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Philip Bond[1]

Are cheap talk and hard evidence both needed in the courtroom?

Abstract: In a recent paper, Bull and Watson (2004) present a formal model of verifiability in which cheap messages are shown to play no role. The current paper characterizes situations where this conclusion does, and does not, hold. In particular, I show that cheap messages and the possibility court-imposed Pareto-dominated outcomes are *complements*: while neither individually expands the set of implementable allocations, they do so when used in combination.

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Introduction

In a recent stimulating paper, Bull and Watson (2004, henceforth BW) present a formal model of verifiability. In each state players possess a set of documents which they decide whether or not to disclose to a court. Any document that exists in all states is said to be *cheap*. All non-cheap documents are referred to as *hard-evidence*. In the setting they study, BW establish that only hard-evidence matters for verifiability.

In real-world courts, hard-evidence and cheap messages clearly coexist. BW's analysis is restricted to the case in which agents are risk-neutral, and the outcomes which the agents wish the court to enforce are balanced monetary transfers. In this paper I generalize BW's analysis, and characterize conditions under which both hard-

[1]University of Pennsylvania, Wharton Finance Department, Steinberg Hall-Dietrich Hall, 3620 Locust Walk, Philadelphia, PA 19104-6367, Email: pbond@wharton.upenn.edu. I thank Mitchell Berlin, Jesse Bull, Hulya Eraslan, Yaron Leitner, Joel Watson, and an anonymous referee and associate editor for helpful comments. Any remaining errors are my own.

evidence and cheap messages are called for. Specifically, I establish the following:
(I) If parties can renegotiate away from any Pareto dominated outcome to a Pareto efficient one, then there is no loss in restricting mechanisms to depend only on hard-evidence.
(II) Under fairly mild conditions on the structure of hard-evidence, when cheap messages are not used the ability of parties to renegotiate away from Pareto dominated outcomes makes no difference to the set of implementable allocations.
(III) However, non-renegotiable outcomes and mechanisms that make use of cheap messages can be usefully combined in the following sense: more allocations are implementable when both devices are employed than if only one is.

Combined, (I) - (III) say that cheap messages and non-renegotiable outcomes are complements: while neither individually expands the set of implementable allocations, they do so when used in combination.

The model

In each state $a \in A$ each of players $i = 1, 2$ is endowed with a state-contingent set of *documents* $D_i(a)$. Write $D_i = \cup_{a \in A} D_i(a)$ for the set of all documents possessed by player i in at least one state. Without loss, assume $D_1 \cap D_2 = \varnothing$. Player i can *positively distinguish* state a from state b if $D_i(a) \not\subset D_i(b)$. A document $d \in D_1 \cup D_2$ is *cheap* if it exists in every state. All non-cheap documents are *hard*. Let D_i^H be player i's set of hard documents, i.e., the set of non-cheap documents possessed by player i in at least one state. Let $\mathcal{D}$ denote the set of all possible document disclosures. Both the state space A and document sets D_1, D_2 are finite.

Let X be the set of all possible outcomes. Each player $i = 1, 2$ has a complete preference ordering over X, given by $\succcurlyeq_i$. Preferences do not depend on the state a. BW restrict attention to the case in which X is the set of balanced transfers between players 1 and 2, and in which both players are risk-neutral.[2]

The players want a court to enforce a state contingent *outcome function*, $g : A \rightarrow X$. The state a is observed by both players $1, 2$, but not by the court. The court is free to enforce any outcome $x \in X$. The court can condition its actions only on what documents the players choose to disclose. Let $\beta_i : A \rightarrow \mathcal{D}$ denote the *document disclosure strategy* of player i. In this setting, BW define an outcome function g as being *implementable* if there exists some function $m : \mathcal{D} \rightarrow \Re$, and a pair of document disclosure strategies β_1, β_2, such that (I) $g(a) \approx_i m(\beta_1(a) \cup \beta_2(a))$ for $i = 1, 2$, all $a \in A$, and (II) the document disclosure strategies form a Nash equilibrium of the game defined by m. The function m is termed the *contract.* Further, an outcome function g is *uniquely* implementable if there exists a contract m such that for any equilibrium (β_1, β_2) of the disclosure game induced by m, both players $1, 2$ are indifferent between $g(a)$ and the outcome $m(\beta_1(a) \cup \beta_2(a))$ for all $a \in A$.[3]

Among BW's main results is Theorem 2, which states that an outcome function g can be implemented if and only if it can be implemented using a contract m that induces full disclosure, i.e., if given m then $\beta_i(a) = D_i(a)$ is an equilibrium. As BW observe, an immediate consequence of Theorem 2 is that "cheap documents play no role in implementation here." In other words, if an outcome function g can be implemented, it can implemented using a contract m that does not depend on whether cheap documents are presented.

As a benchmark for use throughout the paper, observe that if no hard documents

[2]In a second paper, Bull and Watson (Forthcoming) consider a model in which the outcome to be enforced is arbitrary (i.e., is not restricted to be a zero-sum transfer). Their focus is on establishing versions of the revelation principle in this environment, and on relating their model to previous contributions (notably Green and Laffont 1986). See also footnote 7 below.

[3]In BW's zero-sum setting (agents are risk-neutral and outcomes are balanced transfers) all equilibria of a game generate the same payoffs, and so there is no distinction between *weak* and *unique* implementation.

are available (or equivalently, if the contract m makes no use of hard documents) then only completely constant outcome functions g are implementable. This is immediate from standard implementation theory:[4] by assumption, there is no preference reversal across states $a \in A$.

A motivating example

Consider the following example. The state space is $A = \{P, NP\}$, where P can be thought of as the state in which player 2 (the "agent") has performed some pre-specified task for player 1 (the "principal"). Likewise, NP can be thought of as indicating that player 2 has not performed the task. The outcome set X is the set of lotteries over balanced transfers between the two players. Let $(x, -x)$ refer to the degenerate lottery in which player 1 receives a transfer of x from player 2.

The outcome function g that the players would like a court to enforce is: player 2 pays a fine $F > 0$ to player 1 in the non-performance state NP. In state P no transfer is to take place. Thus $g(NP) = (F, -F)$ and $g(P) = (0, 0)$.

The only hard-evidence consists of a document d_1 that is possessed by player 1 in state P. In other words, player 1 possesses decisive proof that player 2 has performed the task.[5] Let C_i denote the set of cheap documents possessed by player i.

Remark 1: If both players are risk-neutral, the outcome function g cannot be weakly implemented.

This is an immediate application of BW's Theorem 1: player 1 prefers $g(NP)$ to $g(P)$, but player 1 cannot positively distinguish state NP, and player 2 cannot positively distinguish state P.

[4] See, e.g., Maskin (1999).

[5] For example, the task might consist of manufacturing and delivering a widget. In this case, the widget itself can serve as the document d_1.

Remark 2: For any level of state-invariant risk aversion, the outcome function g cannot be weakly implemented without the use of cheap documents.

Suppose that it were in fact possible to implement the desired transfer. Since we are ruling out the use of cheap documents, the only possible specification of the contract is $m(\{d_1\}) = (0, 0)$ and $m(\varnothing) = (F, -F)$. But then player 1 always prefers to keep the document hidden, and receive the fine F. Consequently implementation fails.

Remark 3: The outcome function g can sometimes be uniquely implemented using a combination of cheap documents and hard-evidence.

Suppose now that at least one of the two players is risk-averse. Let $\tilde{l}$ be a lottery such that player 1 prefers the transfer 0 to the lottery $\tilde{l}$, and player 2 prefers making a certain transfer of F to the lottery $\tilde{l}$. Such a lottery exists either if F is small enough (holding risk aversion fixed), of if one of the players is sufficiently risk-averse (holding F fixed).

Suppose that player 1 has no cheap documents, while player 2 has a single cheap document c_2. Then the following contract m uniquely implements g:

	$\varnothing$	$\{c_2\}$
$\{d_1\}$	$(0, 0)$	$(F + \varepsilon, -F - \varepsilon)$
$\varnothing$	$\tilde{l}$	$(F, -F)$

This is easily seen. First, suppose the state is P, so that player 1 possesses the document d_1. It is a dominant strategy for player 1 to disclose d_1. Given this, player 2 is best off disclosing nothing. The equilibrium outcome is $(0, 0)$. Second, suppose the state is NP. In this case only player 2 has a disclosure decision to make. He discloses c_2, resulting in outcome $(F, -F)$.

Intuitively, player 2's cheap document c_2 can be interpreted as a statement "I admit that the state is NP." The contract m is structured so that if neither player 1 discloses his hard document d_1, nor player 2 makes his cheap report c_2, then both players are punished.

Finally, it is worth pointing out that the contract m is not unrealistic. If player 1 claims non-performance (i.e., does not disclose d_1) and player 2 claims performance (i.e., does not admit non-performance by disclosing c_2), then the resulting lottery $\tilde{l}$ can be thought of as representing uncertainty about the court's final ruling.

Remainder of paper

In this example, cheap messages and off-equilibrium path lotteries are complements. If used individually, neither allows for the outcome function g to be implemented; while if used togther, implementation of g is possible. The remainder of the paper generalizes these observations. Specifically, Propositions 1, 2 and 3 generalize Remarks 1, 2 and 3.

Implementation under renegotiation-proofness

As noted, BW conduct their analysis for the case in which players are risk-neutral, and the outcomes that the players wish the court to enforce are balanced monetary transfers. As such, all outcomes are renegotiation proof. I start by generalizing their analysis to the case in which preferences and outcome sets X are arbitrary, but the players are able to renegotiate all outcomes.

Formally, renegotiation takes place according to the (unmodelled) process $h : X \to X$. That is, the players renegotiate outcome x to outcome $h(x)$. The only assumptions I make about the renegotiation process h are that (I) the outcome $h(x)$ is Pareto optimal for all pre-renegotiation outcomes $x \in X$, and (II) both players weakly prefer $h(x)$ to x, i.e., $h(x) \succsim_i x$ for $i = 1, 2$. Note that if x is itself Pareto optimal, then the second of these assumptions implies $h(x) \approx_i x$ for $i = 1, 2$.

After taking account of renegotiation, if documents D are disclosed the contract m produces a final outcome $h(m(D))$. Define $m^h \equiv h \circ m$.

I establish the following analogue of BW's Theorem 1. Given that renegotiation results in a Pareto optimal outcomes, I restrict attention to the implementation of outcome functions g that are Pareto efficient (in terms of the outcomes they call for).

Proposition 1. *Suppose an outcome function g is Pareto efficient. Then g is uniquely implementable if and only if whenever $g(a) \succ_1 g(b)$ then either player 1 can positively distinguish a from b (i.e.,$D_1(a) \not\subset D_1(b)$) , or player 2 can positively distinguish b from a (i.e.,$D_2(b) \not\subset D_2(a)$).*

Proof of Proposition 1: The proof is similar to BW's proof of Theorem 1, and is relegated to the appendix. The main complication relative to their proof is establishing uniqueness of equilibrium. ∎

The proof of the sufficiency half of Proposition 1 is constructive, in that it exhibits a contract m that implements the outcome function g. Cheap documents do not play an essential role in the contract m:

Corollary 1. *Suppose an outcome function g is Pareto efficient. Then g is uniquely implementable if and only if it is uniquely implementable by a contract that makes no use of cheap documents.*

Proof of Corollary 1: See appendix. ∎

Hard evidence and implementability

Proposition 1 extends BW's Theorem 1 to any setting in which players can efficiently renegotiate away from Pareto dominated outcomes. Remark 2 in the opening example above also suggests a second class of circumstances in which Theorem 1 may hold, namely those in which the outcome function g specifies Pareto optimal outcomes for all states $a \in A$, and the contract m uses *only hard documents.*

In general, Remark 2 does not hold unless further restrictions are placed on the economy. To motivate these restrictions, consider the following two examples in which an outcome function is implementable using only hard documents, despite failing the condition of BW's Theorem 1. In both examples, X is the set of consumption lotteries, and at least one of the players is risk-averse.

Example 1. *As in the opening example, there are two states,* $A = \{P, NP\}$*, and the outcome function* g *to be implemented is: player* 2 *pays player* 1 *an amount* F *in state* NP *and an amount* 0 *in state* P*. Player* 1 *possesses a hard document* d_1 *in state* P*, while player* 2 *possesses a hard document* d_2 *in state* NP*. There are no cheap documents.*

Provided F *is sufficiently small, the outcome function* g *can be implemented using a contract* m *that specifies the lottery* $\tilde{l}$ *if no documents are disclosed (*$\tilde{l}$ *is defined as in the opening example), the transfer* F *from player* 2 *to* 1 *if* d_2 *is disclosed, and no transfer if* d_1 *is disclosed.*

Example 2. *There are four states, which for convenience we identify with the document allocations:* $A = \{\varnothing, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$*. When it exists, document* d_i *is possessed by player* i*. There are no cheap documents. Consider the outcome function*

$$g(a) = \begin{cases} (0,0) & \text{if } a = \varnothing \text{ or } \{d_1\} \\ (-F_1, F_1) & \text{if } a = \{d_2\} \\ (F_2, -F_2) & \text{if } a = \{d_1, d_2\} \end{cases}.$$

where $F_1, F_2 > 0$*. Note that player* 1 *strictly prefers* $g(\{d_1, d_2\})$ *to* $g(\{d_1\})$*, but player* 1 *cannot positively distinguish* $\{d_1, d_2\}$ *form* $\{d_1\}$ *and player* 2 *cannot positively distinguish* $\{d_1\}$ *from* $\{d_1, d_2\}$*.*

Provided F_2 is sufficiently small there exists a lottery $\tilde{l}$ such that player 1 prefers $(0,0)$ to $\tilde{l}$ and player 2 prefers outcome $(F_2, -F_2)$ to $\tilde{l}$. In this case, the outcome function g can be implemented using the following contract:

$$m(D_1 \cup D_2) = \begin{cases} (0,0) & \text{if } D_1 \cup D_2 = \varnothing \\ \tilde{l} & \text{if } D_1 \cup D_2 = \{d_1\} \\ (-F_1, F_1) & \text{if } D_1 \cup D_2 = \{d_2\} \\ (F_2, -F_2) & \text{if } a = \{d_1, d_2\} \end{cases}.$$

Observe that in equilibrium player 1 discloses nothing in state $\{d_1\}$.

How do Examples 1 and 2 differ from the opening example? In Example 1, although player 2's document d_2 is hard, it is used in exactly the same way as the cheap document c_2 in the opening example. In Example 2, there is a sense in which document d_1 is redundant in state $\{d_1\}$: the same outcome function would be implemented by the same contract if states $\varnothing$ and $\{d_1\}$ were combined, and document d_1 were unavailable in this combined state.

Examples of this sort are impossible under the following pair of additional conditions. The first condition is that the *state and evidence environment must be complete*, in the sense defined by BW: for any state $a \in A$ and any subset $D_1' \cup D_2'$ of $D_1(a) \cup D_2(a)$, there exists a state $a' \in A$ such that $D_1(a') \cup D_2(a') = D_1' \cup D_2'$. Note that completeness is not satisfied in Example 1, since there is no state in which the document set is $\varnothing$. For the same reason, completeness is not satisfied in any setting in which cheap documents are available.

The second condition is that there should not exist any "spare" hard documents that can be used as cheap documents. Formally, for a given outcome function g we say that there are *no spare documents* if $g(a) \approx_i g(a')$ (for $i = 1, 2$) implies $D_1(a) \cup D_2(a) = D_1(a') \cup D_2(a')$: in other words, if whenever g specifies equivalent outcomes in different states, the documents available in these states are the same.

This condition clearly fails to hold in Example 2. Conversely, it holds (vacuously) whenever the outcome function g specifies a different outcome in each state.

Observe that both completeness and the no spare document condition are satisfied in the initial example if the only document available is player 1's hard document d_1. Under these conditions the following generalization of Remark 2 holds:

Proposition 2. *Consider a Pareto efficient outcome function g. Suppose that the state and evidence environment is complete, and that there are no spare documents. Then g is implementable if and only if whenever $g(a) \succ_1 g(b)$ then either player 1 can positively distinguish a from b, or player 2 can positively distinguish b from a.*

Proposition 2 differs from Proposition 1 in that the latter requires all outcomes of the contract, both on and off the equilibrium path, to be Pareto efficient, whereas the former requires only that contract outcomes reached in equilibrium be Pareto efficient. Nonetheless, under the conditions stated this greater flexibility does not expand the range of outcome functions that can be implemented. The reason is that — as established in the proof below — the conditions ensure that all possible disclosures arise in equilibrium, and so there are no off-equilibrium outcomes.

Proof of Proposition 2: The proof makes use of the following claim, which is proved below:

Claim: The outcome function g can be implemented only by full-disclosure.

Assume for now that the claim holds. Sufficiency follows exactly as in Proposition 1. (Details are in the appendix.) Necessity is established as follows. Let m be a contract that implements g, and (β_1, β_2) an equilibrium of the disclosure game. Take a pair of states a and b such that player 1 *cannot* positively distinguish a from b and player 2 *cannot* positively distinguish b from a. As such, $D_1(a) \subset D_1(b)$ and $D_2(b) \subset D_2(a)$. The fact that (β_1, β_2) is an equilibrium implies that

$$g(b) \approx_1 m(\beta_1(b) \cup \beta_2(b)) \succcurlyeq_1 m(\beta_1(a) \cup \beta_2(b))$$

and

$$g(a) \approx_2 m(\beta_1(a) \cup \beta_2(a)) \succcurlyeq_2 m(\beta_1(a) \cup \beta_2(b)).$$

By completeness, $\beta_1(a) \cup \beta_2(b) = D_1(a') \cup D_2(a')$ for some state a'. By the above claim, $D_1(a') \cup D_2(a') = \beta_1(a') \cup \beta_2(a')$. It follows that

$$g(a') \approx_i m(\beta_1(a') \cup \beta_2(a')) = m(\beta_1(a) \cup \beta_2(b)).$$

From the Pareto optimality of the outcomes $g(a)$, $g(b)$ and $g(a')$ it follows that $g(b) \succcurlyeq_1 g(a)$, establishing the result.

Proof of Claim: Suppose to the contrary that there exists a contract m, an equilibrium (β_1, β_2) of the induced document disclosure game, and a state $a_0 \in A$ such that $m(\beta_1(a) \cup \beta_2(a)) \approx_i g(a)$ for all $a \in A$; and $\beta_1(a_0) \cup \beta_2(a_0) \subsetneqq D_1(a_0) \cup D_2(a_0)$. Starting from a_0, iteratively define a sequence $a_0, a_1, \ldots, a_N$ such that $D_1(a_k) \cup D_2(a_k) = \beta_1(a_{k-1}) \cup \beta_2(a_{k-1})$ for each $k = 1, \ldots, N$; $\beta_1(a_k) \cup \beta_2(a_k) \subsetneqq D_1(a_k) \cup D_2(a_k)$ for $k < N$; and $\beta_1(a_N) \cup \beta_2(a_N) = D_1(a_N) \cup D_2(a_N)$. Such a sequence exists by completeness and the finiteness of the starting document set $\beta_1(a_0) \cup \beta_2(a_0)$. To complete the proof of the claim, simply observe that

$$\beta_1(a_N) \cup \beta_2(a_N) = D_1(a_N) \cup D_2(a_N) = \beta_1(a_{N-1}) \cup \beta_2(a_{N-1}),$$

and so for $i = 1, 2$,

$$g(a_N) \approx_i m(\beta_1(a_N) \cup \beta_2(a_N)) = m(\beta_1(a_{N-1}) \cup \beta_2(a_{N-1})) \approx_i g(a_{N-1}).$$

This violates the no spare documents condition since strictly more documents are available in state a_{N-1} than state a_N. This completes the proof of the claim. ∎

Implementation when renegotiation is not possible, and cheap documents are available

So far the paper has shown that (under conditions given) the same outcome functions are implementable in each of the following three scenarios: (a) efficient renegotiation of court-enforced outcomes is possible, and no cheap messages are used; (b) efficient renegotiation of court-enforced outcomes is possible, and cheap messages are available; and (c) efficient renegotiation of court-enforced outcomes is *not* possible, but no cheap messages are used. That is, by themselves neither the ability of a court to impose non-renegotiable outcomes, nor to make use of cheap messages, adds anything to implemention possibilities.

We now turn to the case in which both devices are used together — that is, cheap messages are available, and the court can impose outcomes in X that are Pareto dominated without parties renegotiating to an alternate Pareto efficient outcome. A leading example is an economy in which at least one player is risk-averse, the set of outcomes includes lotteries, and in which there is no time between the announcement of the lottery and its realization for renegotiation to occur. A second common example is that in which parties can commit to make transfers to outside "third" parties. To reiterate, the point of interest here is the interaction of a court's ability to impose inefficient outcomes with the use of cheap messages.

In the economy under consideration, unique implementation of $g(a)$ is possible *only if* g is measurable with respect to the document partition, P^D.[6] The reason is standard. Since preferences do not depend on the state $a \in A$, there is no preference reversal across states. It follows that if exactly the same documents are available in states a, a', the equilibrium sets also coincide. As such, the document partition P^D places an upper bound on the set of uniquely implementable outcome functions.

The main result of the current section is that, in many circumstances, this upper bound is achievable. Moreover, from Proposition 2 cheap documents are essential to achieving this upper bound.[7]

[6] States a and a' belong to the same element of the document partition P^D if the same documents are available in the two states, i.e., $D_1(a) \cup D_2(a) = D_1(a') \cup D_2(a')$.

[7] In a related spirit, Bull and Watson (Forthcoming) consider whether multistage mechanisms are required for implementation. They show that multistage mechanisms may be needed even for weak

Formally, Proposition 3 below gives a set of sufficient conditions for an outcome function g that is P^D-measurable to be uniquely implementable. The proof is constructive. First, define $n_i = \max_a \left|D_i(a) \cap D_i^H\right|$, the maximum number of hard documents that player i has available in any state. Next, for a candidate outcome function g, consider a set of outcomes

$$\{z_{kl}(a), x_{1-}(a), x_{2-}(a), x_{--} | a \in A, 0 \leq k \leq n_1, 0 \leq l \leq n_2\}.$$

The outcome function g is uniquely implementable if this set of outcomes satisfies the following three properties. First, for any $a \in A$, k and l,

$$\begin{aligned} z_{00}(a) &= g(a) \\ z_{k,l}(a) &\succ_1 z_{k+1,l}(a) \text{ and } z_{k,l}(a) \succ_2 z_{k,l+1}(a). \end{aligned} \tag{1}$$

That is, holding a and l (respectively, k) fixed, player 1 (respectively, player 2) strictly prefers outcomes $z_{kl}(a)$ with lower k (respectively, lower l).

Second, for any $a, a' \in A$, k and l,

$$\begin{aligned} z_{kl}(a) &\succ_1 x_{1-}(a') \text{ and } z_{kl}(a) \succ_2 x_{2-}(a') \\ x_{2-}(a) &\succ_1 g(a) \text{ and } x_{1-}(a) \succ_2 g(a). \end{aligned} \tag{2}$$

The first component of condition (2) says $x_{1-}(\cdot)$ and $x_{2-}(\cdot)$ can serve as effective punishments for players 1 and 2 respectively. The second component says that $x_{2-}(\cdot)$ and $x_{1-}(\cdot)$ can serve as effective rewards for players 1 and 2 respectively.

implementation if the evidentiary structure fails a condition they term *evidentiary normality* — a condition which holds automatically in the document-disclosure environment of Bull and Watson (2004). While they stress the dynamic aspect of the example they use to establish this claim, the example is also one in which both hard evidence and cheap documents are necessary for (weak) implementation. Their example, however, relies critically on the failure of evidentiary normality, which is a property that they argue elsewhere in the paper is an "intuitive condition" and "is commonly satisfied in reality." In contrast, evidentiary normality holds throughout the current paper.

Third, for any $a \in A$, k and l,

$$z_{kl}(a) \succ_i x_{--} \text{ for } i = 1, 2. \tag{3}$$

That is, outcome x_{--} punishes both players simultaneously.

Proposition 3. *Suppose that $g : A \to X$ is P^D-measurable, and there exist outcomes*

$$\{z_{kl}(a), x_{1-}(a), x_{2-}(a), x_{--} | a \in A, 0 \leq k \leq n_1, 0 \leq l \leq n_2\}$$

satisfying conditions (1) - (3). Then g is uniquely implementable.

The conditions of Proposition 3 are satisfied in many standard economic environments. In particular, they are satisfied in both of the following settings:

(i) Players are risk-neutral over non-negative consumption, and have initial wealth allocations W_1 and W_2. The outcome set X consists of consumption allocations, and transfers *to* (but not from) third-parties are possible. Proposition 3 applies whenever the outcome function g is such that both players receive strictly positive consumption for all possible $a \in A$.

(ii) Players 1,2 are weakly risk-averse over non-negative consumption, with utility functions u_1 and u_2 and initial wealth allocations W_1 and W_2. The outcome set X consists of lotteries over consumption allocations. No transfers to third-parties are possible. For at least one of the two players $i \in \{1, 2\}$, $u_i(y_i) \to -\infty$ as $y_i \to 0$. The outcome function g to be implemented gives strictly positive consumption to both players in every state $a \in A$. (A proof of this claim is available from the author.)

Proof of Proposition 3: We show that g is implementable using a contract m defined as follows:

The contract m requires each player $i = 1, 2$ to disclose documents ($D_i \subset D_i(a)$ in state a), and to additionally make a "cheap" report $c_i \in A$ about the state.[8]

[8]Thus $m : D \times A \times A \to X$.

Throughout the proof, we say that player i's disclosure (D_i) and report (c_i) are *self-consistent* if and only if $D_i \subset D_i(c_i)$, i.e., if the set of documents disclosed are actually available when the state is the one claimed by the player. We say that player i's report is *truthful* in state a if $D_1(c_i) \cup D_2(c_i) = D_1(a) \cup D_2(a)$, i.e., if the state claimed by the player falls within the same element of the document partition P^D as the true state a.

When both players submit self-consistent disclosures and reports, and the reports agree, i.e., $c_1 = c_2 = c$,

$$m(D_1 \cup D_2; c_1, c_2) \equiv z_{|D_1(c)-D_1||D_2(c)-D_2|}(c).$$

If players submit self-consistent disclosures and reports, but the reports do not agree, i.e., $c_1 \neq c_2$, there are two separate cases. First, if either (A) player i's disclosure proves that player j is lying, but player j's disclosure does not prove that player i is lying, i.e., $D_i \not\subset D_i(c_j)$ and $D_j \subset D_j(c_i)$, or (B) according to their own reports, player i is fully disclosing $(D_i = D_i(c_i))$ but player j is not $(D_j \subsetneq D_j(c_j))$, then player i is rewarded and player j is punished,

$$m(D_1 \cup D_2; c_1, c_2) \equiv x_{j-}(c_j).$$

On the other hand, if neither of the above circumstances are met, both players are punished,

$$m(D_1 \cup D_2; c_1, c_2) \equiv x_{--}.$$

Finally, if player i's disclosure and report are self-consistent, but player j's are not, player i is rewarded and player j is punished,

$$m(D_1 \cup D_2; c_1, c_2) \equiv x_{j-}(c_j),$$

while if the disclosure and report of both players fail self-consistency, then both are punished,

$$m(D_1 \cup D_2; c_1, c_2) \equiv x_{--}.$$

Claim 1: It is an equilibrium for players to submit equal and truthful reports, and to fully disclose.

Proof: Consider any state $a \in A$. Suppose that player 2 reports truthfully ($D_1(c_2) \cup D_2(c_2) = D_1(a) \cup D_2(a)$) and fully discloses ($D_2 = D_2(a)$). If player 1 submits the same report, $c_1 = c_2$, and fully discloses, the outcome $z_{00}(c_2) = g(a)$ is implemented. If player 1 instead reports $c_1 = c_2$ but discloses less than $D_1(a)$, outcome $z_{k0}(c_2)$ results, where $k \geq 1$. If player 1 reports $c_1 \neq c_2$ and discloses D_1 either outcome $x_{1-}(c_1)$ or x_{--} results. (It cannot generate outcome $x_{2-}(c_2)$ since certainly $D_1 \subset D_1(c_2) = D_1(a)$ and $D_2 = D_2(c_2)$.) Since player 1 prefers outcome $g(a)$ to any of these alternatives, reporting truthfully and fully disclosing is a best response.

Claim 2: Reporting truthfully and fully disclosing is the only equilibrium.

Proof: Consider any state $a \in A$. Suppose that, contrary to the claim, there is an equilibrium (c_1, D_1, c_2, D_2) such that $D_1(c_i) \cup D_2(c_i) \neq D_1(a) \cup D_2(a)$ and/or $D_i \subsetneqq D_i(a)$ for at least one of $i = 1, 2$. We establish a contradiction by showing that in all cases at least one of the two players has an incentive to deviate.

First, suppose that at least one of the players submits an inconsistent report. If both players submit inconsistent reports, then player 1 can deviate and make a consistent report and disclosure, leading to an outcome $x_{2-}(c_2) \succ_1 g(c_2) \succ_1 x_{--}$. If only one player (player 1, say) submits an inconsistent report then he can deviate and report $\tilde{c}_1 = c_2$ and disclose $\tilde{D}_1 = \varnothing$. This generates an outcome $z_{kl}(c_2)$ for some k, l, which player 1 prefers since $z_{kl}(c_2) \succ_1 x_{1-}(c_1)$.

Second, suppose that both players submit consistent reports, but disagree, $c_1 \neq c_2$. There are three possible outcomes: x_{--}, $x_{1-}(c_1)$ and $x_{2-}(c_2)$. If the outcome is either x_{--} or $x_{1-}(c_1)$ then player 1 can deviate by reporting $\tilde{c}_1 = c_2$ and disclosing $\tilde{D}_1 = \varnothing$. This generates an outcome $z_{kl}(c_2)$ for some k, l, which player 1 prefers since $z_{kl}(c_2) \succ_1 x_{--}$ and $z_{kl}(c_2) \succ_1 x_{1-}(c_1)$. Likewise, if the outcome is $x_{2-}(c_2)$ then player 2 has an incentive to deviate.

Third, suppose that both players submit consistent reports, and agree, $c_1 = c_2 = c$. The outcome under such behavior is $z_{lk}(c)$, for some l, k. Regardless of whether or not $c = a$, less than full disclosure cannot be an equilibrium. For suppose to the contrary that one of the players — player 1, say — does not disclose fully: $D_1 \subsetneqq D_1(a)$. On the one hand, if $D_1(a) \subset D_1(c)$ then player 1 has the incentive to deviate and disclose $\tilde{D}_1 = D_1(a)$. On the other hand, if $D_1(a) \not\subset D_1(c)$ then $c_2 \neq a$. So player 1 has the incentive to report $\tilde{c}_1 = a$ and disclose $\tilde{D}_1 = D_1(a)$, which results in outcome $x_{2-}(c) \succ_1 g(c) \succcurlyeq_1 z_{lk}(c)$. (Outcome $x_{2-}(c)$ arises because player 1 has demonstrated that player 2 is misreporting.)

Finally, full and consistent disclosure with $c_1 = c_2 = c$ but $D_1(c) \cup D_2(c) \neq D_1(a) \cup D_2(a)$ cannot be an equilibrium. For in this case, $D_i = D_i(a) \subset D_i(c)$ for $i = 1, 2$, and at least one of the inclusions must be strict. Without loss, suppose that $D_2 = D_2(a) \subsetneqq D_2(c)$. Then player 1 can deviate and disclose $\tilde{D}_1 = D_1(a)$ and report $\tilde{c}_1 = a$, thereby generating outcome $x_{2-}(c) \succ_1 z_{lk}(c)$. (Outcome $x_{2-}(c)$ arises because player 1 has fully disclosed according to his own report, while player 2 has not.) This completes the proof of Proposition 3. ■

Concluding remarks

BW make an important contribution by formulating a model that highlights the role played by hard-evidence in determining what outcomes are and are not enforceable by an uninformed court. In doing so, they give a formal definition of "verifiability." BW focus exclusively on the case of balanced transfers between risk-neutral players. The resulting document disclosure games are then zero-sum. Under these assumptions, hard-evidence completely eliminates the role of cheap messages in determining what outcomes a court can and cannot enforce.

Hard-evidence clearly plays a role of great importance in real-world legal systems.

However, cheap talk of various kinds also appears important. Lawsuits do not simply consist of the two parties placing their evidence on the table. In this short paper I have explored one possible role of cheap talk — namely its role in expanding the set of outcomes functions that can be implemented by a court. I have demonstrated that hard evidence and cheap talk are complements: neither by itself expands the set of implementable allocation, but the conjunction of the two devices does so.

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Appendix

Proof of Proposition 1

The proof of necessity is exactly the same as in BW: since players renegotiate all outcomes, every outcome eventually attained is Pareto efficient. It is then easily checked that BW's argument applies. The proof of sufficiency is constructive, and makes use of a contract similar to that used by BW. As a preliminary, for all $(E_1, E_2) \in D_1 \times D_2$ define

$$\Lambda(E_1, E_2) = \{a | a \text{ satisfies } D_1(a) \subset E_1 \text{ and } E_2 \subset D_2(a)\}$$

and $\alpha(E_1, E_2) \in \Lambda(E_1, E_2)$ such that $g(\alpha(E_1, E_2)) \succcurlyeq_1 g(a)$ for all $a \in \Lambda(E_1, E_2)$. Next, define a contract m by

$$m(E_1 \cup E_2) = g(\alpha(E_1, E_2)).$$

The set $\Lambda(E_1 \cup E_2)$ is increasing in E_1 and decreasing in E_2. As such, if $E_1 \subset E_1'$ then $g(\alpha(E_1' \cup E_2)) \succcurlyeq_1 g(\alpha(E_1 \cup E_2))$. Likewise, if $E_2 \subset E_2'$ then $g(\alpha^1(E_1 \cup E_2)) \succcurlyeq_1 g(\alpha^1(E_1 \cup E_2'))$. Since g is Pareto efficient, it follows that $g(\alpha^1(E_1 \cup E_2')) \succcurlyeq_2 g(\alpha^1(E_1 \cup E_2))$. Consequently, full disclosure is a weakly dominant strategy for both players.

Exactly as in BW, it can be shown that $m(D_1(a) \cup D_2(a)) \approx_i g(a)$ for $i = 1, 2$, all $a \in A$. Since by assumption g is Pareto efficient, $m^h(D(a)) \approx_i g(a)$ also. Thus full disclosure is an equilibrium; and in this equilibrium, the equilibrium outcome in state a is $g(a)$. The remainder of the proof establishes that *all* equilibria have the same outcomes. This step was unnecessary in BW, since their game was zero-sum.

Fix a state a, and let E_i and E_j denote the players' equilibrium disclosures in that state. Since E_i is best response to E_j,

$$m^h(E_i \cup E_j) \succcurlyeq_i m^h(D_i(a) \cup E_j).$$

On the other hand, since full disclosure is a weakly dominant strategy,

$$m^h(D_i(a) \cup E_j) \succcurlyeq_i m^h(E_i \cup E_j).$$

Together, these imply

$$m^h(D_i(a) \cup E_j) \approx_i m^h(E_i \cup E_j).$$

Moreover, since $m^h(D_i(a) \cup E_j)$ and $m^h(E_i \cup E_j)$ are both Pareto efficient,

$$m^h(D_i(a) \cup E_j) \approx_j m^h(E_i \cup E_j).$$

Again, since full disclosure is weakly dominant,

$$\begin{aligned} m^h(D(a)) &\succsim_1 m^h(E_1 \cup D_2(a)) \approx_1 m^h(E_1 \cup E_2) \\ m^h(D(a)) &\succsim_2 m^h(D_1(a) \cup E_2) \approx_2 m^h(E_1 \cup E_2) \end{aligned}$$

Finally, it must be the case that both $m^h(D(a)) \approx_1 m^h(E_1 \cup E_2)$ and $m^h(D(a)) \approx_2 m^h(E_1 \cup E_2)$, since otherwise the Pareto efficiency of $m^h(E_1 \cup E_2)$ is violated. This completes the proof of Proposition 1. ∎

Proof of Corollary 1:

Simply modify the definition the set $\Lambda(E_1, E_2)$ in the proof of Proposition 1 to

$$\Lambda(E_1, E_2) = \left\{ a | a \text{ satisfies } D_1(a) \cap D_1^H \subset E_1 \cap D_1^H \text{ and } E_2 \cap D_2^H \subset D_2(a) \cap D_2^H \right\}.$$

The remainder of the contract m is unchanged. Under this revised definition, disclosure of cheap documents has no effect on equilibrium outcomes. The remainder of the proof is unchanged. ∎

Proof of Proposition 2 (sufficiency):

The contract m defined in the proof of Proposition 1 uniquely implements g. To see this, it suffices to establish that outcomes mandated under contract m are Pareto efficient: the proof of Proposition 1 then applies. Pareto efficiency of all outcomes in turn follows from the fact that $m(D_1(a) \cup D_2(a)) \approx_i g(a)$ for $i = 1, 2$, together with completeness of the state and evidence environment. ∎