintiff

Suppose for now that the plaintiff is fully rewarded, $R = Z$. Under this specification, it is straightforward to verify that for all penalties Z sufficiently large, the two parties produce the same amount of evidence in equilibrium — both after defendant actions B and G. So in either case, the equilibrium probability that the defendant loses the court case and is punished is $F(0)$.

The only deterrence effect comes from the difference in the defendant's equilibrium expenditures. Again, it is straightforward to show that $x^*_{DB} = x^*_{DG} + c_0$: the defendant spends c_0 more on evidence production after he has taken action B than G.

Given this, for all Z sufficiently large we can conclude that the deterrence provided in this case is simply $\zeta(Z) = c_0$. Raising Z does not have any effect of deterrence.

Deterrence provided by an unincentivized plaintiff

From Proposition 3 it is straightforward to show that reducing the plaintiff's award will increase the deterrence provided in this case — and so provide a level of deterrence unachievable by any punishment Z under full plaintiff awards. In fact, if one moves to the extreme of a completely unincentivized plaintiff, then the deterrence that can be provided is in fact unbounded.

To see this, start by noting that for all Z large enough the defendant's equilibrium expenditures must satisfy the FOC

$$\frac{F'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)} = \frac{1}{Z}$$
$$\frac{F'\left(c^{-1}\left(x^*_{DB}\right)-c^{-1}\left(c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DB}\right)\right)} = \frac{1}{Z}$$

As the penalty Z grows large the resources the defendant devotes to evidence production also grow without bound, i.e., $x^*_{DG}\to\infty$ and $x^*_{DB}\to\infty$ as $Z\to\infty$. Moreover, the defendant always devotes more to evidence production when he has the disadvantage of arguing against the facts. To see this, note that if this were not the case then the FOC could not both hold at equality since we would have

$$\frac{F'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)} < \frac{F'\left(c^{-1}\left(x^*_{DB}\right)-c^{-1}\left(c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DB}\right)\right)}$$

The defendant's chances of winning the case are also higher when he has behaved, i.e. $c^{-1}\left(x^*_{DG}+c_0\right) > c^{-1}\left(x^*_{DB}\right)-c^{-1}\left(c_0\right)$. To see this, note that if this were not the case then the FOC could not both hold at equality since we would have

$$\frac{F'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DG}+c_0\right)\right)} > \frac{F'\left(c^{-1}\left(x^*_{DB}\right)-c^{-1}\left(c_0\right)\right)}{c'\left(c^{-1}\left(x^*_{DB}\right)\right)}$$

From the observations immediately above, the defendant's probability of winning the case converges to $\lim_{y\to\infty}F\left(y\right)$ regardless of his action. Nonetheless, arbitrarily large incentives can be provided by a making Z large enough. The reason is that provided mild assumptions on the curvature of $c\left(\cdot\right)$ and $F\left(\cdot\right)$ are satisfied, the extra amount that the defendant spends after $a=B$ grows without bound as the punishment Z becomes large, i.e., $x^*_{DB}-x^*_{DG}\to\infty$ as $Z\to\infty$.

Proposition 5. ***(Unbounded deterrence from an unincentivized plaintiff)***
Suppose that the marginal cost of evidence production grows without bound ($c'\left(y\right)\to\infty$ as $y\to\infty$) and that F''/F' and c''/c' are both bounded from above; and if $F''/F'\to 0$ then it does so at a slower rate than $c'\to\infty$. Under these conditions, the deterrence provided by an unincentivized plaintiff can be made arbitrarily large by making Z arbitrarily large.[22]

[22]Example 1 with evidence production functions defined by (1) the signal θ distributed logistically satisfies the assumptions on c''/c' and F''/F'.

Proof of Proposition 5: From the FOC, for all Z sufficiently large we have

$$F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)=F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)c'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)$$

By the mean-value theorem, there must exist some

$$\tilde{y}\in\left[c^{-1}\left(x_{DG}^{*}+c_{0}\right),c^{-1}\left(x_{DB}^{*}\right)\right]$$

such that

$$c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)-c'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)=c''\left(\tilde{y}\right)\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)$$

Thus

$$\begin{aligned}&\left(F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)-F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)\right)c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)\\=&-c''\left(\tilde{y}\right)\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)\end{aligned}$$

Since trivially

$$\begin{aligned}F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)&=F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)\\&\quad-\left(F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)-F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)\right)\end{aligned}$$

we have

$$\begin{aligned}&\left(F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)-F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)\right)\\&\times\left(c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)-c''\left(\tilde{y}\right)\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)\right)\\=&-c''\left(\tilde{y}\right)\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)\end{aligned}\tag{21}$$

A second application of the mean-value theorem implies that there exists

$$\hat{y}\in\left[c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right),c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right]$$

such that

$$\begin{aligned}&F'\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)\right)-F'\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(c_{0}\right)\right)\\=&F''\left(\hat{y}\right)\left(c^{-1}\left(x_{DG}^{*}+c_{0}\right)-c^{-1}\left(x_{DB}^{*}\right)+c^{-1}\left(c_{0}\right)\right)\end{aligned}$$

Substituting into equation (21) and dividing both sides by

$$F'\left(c^{-1}\left(x_{DG}^{*}+c_0\right)\right)c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)\left(c^{-1}\left(x_{DG}^{*}+c_0\right)-c^{-1}\left(x_{DB}^{*}\right)\right)$$

gives

$$\begin{aligned}
&\frac{F''(\hat{y})}{F'\left(c^{-1}\left(x_{DG}^{*}+c_0\right)\right)}\left(1+\frac{c^{-1}(c_0)}{c^{-1}\left(x_{DG}^{*}+c_0\right)-c^{-1}\left(x_{DB}^{*}\right)}\right)\\
&\times\left(1-\frac{c''(\tilde{y})}{c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)}\left(c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_0\right)\right)\right)\\
=\;&\frac{c''(\tilde{y})}{c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)} \qquad (22)
\end{aligned}$$

Suppose that contrary the claimed result, there exists some Δ such that $x_{DB}^{*}-(x_{DG}^{*}+c_0)\leq\Delta$ for all Z, no matter how large. Observe that $x_{DB}^{*},x_{DG}^{*}\rightarrow\infty$ as $Z\rightarrow\infty$.

Since $\tilde{y}\leq c^{-1}\left(x_{DB}^{*}\right)$,

$$\frac{c''(\tilde{y})}{c'\left(c^{-1}\left(x_{DB}^{*}\right)\right)}\leq\frac{c''(\tilde{y})}{c'(\tilde{y})},$$

and so the right-hand side of equation (22) is bounded above by assumption.

Yet another application of the mean-value theorem implies that there exists $\check{y}\in[x_{DG}^{*}+c_0,x_{DB}^{*}]$ such that

$$c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_0\right)=\left(x_{DB}^{*}-\left(x_{DG}^{*}+c_0\right)\right)\frac{1}{c'\left(c^{-1}(\check{y})\right)}.$$

Consequently, as $Z\rightarrow\infty$, the gap between the evidence actually produced in the two states must approach 0, i.e., $c^{-1}\left(x_{DB}^{*}\right)-c^{-1}\left(x_{DG}^{*}+c_0\right)\rightarrow 0$. Since $\hat{y}\leq c^{-1}\left(x_{DG}^{*}+c_0\right)$,

$$\left|\frac{F''(\hat{y})}{F'\left(c^{-1}\left(x_{DG}^{*}+c_0\right)\right)}\right|\geq\left|\frac{F''(\hat{y})}{F'(\hat{y})}\right|.$$

CASE: F''/F' is bounded away from 0. It is immediate that the left-hand of (22) is unbounded. This gives the required contradiction and completes the proof.

CASE: $F''/F'\rightarrow 0$. The limit of the left-hand side of (22) is

$$-\lim_{Z\rightarrow\infty}\left|\frac{F''(\hat{y})}{F'\left(c^{-1}\left(x_{DG}^{*}+c_0\right)\right)}\right|\frac{c^{-1}(c_0)\,c'\left(c^{-1}(\check{y})\right)}{x_{DB}^{*}-\left(x_{DG}^{*}+c_0\right)} \qquad (23)$$

Since $c'\left(c^{-1}(\check{y})\right) \geq c'\left(c^{-1}(x^*_{DG} + c_0)\right) \geq c'(\hat{y})$, expression (23) is in turn less than

$$-\lim_{Z\to\infty} \left|F''(\hat{y})/F'(\hat{y})\right| c'(\check{y})\, c^{-1}(c_0)/\Delta.$$

But by assumption this is $-\infty$, giving a contradiction and completing the proof. ∎