How well do financial prices aggregate dispersed information?

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Abstract

I derive and implement a sufficient statistics formula for how well financial prices aggregate dispersed information. The key inputs are closely related to the outputs of the price-dividend predictability literature. The formula follows from market-clearing. Empirical implementation suggests a low level of aggregation: the information of a *single* representative investor is much more informative than the information conveyed by the price. I further derive formulae for the value of a representative investor's information (empirical implementation suggests the value is small); and for the information-implied demand elasticity (empirical implementation yields a highly inelastic value consistent with Gabaix and Koijen 2023's estimate).

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1 Introduction

It is commonplace to talk of financial markets as aggregating the information observed by dispersed investors. Canonical models in financial economics qualitatively capture the mechanism.[1] Separately, existing research estimates market efficiency, in the sense of how well prices forecast future cash flows.[2] What is missing, however, is a *quantitative* examination of the extent to which prices aggregate dispersed information.

The current paper seeks to fill this gap. Specifically, I derive a sufficient statistics formula that answers the question: For what value of N does observing the price but no dispersed information have the same information content as observing the information of N dispersed investors but no price? Implementation of the formula for the case of the aggregate US stock market suggests that the extent of information aggregation is low, and yields an estimate of $N \approx .06$. That is: Directly observing the information of even just a single typical investor (but no price) would be much more informative than observing the price.

Quantifying the extent of information aggregation requires estimates of the information content of market prices and the information held by dispersed investors. The former is readily estimated; as noted, a significant literature has done so in a variety contexts. As such, the innovation here is a sufficient statistics formula for the amount of information held by a representative investor.

I further derive a sufficient statistics formula for the *value* of a representative investor's information. That is: how different are the expected returns of investors who trade with and without the representative investor's information? Implementation of my formulae suggests both that the representative investor possesses significant information, in the sense of reducing the conditional variance of future cash flows by approximately 20%; but possession of this information yields only a small expected return advantage of approximately 3-4 basis points. The reconciliation of this pair of estimates is that most return variance is driven by discount rate innovations, and so superior information about cash flow innovations yields only a small trading advantage.

Finally, the estimates in this paper deliver an estimate for the price elasticity of demand for a financial asset. That is: the impact of a change in price on the quantity of an asset demanded stems from a combination of (i) the change in quantity demanded holding expectations about future cash flows and prices fixed, and (ii) the change in quantity demanded stemming from a shift in expected future prices and cash flows caused by the price change. My estimates of the information possessed by a representative investor and of the information

[1] See Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981).

[2] See, for example, Bai, Philippon and Savov (2016), Kacperczyk, Sundaresan and Wang (2021), Farboodi et al (2022), and Dávila and Parlatore (2025).

contained in asset prices imply an upper bound for the (absolute value of) demand elasticity for the market asset of .14. By way of comparison, Gabaix and Koijen (2023) estimate a price elasticity of $\approx \frac{1}{5}$, which they describe as "surprisingly inelastic." As such, my estimates suggest that the inelasticity identified by Gabaix and Koijen can be entirely accounted for by the information conveyed by prices, and specifically, by the fact that an increase in asset prices signals an increase in expected future cash flows and prices, partially offsetting the direct negative effect of an increase in today's price.

The key formula for the information advantage of the representative investor relative to the econometrician is (see Proposition 1)

$$1 - \frac{\frac{\partial[\text{econometrician's expected return}]}{\partial[\text{price}]} \frac{\partial[\text{price}]}{\partial[\text{future dividend innovation}]}}{\frac{\partial[\text{future return}]}{\partial[\text{future dividend innovation}]}}. \tag{1}$$

In particular, all three terms can be estimated from data on prices and dividends. The economic underpinning of (1) is as follows. The formula measures how much more information a representative investor has about future dividend innovations than does the econometrician. Some of this information is aggregated into the price, as reflected in the term $\frac{\partial[\text{price}]}{\partial[\text{future dividend innovation}]}$. The price-dividend predictability literature has found that high ratios predict low returns; that is, $\frac{\partial[\text{econometrician's expected return}]}{\partial[\text{price}]} < 0$. But for market clearing to hold, innovations to future dividends *cannot* reduce the representative investor's demand for an asset; and hence cannot affect the representative investor's return expectation. Intuitively, the gap between an innovation to future dividends affecting the econometrician's expected return but not the typical investor's expected return reveals a typical investor's information advantage relative to the econometrician about future dividends. Formalizing this argument yields (1).

Formula (1) gives the combined information advantage about future dividends of a typical investor relative to the econometrician. Part of this information advantage stems from signals that are directly informative about future dividends; it is exactly the information content of such signals that I wish to estimate. But part of the investor's information advantage derives instead from a typical investor potentially having knowledge of discount rate innovations beyond that possessed by the econometrician, and this information in turn being useful in interpreting what the market price implies about future dividends. The two sources of information advantage are challenging to fully disentangle. But by imposing the condition that each investor's demand curve is downwards sloping, I obtain bounds the contribution of each source.

In this paper I focus on the case of the aggregate stock market. In this case, and as (1) shows, the inputs for the sufficient statistics formulae that I derive are closely related to the literature that studies the predictability of market returns and dividends using the price-dividend ratio. I operationalize the formulae using the estimates of Binsbergen and Koijen (2010). The formulae I derive make clear the relevance of conditioning on the history of prices and dividends when forming expectations, and a distinguishing feature of Binsbergen and Koijen (2010) is to present a tractable method of doing exactly this. Closely related to these authors' approach, see the survey of Koijen and Van Nieuwerburgh (2011), and Cochrane (2011).

The key ingredient in the analysis is an exploitation of the market clearing condition. In addition, I focus on the first two moments of all relevant distributions; as such, the formulae derived should be viewed as approximations. The formulae hold in a framework general enough to nest standard models in which potentially heterogeneous investors trade for a mix of informational motives and responses to liquidity shocks, discount rate shocks, hedging needs, etc.

Related literature:

This paper is related to the large literature on price-dividend predictability. That literature adopts the (typically implicit) view that investors are symmetrically informed, and possess information unobservable to the econometrician. A separate and primarily theoretical literature has studied the process by which prices come to contain information, and emphasizes the idea that investors observe independent and noisy signals of economic fundamentals, which are aggregated into the price (classical references are Hayek 1945, Grossman 1976, Hellwig 1980). I link these literatures by showing how estimates from the price-dividend predictability literature can be used to infer how much information dispersed investors have.

Related, a significant literature quantifies the information content of prices; see references in footnote 2. In this paper I use estimates of the information content of prices to infer the information possessed by individual investors. My estimate of the information content of aggregate prices is in (62). Separately, there is a relation between the inputs required for the sufficient statistics formulae that I derive and Dávila and Parlatore (2025); see section 8.2.2.

The framework used to derive sufficient statistics formulae is related to Watanabe (2008) and Biais et al (2010). These papers note that uninformed investors will (rationally) experience below-market returns, because they increase their holdings when future returns are low. Glode (2011) and Savov (2014) use related observations to rationalize investment in active mutual funds with negative alphas, along with providing some evidence. The same economic force operates in this paper. Nonetheless, my estimates suggest that the "underperformance" of an uninformed investor is quantitively small.

A key step in my analysis it to characterize the weighted-average amount of information possessed by investors. This aspect of my analysis is complementary to Kadan and Manela (2019) and Farboodi et al (2025), who quantify the value to an investor of a given signal, for example, a macroeconomic employment report or quarterly earnings forecasts. Instead, in this paper I use observed correlations between prices and subsequent returns and dividends to infer how much information the representative investor possesses. Similarly, this aspect of my analysis is complementary to Egan, MacKay, and Yang (2022), who apply revealed-preference arguments to a cross-section of index funds in order to infer the expectations of investors who buy such funds. Relative to Egan, MacKay, and Yang, this paper has the advantage of shedding light on the average information of *all* investors in the market, via the use of market-clearing identities; but the disadvantage of saying nothing about heterogeneity among investors. I note that the evaluation of the extent of information aggregation by prices uses the average information of all investors.

The Probability of Informed Trade (PIN; Easley, Kiefer, and O'Hara 1996) quantifies the fraction of trade stemming from informed traders, and as such measures the extensive margin of information. In contrast, the measure in this paper captures both intensive and extensive margins. The measure in the current paper is also constructed entirely from pricing and dividend data, and does not require the use of order flow information, and as such is robust to changes in trading patterns (such as the proliferation of trading venues and the increasing prevalence of high-frequency traders). Also related, Kyle's lambda (Kyle 1985) is frequently estimated, and used as a proxy for the prevalence of informed trade; but it is challenging to relate the estimated value to a cardinal measure of the amount of informed trade.

Kurlat (2019) derives and implements a sufficient statistics formula for the ratio of private to social value of information in what is essentially the origination part of financial markets. This paper instead examines the amount and private value of information in a secondary financial market. Bond and García (2021) theoretically characterize the social value of private information in a related setting.

2 Framework

I derive sufficient statistics formulae using a general framework in which investors trade both because of differing expectations about returns, and because of shocks to desired holdings that resemble discount rate shocks. The framework is closely related to the canonical models of Grossman and Stiglitz (1980), Hellwig (1980), and especially Diamond and Verrecchia (1981). Following these papers, all random variables below are normal. As such, all results should be interpreted as approximations based on the first two moments of distributions.

The framework features a single risky asset. In the empirical implementation I will consider the S&P 500 index. I conjecture that many of the insights of the paper can be extended to multi-asset models.

A unit continuum of investors, indexed by i, trade a risky asset and a risk-free asset. The gross return of the risk-free asset is R_t, and the price of the risky asset at date t is P_t. The risk-free asset is in zero supply, and the supply of the risky asset is normalized to 1. The risky asset pays dividends D_t at the start of each period t. The (absolute) excess return on the risky asset from date t to $t+1$ is

$$X_{t+1} \equiv P_{t+1} + D_{t+1} - R_t P_t.$$

Write $\mathcal{I}_{i,t}$ for investor i's information at date t, detailed below. Let investor i's demand $q_{i,t}$ for the asset be a function of the expected return, $E\left[X_{t+1}|\mathcal{I}_{i,t}\right]$, and factors unrelated to returns, $Z_t + u_{i,t}$:

$$q_{i,t} = A_i E\left[X_{t+1}|\mathcal{I}_{i,t}\right] - B_i \left(Z_t + u_{i,t}\right). \tag{2}$$

In (2), A_i and B_i are potentially equilibrium objects. In particular, in the standard mean-variance framework, A_i depends on the combination of investor i's risk tolerance and $var\left[X_{t+1}|\mathcal{I}_{i,t}\right]$, where the latter is an equilibrium object.

The term Z_t is an aggregate "taste" shock to investors' desired asset holdings. As such, Z_t shifts prices independent of expectations of dividends. Following the literature, I will typically refer to Z_t as a discount rate shock. Similarly, $u_{i,t}$ is an investor specific shock to desired holdings.

At date t, investors receive private and noisy signals of next period's dividend D_{t+1} (see (4) below).

Write $\mathcal{H}_t$ for the history of exogenous cash flows and aggregate discount rate shocks, i.e.,

$$\mathcal{H}_t = \left\{D_t, Z_{t-1}, D_{t-1}, Z_{t-2}, D_{t-2}, \ldots\right\}.$$

Information in $\mathcal{H}_t$ is public at the date t trading stage.[3] Define the innovations

$$\begin{aligned} \epsilon_{D,t} &= D_{t+1} - E\left[D_{t+1}|\mathcal{H}_t\right] \\ \epsilon_{Z,t} &= Z_t - E\left[Z_t|\mathcal{H}_t\right]. \end{aligned}$$

I highlight the timing convention in $\epsilon_{D,t}$, which arises because investors observe noisy signals about D_{t+1} at date t, implying that the date t price P_t contains information about D_{t+1}.

[3] The lagged aggregate value of the discount rate term Z_{t-1} can be inferred from the history of dividends and equilibrium prices.

Assume $\epsilon_{D,t}$ and $\epsilon_{Z,t}$ are normally distributed and i.i.d. As noted, the normality assumption means that all results should be interpreted as approximations based on the first two moments of distributions. For use throughout, note that

$$\begin{aligned} E\left[\epsilon_{D,t}\right] &= E\left[\epsilon_{D,t}|\mathcal{H}_t\right] = 0 \\ var\left[\epsilon_{D,t}\right] &= var\left[\epsilon_{D,t}|\mathcal{H}_t\right], \end{aligned} \tag{3}$$

with parallel identities for $\epsilon_{Z,t}$, by the laws of total expectation and variance, respectively.

The assumption that $\epsilon_{D,t}$ and $\epsilon_{Z,t}$ are uncorrelated is important. While the analytical results below can be generalized to allow for correlation (notes available upon request), these generalizations are much harder to empirically implement. The assumption that dividend and discount rate innovations are uncorrelated fits well with how the literature has conceived of the origins of discount rate fluctuations (e.g., see review in Cochrane 2011). Even in a consumption-based asset pricing paper such as Bansal and Yaron (2004), discount rate fluctuations stem largely from fluctuations in cash flow volatility that are assumed to be uncorrelated with cash flow innovations.

Investor i observes at date t a private signal of date $t+1$ dividends,

$$y_{i,t} = D_{t+1} + \epsilon_{i,t}, \tag{4}$$

where $\epsilon_{i,t} \sim N\left(0, \tau_i^{-1}\right)$. In particular, τ_i is the precision of investor i's private signal. Note that the shock $Z_t + u_{i,t}$ both directly affects investor i's asset demand, and also serves as a signal about Z_t.

Regarding the process $\begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix}$, I impose only:

Assumption 1 *In equilibrium, the price innovation is linear in the dividend and discount rate innovations:*

$$P_t - E\left[P_t|\mathcal{H}_t\right] = c_D \epsilon_{D,t} + c_Z \epsilon_{Z,t}. \tag{5}$$

Appendix D establishes that if $\begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix}$ follows a VAR(1) process then there is indeed an equilibrium that satisfies the linearity condition (5). But for both transparency and generality, I establish all results starting directly from Assumption 1, and without imposing further conditions on the process $\begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix}$.

For the purposes of Proposition 3, regarding the value of information, and subsection 9.4, regarding the implied price elasticity of demand for the aggregate asset, I also use:

Assumption 2 *The unconditional mean of Z_t is zero: $E[Z_t] = 0$.*

Investor i's information $\mathcal{I}_{i,t}(\tau_i)$ consists of $\mathcal{H}_t$, combined with prices $\{P_t, P_{t-1}, \ldots\}$, private shocks to asset demand, $\{Z_t + u_{i,t}, Z_{t-1} + u_{i,t-1}, \ldots\}$, and private dividend signals $\{y_{i,t}, y_{i,t-1}, \ldots\}$., each of which has a precision τ_i. I drop the argument τ_i when the meaning is clear. As noted, the inclusion of the lagged realization of the aggregate discount rate, i.e., Z_{t-1}, in $\mathcal{I}_{i,t}$ reflects the fact that this realization can be inferred from the history of dividend and price realizations. But an individual investor i does not know the contemporaneous aggregate discount rate shock; instead, investors know only their own trading preferences, $Z_t + u_{i,t}$. The special case $\mathcal{I}_{i,t}(0)$ is the information of an *uninformed* investor who lacks signals about future dividends (though still benefits from information about his/her own trading preferences).

In addition to $\mathcal{I}_{i,t}(\tau_i)$ and $\mathcal{H}_t$, define the econometrician's information set of

$$\mathcal{J}_t \equiv \mathcal{H}_t \cup \{P_t, P_{t-1}, \ldots\}.$$

As written, the only public signals are the realizations of prices and dividends. It is straightforward to extend the framework to incorporate additional public signals, such as public macroeconomic announcements. Under such an extension, the required inputs would be the outputs of predictability regressions that incorporate the same set of public announcements.

3 Measuring information aggregation

3.1 Information aggregation

Lemma 3 below establishes that equilibrium prices coincide with those in a representative-agent economy in which each individual investor i observes a signal of form (4), where $\epsilon_{i,t}$ has variance $\bar{\tau}^{-1}$. By the standard formula,

$$var\left[\epsilon_{D,t}|y_{i,t}, \mathcal{H}_t\right]^{-1} = var\left[\epsilon_{D,t}|\mathcal{H}_t\right]^{-1} + \bar{\tau},$$

or equivalently (using (3)),

$$\bar{\tau} = var\left[\epsilon_{D,t}|y_{i,t}, \mathcal{H}_t\right]^{-1} - var\left[\epsilon_{D,t}\right]^{-1}.$$

Analogously, the precision of the information in the price is

$$\tau_{price} \equiv var\left[\epsilon_{D,t}|P_t, \mathcal{H}_t\right]^{-1} - var\left[\epsilon_{D,t}\right]^{-1}. \tag{6}$$

The main result in the paper it to deliver sufficient statistics that bound the value of

$$\frac{\tau_{price}}{\bar{\tau}}. \tag{7}$$

The ratio (7) is a measure of how effectively the price aggregates information. Specifically, it measures the number of independent signals of precision $\bar{\tau}$ an investor would have to observe to be indifferent between (a) observing these signals, but not the price, and (b) observing the price but nothing else. A ratio $\frac{\tau_{price}}{\bar{\tau}} = 0$ corresponds to a complete absence of information aggregation. Conversely, a ratio $\frac{\tau_{price}}{\bar{\tau}} = \infty$ corresponds to perfect information aggregation.

3.2 Roadmap

As a high-level roadmap to the paper: The precision of the price is straightforwardly estimated; indeed, a significant existing literature does so. The challenge is to estimate $\bar{\tau}$. There are two steps.The first step is to measure the total information advantage of an average investor relative to the econometrician,

$$\left(\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}\right)^{-1}. \tag{8}$$

This total information advantage reflects a combination of an investor's direct private signal about the dividend innovation $\epsilon_{D,t}$; and an investor's superior ability to extract information from the price P_t that arises from the fact that an investor potentially has information about the discount rate that is unavailable to the econometrician, and as such is able to extract more information from the price than the econometrician is able to. Conceptually, (8) is measured as follows. Consider a positive innovation to the dividend D_{t+1}. Investors observe noisy signals of this innovation at date t. By market clearing, such an innovation leaves the average expected return of investors, viz. $\int E\left[X_{t+1}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right] di$, unchanged. But to the extent to which the innovation is incorporated into the date t price P_t it raises this price. An econometrician observing a higher price attaches some weight to the price increase stemming from a change in the discount rate Z_t. Accordingly, the econometrician's expectation of the return, $E\left[X_{t+1}|\mathcal{J}_t\right]$, falls. The gap between the decline in $E\left[X_{t+1}|\mathcal{J}_t\right]$ and the constancy of $\int E\left[X_{t+1}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right] di$ allows a quantification of how much more the representative-agent knows than the econometrician, i.e., (8).

The second step is to express $\bar{\tau}$ in terms of the representative-agent's total information advantage, (8). The key challenge is that (8) reflects both the signal $y_{i,t}$ of $\epsilon_{D,t}$, which has precision $\bar{\tau}$; and an individual investor's knowledge of $Z_t + u_{i,t}$. Here, the key step is to use the restriction that individual demand decreases in the price to derive an upper bound on the information contained in $Z_t + u_{i,t}$.

4 Conditional distributions

I start by collecting results on the conditional distributions of $\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}$ under different conditioning information. In particular, the moments of $\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}$ conditional on $\mathcal{I}_{i,t}$ and $\mathcal{J}_t$ have simple linear relations.

An input for the results in this section is the observation that, because $\mathcal{I}_{i,t}$ and $\mathcal{J}_t$ both include the date t price P_t, and because the price is determined by the innovations $\epsilon_{D,t}$ and $\epsilon_{Z,t}$, the variance-covariance matrix of $\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}$ conditional on each of these information sets is singular. Formally, this follows from (using $\mathcal{J}_t$ as an example)

$$var\left[\epsilon_{Z,t}|\mathcal{J}_t\right] = \left(\frac{c_D}{c_Z}\right)^2 var\left[\epsilon_{D,t}|\mathcal{J}_t\right] \tag{9}$$

$$cov\left[\epsilon_{Z,t},\epsilon_{D,t}|\mathcal{J}_t\right] = -\frac{c_D}{c_Z} var\left[\epsilon_{D,t}|\mathcal{J}_t\right]. \tag{10}$$

The results in this section follow from this observation, combined with manipulation of standard updating rules for normal random variables. In particular, it follows that

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]} = \frac{var\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{Z,t}|\mathcal{J}_t\right]} = \frac{cov\left[\epsilon_{D,t},\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{cov\left[\epsilon_{D,t},\epsilon_{Z,t}|\mathcal{J}_t\right]}$$

and so

$$var\left[\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}|\mathcal{I}_{i,t}\right] = \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]} var\left[\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}|\mathcal{J}_t\right].$$

That is, the ratio

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}$$

measures an investor's information advantage over the econometrician not just with respect to $\epsilon_{D,t}$, but with respect to $\epsilon_{Z,t}$ also.

A key ingredient for equilibrium relations is how the sensitivity of the conditional expectation of $\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}$ to its true value depends on the information set. The relation is most easily expressed in terms of forecast errors:[4]

[4]For a related instance of the convenience afforded by forecast errors see Coibion and Gorodnichenko

Lemma 1

$$\frac{\partial}{\partial \epsilon_{D,t}}\left(E\left[\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)|\mathcal{I}_{i,t}\right]-\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)\right) = \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]}\frac{\partial}{\partial \epsilon_{D,t}}\left(E\left[\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)|\mathcal{J}_{t}\right]-\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)\right) \quad (11)$$

$$\frac{\partial}{\partial \epsilon_{Z,t}}\left(E\left[\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)|\mathcal{I}_{i,t}\right]-\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)\right) = \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]}\frac{\partial}{\partial \epsilon_{Z,t}}\left(E\left[\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)|\mathcal{J}_{t}\right]-\left(\begin{array}{c}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{array}\right)\right) \quad (12)$$

I am ultimately interested in estimating $\bar{\tau}$, the precision of the signal about the dividend innovation $\epsilon_{D,t}$ that is observed by the representative agent. By the standard formula,

$$var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1} = var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}+\bar{\tau}, \quad (13)$$

which rearranges to

$$\bar{\tau}=\left(\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]^{-1}}-\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]^{-1}}\right)var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]^{-1}. \quad (14)$$

Equation (14) gives $\bar{\tau}$ in terms of the representative agent's information advantage relative to the econometrician, $\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]^{-1}}$, which can be estimated, see Proposition 1; the information contained in the price, $var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]^{-1}$, which can likewise be estimated; and the information advantage of an uninformed investor ($\tau_i = 0$) relative to the econometrician, which can be bounded.

In order to decompose an investor's information advantage over an econometrician into contributions from signals about the dividend from signals about the discount rate, I make use of the fact that demand curves slope down. Operationalizing this requires characterizing how an investor's expectations respond to a change in the price, holding other elements of the information set fixed:

Lemma 2

$$\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)} = \frac{1}{\frac{\partial P_t}{\partial \epsilon_{D,t}}}\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]}\left(\left(\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]}\right)^{-1}-\frac{var\left[\epsilon_{D,t}|\mathcal{J}_{t}\right]}{var\left[\epsilon_{D,t}\right]}\right) \quad (15)$$

$$\frac{\partial E\left[X_{t+1}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)} = \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)}\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}-\frac{\partial P_t}{\partial \epsilon_{D,t}}\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\right)+\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}. \quad (16)$$

To build intuition for (15), note first that if an investor has the same information as the

(2015).

econometrician[5] then (15) simplifies to

$$\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} = \frac{1}{\frac{\partial P_t}{\partial \epsilon_{D,t}}}\left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{var\left[\epsilon_{D,t}\right]}\right),$$

capturing the standard idea that since the price P_t is a noisy signal of $\epsilon_{D,t}$, an increase in the price originating from a positive shock to $\epsilon_{D,t}$ increases the expected value of $\epsilon_{D,t}$ by less than the shock. Relative to this baseline, an improvement in an investor's information about discount rates (lower $var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]$) increases the amount of updating, while an improvement in an investor's information about dividends (lower $var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]$) reduces updating from the price.

To build intuition for (16), it is useful to consider the case in which Z_t is i.i.d. In this case, shocks to Z_t only affect the contemporaneous price P_t, and (16) simplifies to the intuitive expression

$$\frac{\partial E\left[X_{t+1}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} = \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\left(\frac{\partial E\left[P_{t+1}\right]}{\partial \epsilon_{D,t}} + 1\right) - R.$$

Equation (16) extends this to the more general case in which Z_t is serially correlated.

5 The information advantage of the representative agent

This section derives a formula that gives the information advantage of the representative agent in terms of sufficient statistics that can be empirically estimated. The key step in obtaining the formula is the market clearing condition, combined with Lemma 1.

5.1 Equivalence to a representative-agent economy

The general framework allows for a great deal of heterogeneity of investors; in particular, it places no restrictions on the distribution of characteristics (A_i, B_i, τ_i) over the investor population.

Nonetheless, the equilibrium price coincides with the equilibrium price in an economy in which agents are ex ante identical, i.e., share a common $\left(\bar{A}, \bar{B}, \bar{\tau}\right)$. This representative-agent characterization of the economy is simply a modest generalization of various results in the existing literature.

The market-clearing condition is

$$\int_i q_{i,t} di = 1, \tag{17}$$

[5] That is: $\mathcal{I}_{i,t}\left(\bar{\tau}\right) = \mathcal{I}_{i,t}\left(0\right) = \mathcal{J}_t$.

which implies

$$\int_i \frac{\partial q_{i,t}}{\partial \epsilon_{D,t}} di = 0 \tag{18}$$

$$\int_i \frac{\partial q_{i,t}}{\partial \epsilon_{Z,t}} di = 0. \tag{19}$$

A trivial decomposition of $\frac{\partial q_{i,t}}{\partial \epsilon_{D,t}}$ and $\frac{\partial q_{i,t}}{\partial \epsilon_{Z,t}}$ is

$$\frac{\partial q_{i,t}}{\partial \epsilon_{D,t}} = A_i \frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - A_i \frac{\partial}{\partial \epsilon_{D,t}} \left(X_{t+1} - E\left[X_{t+1} | \mathcal{I}_{i,t} \right] \right) \tag{20}$$

$$\frac{\partial q_{i,t}}{\partial \epsilon_{Z,t}} = \left(A_i \frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} - B_i \right) - A_i \frac{\partial}{\partial \epsilon_{Z,t}} \left(X_{t+1} - E\left[X_{t+1} | \mathcal{I}_{i,t} \right] \right). \tag{21}$$

Here, the first term in each expression is the change in asset demand of investor who perfectly forecasts the return X_{t+1}, and the second term represents the "underreaction" stemming from imperfect information. Decompositions (20) and (21) combine with Lemma 1 to yield

$$\frac{\partial q_{i,t}}{\partial \epsilon_{D,t}} = A_i \frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - A_i \frac{var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(\tau_i \right) \right]}{var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(0 \right) \right]} \frac{\partial}{\partial \epsilon_{D,t}} \left(X_{t+1} - E\left[X_{t+1} | \mathcal{I}_{i,t}\left(0 \right) \right] \right) \tag{22}$$

$$\frac{\partial q_{i,t}}{\partial \epsilon_{Z,t}} = \left(A_i \frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} - B_i \right) - A_i \frac{var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(\tau_i \right) \right]}{var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(0 \right) \right]} \frac{\partial}{\partial \epsilon_{Z,t}} \left(X_{t+1} - E\left[X_{t+1} | \mathcal{I}_{i,t}\left(0 \right) \right] \right). \tag{23}$$

Substituting into the market-clearing conditions (18) and (19) delivers the following representative agent result.

Lemma 3 *The equilibrium price coefficients c_D and c_Z coincide with those in an economy in which all investors are ex ante identical, i.e., $\left(A_i, B_i, \tau_i \right) = \left(\bar{A}, \bar{B}, \bar{\tau} \right)$ for all i, where*

$$var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(\bar{\tau} \right) \right] = \frac{\int A_i var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\left(\tau_i \right) \right]}{\int A_i}$$

$$\frac{\bar{B}}{\bar{A}} = \frac{\int B_i}{\int A_i}.$$

In words, Lemma 3 says that the precision $\bar{\tau}$ of the representative agent's information is the weighted average of the cross-section of signal precisions, where the weights are the coefficients A_i that determine the sensitivity of investor i's trade to expectations about the excess return.

5.2 Measuring the information of the representative agent

The main result of this section is that the precision of the representative agent's information advantage over the econometrician can be expressed in terms of sufficient statistics that can be estimated using only aggregate data.

The key step is the application of market-clearing. Condition (18) for the representative investor is

$$\bar{A}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \bar{A}\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}\frac{\partial}{\partial \epsilon_{D,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{J}_t\right]\right) = 0, \tag{24}$$

which rewrites to

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]} = \frac{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}{\frac{\partial}{\partial \epsilon_{D,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{J}_t\right]\right)}.$$

By construction, the dividend innovation $\epsilon_{D,t}$ affects the econometrician's information set $\mathcal{J}_t$ only via the price P_t. Hence:

Proposition 1 *The representative agent's informational advantage is given by*

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}} = 1 - \frac{\frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t}\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}.$$

Proposition 1 says that the representative agent's informational advantage over the econometrician can be inferred from the objects $\frac{\partial P_t}{\partial \epsilon_{D,t}}$, $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$, and $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$.

In words, $\frac{\partial P_t}{\partial \epsilon_{D,t}}$ is the relation between today's price and the innovations to next period's dividend. It is related to ability of prices to forecast future dividends. Likewise, $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$ is related to the ability of today's price to forecast next period's return. Finally, $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$ is a combination of $\frac{\partial P_t}{\partial \epsilon_{D,t}}$, the direct effect of the dividend innovation $\epsilon_{D,t}$ on D_{t+1}, which is simply 1, and the effect of the dividend innovation $\epsilon_{D,t}$ on P_{t+1}, which is determined largely by the persistence of dividend innovations.

Section 8 relates these objects to dividend-price predictability.

The economic idea behind Proposition 1 is as follows. Suppose that higher prices today lead an econometrician to forecast lower returns, as the price-dividend ratio literature suggests. But by definition, the average investor's holding doesn't change, since the average investor must continue to hold the market supply. For this to happen, it must be the case that the average investor's expectation differs from the econometrician's. In particular, the average investor must observe private signals that are on average positive when prices are high. Proposition 1 quantifies this statement.

6 Bounds for information aggregation

The measure of information aggregation is $\frac{\tau_{price}}{\bar{\tau}}$. From (6) and (14), this equals

$$\frac{\tau_{price}}{\bar{\tau}} = \frac{\frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}{var\left[\epsilon_{D,t}\right]^{-1}} - 1}{\left(\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}} - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}\right)\frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}{var\left[\epsilon_{D,t}\right]^{-1}}}. \tag{25}$$

The econometrician's information $\frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}{var\left[\epsilon_{D,t}\right]^{-1}}$ is straightforwardly measured; conceptually, the key input is the R^2 of regressing the dividend innovation on the lagged price. By Proposition 1, the representative agent's information advantage relative to the econometrician, $\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}$, can be expressed in terms of observable sufficient statistics. This leaves the term $\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}$, which measures an uninformed investor's information advantage relative to the econometrician. To proceed, I bound this term.

A lower bound for $\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}$ is simply 1, i.e., an uninformed investor doesn't enjoy any information advantage relative to an econometrician. To obtain an upper bound, I make use of the fact that an investor's demand curve should be downwards sloping as a function of the price., i.e.,

$$\frac{\partial E\left[X_{t+1}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} < 0. \tag{26}$$

Intuitively, inequality (26) provides an upper bound on $\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}$ because if an uninformed investor is much more informed than the econometrician, by definition this means that an uninformed investor can extract much more information from a price change by making use of his/her information of his/her own trading preferences. Substituting in Lemma 2, the downwards-sloping demand condition (26) delivers:[6]

[6]The upper bound (27) for an uninformed investor's information advantage requires $\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{D,t}}} > \frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}$. In the empirical implementation of Section 9 this is indeed the case. In general, one would expect this inequality to hold, because $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$ and $\frac{\partial P_t}{\partial \epsilon_{D,t}}$ are both naturally positive (unless dividend innovations are followed by reversals so strong that $P_{t+1} + D_{t+1}$ declines), while $\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}$ and $\frac{\partial P_t}{\partial \epsilon_{Z,t}}$ naturally take opposite signs (unless the discount rate innovation is very persistent).

Proposition 2 *If the representative investor's demand curve slopes down then*

$$1 \leq \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}} < \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{var\left[\epsilon_{D,t}\right]} - \frac{\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}}{\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{D,t}}} - \frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}} \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}. \tag{27}$$

Importantly, the quantities in the RHS of (27) are observable. The only term not already discussed is the ratio $\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}$, corresponding to the ratio of how an innovation to the discount rate affects prices and returns, respectively. Discount rate innovations aren't directly observed, and instead are inferred from residuals. In particular, the variance terms $\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ and $\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ can both be inferred from the variance in returns and prices that is unexplained by dividend innovations. Given estimates of these variances, the ratio $\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}$ is given by

$$\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}} = -\sqrt{\frac{\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]}{\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]}}. \tag{28}$$

The negative sign in (28) reflects the fact that the shock $\epsilon_{Z,t}$ pushes P_t and X_{t+1} in opposite directions, *unless* the process $\{Z_t\}$ is so strongly persistent that the effect of $\epsilon_{Z,t}$ on P_{t+1} outweighs its effect on P_t. The quantitative implementation in Section 8 confirms that that (28) indeed has a negative sign.

7 The value of private information

My primary focus is to estimate the extent to which prices aggregate individual information. The tools developed also deliver an estimate of the value of the representative agent's private information. Specifically, I calculate by how much giving an uninformed investor access to the representative agent's information would increase the uninformed investor's expected return. Observe that this exercise is well-defined regardless of whether or not uninformed investors are actually present in the market.

The benefit of focusing on return differentials rather than, for example, willingness-to-pay measures of utility differences, is that it is possible to give a sufficient statistics formula for the return differential. In contrast, I have been unable to find a sufficient statistics formula for willingness-to-pay in which the components can be estimated. In particular, a utility-based measure would require estimates of investor risk aversion.

To isolate the value of information, I evaluate this expected return differential for an investor who resembles the representative investor in other dimensions. Concretely, I consider an investor with characteristics

$$(\bar{A}, \bar{B}) = \left(\int_i A_i di, \int_i B_i di \right), \tag{29}$$

and compare the expected return from investment strategies made under the information of the representative agent,

$$q_{i,\bar{\tau},t} \equiv \bar{A} E \left[X_{t+1} | \mathcal{I}_{i,t} \left(\bar{\tau} \right) \right] - \bar{B} \left(Z_t + u_{i,t} \right),$$

and under the information of the uninformed investor,

$$q_{i,0,t} \equiv \bar{A} E \left[X_{t+1} | \mathcal{I}_{i,t} \left(0 \right) \right] - \bar{B} \left(Z_t + u_{i,t} \right).$$

Getting access to the average investor's information would raise an uninformed investor's expected return by a fraction

$$V \equiv \frac{E \left[q_{i,\bar{\tau},t} X_{t+1} \right]}{E \left[q_{i,0,t} X_{t+1} \right]}. \tag{30}$$

As a first step in evaluating V, note that an investor with characteristics $\left(\bar{A}, \bar{B}, \bar{\tau} \right)$ is a representative agent for the economy, in the sense of Lemma 3. Moreover, the market clears, so that

$$\int q_{i,\bar{\tau},t} di = E \left[q_{i,\bar{\tau},t} | \mathcal{H}_t \right] = 1. \tag{31}$$

Consequently, the investment strategy $q_{i,\bar{\tau},t}$ is independent of the aggregate shocks $\epsilon_{D,t}$ and $\epsilon_{Z,t}$. This is a version of the "average investor theorem" of Sharpe (1991).

Corollary 1 $\frac{\partial q_{i,\bar{\tau},t}}{\partial \epsilon_{D,t}} = \frac{\partial q_{i,\bar{\tau},t}}{\partial \epsilon_{Z,t}} = 0.$

Corollary 1 implies that the representative agent's asset holding is uncorrelated with the return. Moreover, by the law of iterated expectation,

$$E \left[q_{i,\bar{\tau},t} | \mathcal{H}_t \right] = E \left[q_{i,0,t} | \mathcal{H}_t \right] = \bar{A} E \left[X_{t+1} | \mathcal{H}_t \right] - \bar{B} E \left[Z_t | \mathcal{H}_t \right]. \tag{32}$$

In words: While an uninformed and informed investor make different investment decisions, in expectation the gap between their asset holdings is zero.

Substitution of Corollary 1, (31) and (32) into (30) yields (see proof of Proposition 3 for details)

$$V = \frac{E \left[X_{t+1} \right]}{E \left[X_{t+1} \right] + cov \left[q_{i,0,t}, X_{t+1} | \mathcal{H}_t \right]}. \tag{33}$$

Expression (33) relates the value of information to the covariance between an uninformed investor's asset position, and returns. Expanding, this covariance is given by

$$cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right] = \frac{\frac{\partial q_{i,0,t}}{\partial \epsilon_{D,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right] + \frac{\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]. \tag{34}$$

The key challenge in evaluating (34) is that while $\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ can be estimated (see discussion following Proposition 2) this still leaves the term $\frac{\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}$.

The key step in characterizing the value of information is to use the market-clearing condition (19) to infer $\frac{\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}$. Doing so yields a sufficient statistics expression for the value of information V:

Proposition 3 *Under Assumption 2, the value of information V is given by (33), where*

$$cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right] = \left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}\right)\left(1 - \frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}}\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\right)\frac{\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]}{E\left[X_{t+1}\right]}. \tag{35}$$

Note that the role of Assumption 2 is that it implies (via the unconditional expectation of the market-clearing condition (17)) that the parameter $\bar{A}$ entering the representative agent's demand is the reciprocal of the unconditional market risk premium, i.e.,

$$\bar{A}E\left[X_{t+1}\right] = 1. \tag{36}$$

The covariance term (35) can be estimated. For the first term, note that the information advantage of the representative agent relative to uninformed investor simply equals the ratio of the information advantages of the representative agent and uninformed investor to the econometrician:

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}} = \frac{\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}}{\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}}. \tag{37}$$

The numerator in the RHS of (37) is given directly by Proposition 1, while Proposition 2 gives bounds for the denominator.

For the second term: Both $\frac{\partial P_t}{\partial \epsilon_{D,t}}$ and $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$ can be estimated, and indeed enter Proposition 1, while the ratio $\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}$ can be estimated using (28).

Finally, the third term in (37) is simply the variance of date $t+1$ returns attributable to shocks to date $t+1$ dividends.

The estimated value of the covariance (35) is negative, meaning that uninformed investors end up negatively timing the market, in the sense of increasing their asset holdings when future returns are low. To see why negative covariance arises, consider a date t shock to either discount rates or dividends that increases the return from date t to $t+1$. In response to this shock, uninformed investors' return-expectations rise less than those of informed investors (see Lemma 1). By market clearing, the average investor's desired asset holding cannot respond (Corollary 1). It follows that an uninformed investor's asset holding must fall.

As noted, the negative covariance between uninformed asset holdings and subsequent returns is closely related to results in Watanabe (2008) and Biais et al (2010). It occurs even though the uninformed investor has fully Bayesian expectations about future returns.

As a final comment on Proposition 1: Recall that the value of information V is based on an uninformed investor who responds to return expectations and discount rate shocks Z_t+u_{it} in the same way as the average investor in the economy, i.e., has characteristics $\bar{A}$ and $\bar{B}$ given by (29). However, in many underpinnings of asset demand (2), the coefficient A_i on return expectations is a joint function of the endogenous perceived return variance, $var\left[X_{t+1}|\mathcal{I}_{i,t}\right]$, and exogenous risk aversion. Moreover, the same is potentially true of the coefficient B_i on discount rate shocks. As such, the measure V doesn't account for the increase in the average size of an uninformed investor's average position that may accompany getting access to better information and thereby reducing perceived return variance.

Two points are worth making here. First, incorporating this effect into the measure V would require substantially stronger assumptions about investors' asset demands than I have made so far. That is: How exactly does an investor's asset demand depend on perceived return variance? Second: Although estimates of the reduction in the perceived variance of dividend innovations $\epsilon_{D,t}$ turn out to be relatively large (Section 9), estimates of the reduction in the perceived variance of returns X_{t+1} are much smaller, because estimates indicate that most return variance is driven by date $t+1$ discount rate shocks (Section 8). As such, incorporating the reduction-in-perceived-variance effect into V is likely to have only a small impact.

8 Predictability empirics

The key quantities required to operationalize Propositions 1, 2 and 3 are tightly related to the outputs of empirical analysis that studies the predictability of returns and dividends, emphasizing the role of the price-dividend ratio.

The literature is sizable. In this paper, I use of the estimates of Binsbergen and Koijen (2010), henceforth BK, to illustrate the methodology developed in Section 5. As the preceding analysis indicates, the interpretation of today's prices requires incorporating the history of prices and dividends in order to infer the history of discount rate shocks. An important advantage of BK's estimates is that, as they write, "Our latent variables approach aggregates the whole history of price-dividend ratios and dividend growth rates to estimate expected returns and expected growth rates."

8.1 Estimated VAR

BK estimate an empirical model with exogenous shocks to dividend growth, expected return, and realized dividends. As emphasized by BK and Cochrane (2008), an implication of the present value identity (Campbell and Shiller 1988) is that such an empirical model is indistinguishable from an alternative one with exogenous shocks to dividend growth and the price-dividend ratio. I will work with this latter specification because it is stated in terms of observables, and as such is closer to quantities needed as inputs for Propositions 1, 2 and 3.

Specifically, write $r_{t+1} = \ln \frac{P_{t+1}+D_{t+1}}{P_t}$ and $\Delta d_{t+1} = \ln \frac{D_{t+1}}{D_t}$ for the asset log return and dividend log growth rate between dates t and $t+1$. Write $pd_t = \log \frac{P_t}{D_t}$ for the log price-dividend ratio, along with $\bar{pd}$ for its steady state value, and $\rho = \frac{\exp\left(\bar{pd}\right)}{1+\exp\left(\bar{pd}\right)}$. Let $\nu_{d,t+1}$ and $\nu_{pd,t+1}$ denote the unforecastable (to the econometrician) innovations to Δd_{t+1} and pd_{t+1}:

$$
\begin{aligned}
\Delta d_{t+1} &= E\left[\Delta d_{t+1} | \mathcal{J}_t\right] + \nu_{d,t+1} \\
pd_{t+1} &= E\left[pd_{t+1} | \mathcal{J}_t\right] + \nu_{pd,t+1}.
\end{aligned}
$$

Denote by μ_t and g_t the econometrician's date t expectations about returns and dividend growth:

$$
\begin{aligned}
\mu_t &= E\left[r_{t+1} | \mathcal{J}_t\right] \\
g_t &= E\left[\Delta d_{t+1} | \mathcal{J}_t\right].
\end{aligned}
$$

BK assume that μ_t and g_t follow AR1 processes:

$$
\mu_{t+1} = \bar{\mu} + \phi_\mu \left(\mu_t - \bar{\mu}\right) + \nu_{\mu,t+1} \tag{38}
$$

$$
g_{t+1} = \bar{\Delta d} + \phi_g \left(g_t - \bar{\Delta d}\right) + \nu_{g,t+1}. \tag{39}
$$

From the present value identity, the innovation $\nu_{pd,t+1}$ is a function of $\nu_{\mu,t+1}$ and $\nu_{g,t+1}$ (see appendix for details):

$$\nu_{pd,t+1} = \frac{\nu_{g,t+1}}{1-\rho\phi_g} - \frac{\nu_{\mu,t+1}}{1-\rho\phi_\mu}. \tag{40}$$

Since the econometrician observes only dividends and prices, the innovations $\nu_{g,t+1}$ and $\nu_{\mu,t+1}$ must be functions of $\nu_{pd,t+1}$ and $\nu_{d,t+1}$:[7]

$$\begin{aligned} \nu_{g,t+1} &= a_{pd}\nu_{pd,t+1} + a_d\nu_{d,t+1} \\ \nu_{\mu,t+1} &= b_{pd}\nu_{pd,t+1} + b_d\nu_{d,t+1}, \end{aligned} \tag{41}$$

where, from (40), b_{pd} and b_d satisfy

$$\begin{aligned} \frac{a_{pd}}{1-\rho\phi_g} - \frac{b_{pd}}{1-\rho\phi_\mu} &= 1 \\ \frac{a_d}{1-\rho\phi_g} - \frac{b_d}{1-\rho\phi_\mu} &= 0. \end{aligned}$$

The estimated system is[8]

$$g_{t+1} - \bar{\Delta d} = \phi_g\left(g_t - \bar{\Delta d}\right) + a_{pd}\nu_{pd,t+1} + a_d\nu_{d,t+1} \tag{42}$$

$$\Delta d_{t+1} = g_t + \nu_{d,t+1} \tag{43}$$

$$pd_{t+1} - \bar{pd} = \frac{\phi_g - \phi_\mu}{1-\rho\phi_g}\left(g_t - \bar{\Delta d}\right) + \phi_\mu\left(pd_t - \bar{pd}\right) + \nu_{pd,t+1}. \tag{44}$$

The system has ten parameters,

$$\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu, a_{pd}, a_d, \sigma^2_{pd}, \sigma^2_d, \sigma_{pd,d}\right\}.$$

Appendix B details how to recover estimates of these ten values from the estimates reported in BK. The first five parameters $\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu\right\}$ coincide with the BK values. Using the estimates reported in the first column of BK's Table II yields the values reported in Table 1.

BK's estimates are based on nominal annual returns and nominal annual dividend growth rates. Accordingly, the appropriate risk free rate R_t is a nominal annual rate. I use a risk free rate of 2% in the calculations below.

[7]Linearity here follows from Cochrane (2008, p. 11).

[8]Equation (44) follows from the present value identity; see appendix for details.

Parameter	Estimated value
$\bar{\Delta d}$	0.062
$\bar{pd}$	3.571
ρ	0.969
ϕ_g	0.354
ϕ_μ	0.932
a_{pd}	0.0482
a_d	0.3952
σ_{pd}	0.1596
σ_d	0.0576
$\frac{\sigma_{pd,d}}{\sigma_{pd}\sigma_d}$	−0.3118
$\bar{\mu}$	0.090
b_{pd}	−0.0898
R^2 of Δd_{t+1} regressed on $\mathcal{J}_t$	13.9%

Table 1: Parameter estimates recovered by BK

8.2 From predictability estimates to inputs for Proposition 1

The evaluation of Proposition 1 requires estimates of $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$, $\frac{\partial P_t}{\partial \epsilon_{D,t}}$, and $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$. See Table 2, which also summarizes the corresponding economic quantities and estimated values. Recall that $\epsilon_{D,t}$ is the innovation to date $t+1$ dividends, though investors observe noisy signals at date t.

Term	Description	Estimate	Result(s)
$\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$	Expected return and price	$-.0983$	Props 1, 2, 3
$\frac{\partial P_t}{\partial \epsilon_{D,t}}$	Price and next-period dividend	12.1	Props 1, 2, 3
$\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$	Realized return and dividend	4.24	Props 1, 2, 3
$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$	Variance of returns due to $\epsilon_{Z,t}$	$P_{t-1}^2 \times .028^2$	Props 2, 3
$\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$	Variance of price due to $\epsilon_{Z,t}$	$P_{t-1}^2 \times .160^2$	Props 2, 3
$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]$	Variance of returns due to $\epsilon_{D,t}$	$P_{t-1}^2 \times .011^2$	Prop 3
$E\left[X_{t+1}\right]$	Equity premium	$P_{t-1} \times .0915$	Prop 3

Table 2: Quantities to estimate; estimates; and results for which inputs are relevant

In particular, $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$ is closely related to the estimated parameter b_{pd}, i.e., the relation between the expected return ($\nu_{\mu,t}$) and the price-dividend ratio ($\nu_{pd,t}$), holding dividends fixed ($\nu_{d,t} = 0$). Similarly, $\frac{\partial P_t}{\partial \epsilon_{D,t}}$ is closely related to a_{pd}, i.e., the relation between expectations about the dividend growth rate ($\nu_{g,t}$) and today's price ($\nu_{pd,t}$), holding current dividends fixed ($\nu_{d,t} = 0$). Below, I sketch some key steps; the appendix contains all omitted details.

8.2.1 The term $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$

The term $\frac{\partial E[X_{t+1}|\mathcal{J}_t]}{\partial P_t}$ captures how an increase in the price P_t affects the econometrician's expected return from t to t+1. From (41), its empirical counterpart is b_{pd}, which captures how an innovation in the price-dividend ratio affects the expected return, holding the dividend constant. The estimated value of b_{pd} is

$$b_{pd} = \frac{\partial E\left[r_{t+1}|\mathcal{J}_t\right]}{\partial \ln P_t} = -.0898, \tag{45}$$

i.e., a 1% increase in prices is associated with a decline in the econometrician's expected return of 9 basis points. Converting from percent-to-percent to dollar-to-dollar changes gives

$$\frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t} \approx -.0993. \tag{46}$$

That is: a price increase of \$1 reduces the (\$) expected return by approximately \$0.1.

8.2.2 The term $\frac{\partial P_t}{\partial \epsilon_{D,t}}$

The term $\frac{\partial P_t}{\partial \epsilon_{D,t}}$ captures how an innovation to the date $t+1$ dividend D_{t+1} affects the date t price. Its closest empirical counterpart is a_{pd}, which captures how an innovation in the price-dividend ratio affects the expected dividend growth rate, and has an estimated value

$$a_{pd} = \frac{\partial E\left[\ln D_{t+1}|\mathcal{J}_t\right]}{\partial \ln P_t} = 0.0482,$$

i.e., a 1% increase in prices is associated with an increase in the econometrician's expected dividend growth rate of 5bp.

I first relate a_{pd} to $\left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}}$, and then second relate $\left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}}$ to the desired term $\frac{\partial P_t}{\partial \epsilon_{D,t}}$. The second step is a simply a shift from percentage changes to level changes. The first step corresponds to switching from "does today's price predict future dividends" to "do future dividends predict today's price"? Note that this distinction is closely related to Dávila and Parlatore (2025), who derive a measure of price informativeness for which the key empirical input is the R^2 of a regression of date t price innovations on date $t+1$ cash flow innovations. The input required for my sufficient statistics formulae, $\frac{\partial P_t}{\partial \epsilon_{D,t}}$, likewise corresponds to the regression coefficient from this same regression.

For the first step:

$$\left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}} = \frac{cov\left[pd_t, \Delta d_{t+1} | \Delta d_t, \mathcal{J}_{t-1}\right]}{var\left[\Delta d_{t+1} | \Delta d_t, \mathcal{J}_{t-1}\right]} = \frac{1}{a_{pd}} \frac{a_{pd}^2 var\left[\nu_{pd,t} | \nu_{d,t}\right]}{var\left[\nu_{d,t+1}\right] + a_{pd}^2 var\left[\nu_{pd,t} | \nu_{d,t}\right]}. \tag{47}$$

The final fraction in (47) is the fraction of the residual variance of Δd_{t+1} after controlling for Δd_t and $\mathcal{J}_{t-1}$ that is explained by further controlling for pd_t. If this ratio is 1, i.e., pd_t and Δd_{t+1} are perfectly correlated conditional on Δd_t and $\mathcal{J}_{t-1}$, then $\left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}}$ is simply the reciprocal of a_{pd}. Evaluating

$$\frac{a_{pd}^2 var\left[\nu_{pd,t} | \nu_{d,t}\right]}{var\left[\nu_{d,t+1}\right] + a_{pd}^2 var\left[\nu_{pd,t} | \nu_{d,t}\right]} = 0.0159. \tag{48}$$

The low value of this last term reflects the standard result that today's price contains very limited predictive power for tomorrow's dividend. Returning to (47),

$$\left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}} = \frac{0.0159}{0.0482} = 0.329, \tag{49}$$

i.e., a 100% increase in date $t+1$ dividends suggests that date t prices were 33% higher.

For the second step, viz., converting the elasticity estimate (49) to the required dollar-dollar estimate:

$$\frac{\partial P_t}{\partial \epsilon_{D,t}} = \left.\frac{\partial P_t}{\partial D_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}} = \frac{P_t}{D_{t+1}} \left.\frac{\partial \log P_t}{\partial \log D_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}} = \frac{P_t}{D_{t+1}} \left.\frac{\partial pd_t}{\partial \Delta d_{t+1}}\right|_{\Delta d_t, \mathcal{J}_{t-1}} \approx 12.1. \tag{50}$$

8.2.3 The term $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$

The term $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$ captures how an innovation to the date $t+1$ dividend D_{t+1} affects the excess return between dates t and $t+1$. There are two distinct channels. First, an innovation to D_{t+1} is partially reflected in the date t price P_t, as quantified immediately above. Second, the remaining part of the innovation, which the econometrician observes only at date $t+1$, directly affects the price and dividend at that date, $P_{t+1} + D_{t+1}$.

Formally, these two channels correspond to the following decomposition of $\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}$ into effects stemming from anticipated and unanticipated elements of $\epsilon_{D,t}$:

$$\begin{aligned}
\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} &= \frac{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{\partial \epsilon_{D,t}} \frac{\partial X_{t+1}}{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]} + \frac{\partial \left(\epsilon_{D,t} - E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial \epsilon_{D,t}} \frac{\partial X_{t+1}}{\partial \left(\epsilon_{D,t} - E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)} \\
&= \frac{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{\partial \epsilon_{D,t}} \frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]} + \frac{\partial \left(\epsilon_{D,t} - E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial \epsilon_{D,t}} \frac{\partial \left(P_{t+1} + D_{t+1}\right)}{\partial \left(\epsilon_{D,t} - E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)} \\
&= \frac{\partial P_t}{\partial \epsilon_{D,t}} \frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t} + \frac{\partial \left(\epsilon_{D,t} - E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial \epsilon_{D,t}} \left.\frac{\partial \left(P_{t+1} + D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t}. \qquad (51)
\end{aligned}$$

Note that the final equality follows from the fact that the innovation $\epsilon_{D,t}$ affects the econometrician's date t expectation only via the price P_t.

Both elements of the first term of (51), corresponding to the effect of the anticipated component of $\epsilon_{D,t}$, are calculated above.

Next, consider the second term of (51), which corresponds to the effect of the unanticipated component of $\epsilon_{D,t}$. The key element here is $\frac{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{\partial \epsilon_{D,t}}$, and its empirical counterpart is the R^2 of the regression of Δd_{t+1} on the econometrician's information set $\mathcal{J}_t$, which BK estimate to be 13.9%.

Specifically (using Lemma A.3 in Bond and García (2022)):

$$\frac{\partial E\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{\partial \epsilon_{D,t}} = 1 - \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{var\left[\epsilon_{D,t}\right]} \approx R^2 \text{ of regressing } \Delta d_{t+1} \text{ on } \mathcal{J}_t. \qquad (52)$$

For the remaining element of the second term in (51),

$$\left.\frac{\partial \left(P_{t+1} + D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t} = 1 + e^{pd_{t+1}} \left(\left.\frac{\partial pd_{t+1}}{\partial \Delta d_{t+1}}\right|_{\mathcal{J}_t} + 1 \right) = 1 + e^{pd_{t+1}} \left(\frac{cov\left[\nu_{pd,t+1}, \nu_{d,t+1}\right]}{var\left[\nu_{d,t+1}\right]} + 1 \right).$$

Evaluating,

$$\left.\frac{\partial \left(P_{t+1} + D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t} \approx 1 + 39.1 \times \left(-0.3118 \times \frac{0.1596}{0.0576} + 1 \right) = 6.31. \qquad (53)$$

Putting everything together,

$$\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} \approx 12.1 \times (-0.0993) + (1 - 0.139) \times 6.31 = 4.24.$$

Perhaps as one would expect, the main impact of the dividend innovation $\epsilon_{D,t}$ stems from the unanticipated component, and specifically, its effect on the date $t+1$ dividend and price.

8.3 From predictability estimates to inputs for Proposition 2

Relative to Proposition 1, Proposition 2 additionally requires an input for the ratio $\frac{\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}$, which from (28) in turn requires inputs for the return and price variance stemming from discount rate shocks, viz., $\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ and $\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ respectively.

8.3.1 The term $\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$

The term $\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ measures the variance in date $t+1$ returns that is attributable to innovations to the date t discount rate. It is estimated from

$$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right] = var\left[E\left[X_{t+1}|D_{t+1}, \mathcal{J}_t\right]|D_{t+1}, D_t, \mathcal{J}_{t-1}\right]. \quad (54)$$

Note in particular that (54) *doesn't* incorporate information from the realizations of date $t+1$ prices, which depend on the date $t+1$ innovation to the discount rate. Evaluating,

$$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right] \approx P_{t-1}^2 \times .0028^2.$$

Hence date t discount rate innovations contribute little to the variance of the return X_{t+1}. The reason is that, empirically, the price-dividend ratio is highly persistent, with the estimated autoregressive parameter ϕ_μ equalling 0.932. Hence a shock to the date t discount rate shifts both P_t and P_{t+1} by roughly equal amounts, leaving the expected return largely unaffected.

8.3.2 The term $\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$

The term $\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ measures the variance in the date t price that is attributable to innovations to the date t discount rate. Evaluating (see appendix)

$$\left(\frac{\partial P_t}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right] = var\left[P_t|\epsilon_{D,t}, \mathcal{H}_t\right] = var\left[P_t|D_{t+1}, D_t, \mathcal{J}_{t-1}\right] \approx P_{t-1}^2 \times .160^2. \quad (55)$$

The approximate empirical counterpart[9] of (55) is the conditional variance $var\left[pd_t|\Delta d_{t+1}, \Delta d_t, \mathcal{J}_{t-1}\right]$ of the price-dividend ratio, where the conditioning set includes both the econometrician's

[9] As shown in the appendix, $var\left[P_t|D_{t+1}, D_t, \mathcal{J}_{t-1}\right]$ and $var\left[pd_t|\Delta d_{t+1}, \Delta d_t, \mathcal{J}_{t-1}\right]$ differ by a factor of approximately $\left(P_{t-1}e^{\bar{\Delta d}}\right)^2 \approx \left(P_{t-1} \times 1.064\right)^2$.

date $t-1$ information and the realizations of date t and $t+1$ dividends. Evaluating (see (98) in appendix) gives

$$var\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\approx .150^2. \tag{56}$$

In this case, a much simpler approach also yields an approximately similar result. Empirically, date t dividend fluctuations drive little of the fluctuation in date t price-dividend ratios; and future dividend growth is only weakly forecast by date t price-dividend ratios. Consequently, conditioning on Δd_{t+1} and Δd_t in (56) makes little difference, and hence

$$var\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\approx var\left[pd_t|\mathcal{J}_{t-1}\right]=var\left[\nu_{pd,t}\right]=.160^2.$$

A final point to note is that the appendix's evaluation of $\left(\frac{\partial X_{t+1}}{\partial\epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ and $\left(\frac{\partial P_t}{\partial\epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right]$ confirms that $\frac{\partial X_{t+1}}{\partial\epsilon_{Z,t}}$ and $\frac{\partial P_t}{\partial\epsilon_{Z,t}}$ have opposite signs,[10] a result already reflected in the negative sign in (28) (see earlier discussion).

8.4 From predictability estimates to inputs for Proposition 3

Relative to Propositions 1 and 2, Proposition 3 additionally requires inputs for the return variance due to dividend shocks, $\left(\frac{\partial X_{t+1}}{\partial\epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]$, and for the equity premium, $E\left[X_{t+1}\right]$.

8.4.1 The term $\left(\frac{\partial X_{t+1}}{\partial\epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]$

The term $\left(\frac{\partial X_{t+1}}{\partial\epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]$ measures the variance in date $t+1$ returns that is attributable to innovations to date $t+1$ dividends. By (51), this variance can be decomposed into the portions due to the innovation that is incorporated into the date t price, and the remaining portion that is observed by the econometrician only at date $t+1$:

$$\begin{aligned}\left(\frac{\partial X_{t+1}}{\partial\epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right] &= \left(\frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t}\right)^2 var\left[\frac{\partial P_t}{\partial\epsilon_{D,t}}\epsilon_{D,t}\right]\\ &+ \left(\left.\frac{\partial\left(P_{t+1}+D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t}\right)^2 var\left[\frac{\partial\left(\epsilon_{D,t}-E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial\epsilon_{D,t}}\epsilon_{D,t}\right].\end{aligned} \tag{57}$$

Evaluating,

$$\left(\frac{\partial X_{t+1}}{\partial\epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]\approx P_{t-1}^2\times .011^2. \tag{58}$$

This variance is overwhelmingly driven by the second term in (57). That is: BK's estimates suggest that the variance in date $t+1$ returns that is attributable to variance in the date

[10]Specifically, see (99), and the fact that the quantitative evaluation of (100) is negative.

$t+1$ dividend innovation is almost entirely attributable to dividend innovations that *aren't* incorporated into the date $t+1$ price.

Remark (Aside): Combining the variance to returns X_{t+1} stemming from dividend shocks $\epsilon_{D,t}$ and discount rate shocks $\epsilon_{Z,t}$ gives

$$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right] + \left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)^2 var\left[\epsilon_{Z,t}\right] \approx P_{t-1}^2 \times .0113^2. \quad (59)$$

Expressed in percentage terms, date $t+1$ innovations and date t discount rate innovations generate a standard deviation of returns of approximately 1%. Why is this value so much lower than the total standard deviation of returns, which is on the order of $15\% - 20\%$? The reason is that (59) omits the return variation stemming from date $t+1$ discount rate innovations. Evaluating, this source of return variation is

$$\left(\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t+1}}\right)^2 var\left[\epsilon_{Z,t+1}\right] \approx P_{t-1}^2 \times .170^2. \quad (60)$$

In other words, most of the variation in returns from date t to $t+1$ stems from date $t+1$ discount rate innovations; and the estimated value of this variation is consistent with total return variation lying in the $15-20\%$ range.

8.4.2 The term $E\left[X_{t+1}\right]$

Finally, I consider the term $E\left[X_{t+1}\right]$. This is simply the equity premium. While many estimates are available, for consistency I use one based on the same BK estimates as the other terms. The key input into the calculation is, as one would expect, BK's estimate of $\bar{\mu}$, the unconditional mean of the log return. Evaluating:

$$E\left[X_{t+1}\right] \approx P_{t-1} \times .0915.$$

9 Estimates

Finally, I use the estimated values to operationalize Propositions 1, 2 and 3, dealing with, respectively, the information advantage of the representative agent over the econometrician; the extent to which prices aggregate information; and the value of the representative agent's information. I further use these estimates to obtain an information-induced demand elasticity.

9.1 The representative agent's information advantage

The representative agent's information advantage is, by Proposition 1,

$$\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}} \approx 1-\frac{-.0983\times 12.1}{4.24}=1.28. \tag{61}$$

That is: The representative agent's private information reduces the conditional variance of the next dividend by approximately $1-\frac{1}{1.28}\approx 22\%$.

9.2 Information aggregation

First, I evaluate the information in the price. To do so, I make use of

$$\begin{aligned} var\left[\epsilon_{D,t}|\mathcal{J}_t\right]=var\left[D_t e^{\Delta d_{t+1}}|\mathcal{J}_t\right] &\approx \left(D_t e^{\bar{\Delta d}}\right)^2 var\left[\Delta d_{t+1}|\mathcal{J}_t\right]=\left(D_t e^{\bar{\Delta d}}\right)^2 var\left[\nu_{d,t+1}\right], \\ var\left[\epsilon_{D,t}|\Delta d_t,\mathcal{J}_{t-1}\right]=var\left[D_t e^{\Delta d_{t+1}}|\Delta d_t,\mathcal{J}_{t-1}\right] &\approx \left(D_t e^{\bar{\Delta d}}\right)^2\left(var\left[\nu_{d,t+1}\right]+a_{pd}^2 var\left[\nu_{pd,t}|\nu_{d,t}\right]\right), \end{aligned}$$

where the latter equation follows from (96) in the appendix. Evaluating,

$$\frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}{var\left[\epsilon_{D,t}|\Delta d_t,\mathcal{J}_{t-1}\right]}\approx 1.016. \tag{62}$$

That is: The information in the price reduces the conditional variance of the next dividend by approximately 1.6%. The low magnitude corresponds to the widespread finding that aggregate prices have little predictive power for aggregate dividends.

Second, I use Proposition 2 to bound the amount of information of an uninformed investor. Evaluating,

$$1\leq\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1}}\leq 1.046. \tag{63}$$

Comparing the upper bound in (63) to (61) indicates that the large majority of the representative agent's information advantage stems from information about dividend shocks as opposed to information about discount rate shocks.

Substitution into (25) delivers an estimate for the extent of information aggregation:

$$.056\leq\frac{\tau_{price}}{\bar{\tau}}\leq .067. \tag{64}$$

That is: The extent of information aggregation is low. Observing just one private signal about the dividend D_{t+1}, but not seeing the date t, conveys much more information than seeing the date t price but no private signal.

The economic force behind result (64) is the empirically high variance of discount rates shock—again, a central finding of prior research. This variance works against the effective aggregation of information in prices.

9.3 The value of the representative agent's information

The value of the representative agent's information is given by Proposition 3. As first step, note that (61) and (63) imply that the representative agent's information advantage relative to an uninformed investor satisfies

$$1.23 \leq \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}} \leq 1.28. \tag{65}$$

Evaluating Proposition 3 yields the following bounds for the value of the representative agent's information advantage V:

$$1.0034 \leq V \leq 1.0042. \tag{66}$$

Expressed in return rates, this corresponds to an uninformed investor experiencing a return penalty of approximately 3-4 basis points relative to the return of the representative agent.

Estimate (65) suggests a sizable information gap between the representative agent and an uninformed investor. Why, then, is the estimated value of this information in (66) so small? To gain insight, it is useful to return to (34) and evaluate the magnitude of the first term on the RHS. This term corresponds to the advantage that the representative investor enjoys relative to an uninformed investor in timing dividend innovations—which is what the representative agent's informational advantage relates to. From (20) and (22), and using Corollary 1,

$$\frac{\partial q_{i,0,t}}{\partial \epsilon_{D,t}} = \bar{A}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \left(\bar{A}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \frac{\partial q_{i,\bar{\tau},t}}{\partial \epsilon_{D,t}}\right)\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}} = \bar{A}\left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}\right)\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}.$$

Hence the first term on the RHS of (34) evaluates to

$$\left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]^{-1}}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1}}\right)\frac{\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\right)^2 var\left[\epsilon_{D,t}\right]}{E\left[X_{t+1}\right]}. \tag{67}$$

Expression (67) differs only from (35) only via the multiplicative term

$$1-\frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}}{\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\approx 1.051.$$

That is: Although the representative agent's direct informational advantage about dividend innovations *also* generates an informational advantage about discount rate innovations, the quantitative value of this advantage is an order of magnitude smaller than that stemming from the direct advantage of better information about dividend innovations. Building on this insight, the estimated information advantage is small because the return variance that stems from dividend innovations is itself quantitively small—from (58), the standard deviation of returns stemming from dividend innovations is about 1%.

9.4 The implied price elasticity of demand

In a recent work, Gabaix and Koijen (2023) suggest that the price elasticity of demand for the aggregate stock market is "surprisingly inelastic."

The demand-based asset pricing literature (Koijen and Yogo 2019) gives a number of motivations for the limited elasticity of asset demand; see, for example, the discussion in Davis et al (2025). Relatively little discussed, however, is the possibility that demand elasticity stems from the information-content of asset prices, viz., a decline in the asset price makes an asset more attractive holding future cash flows and prices constant; but makes an asset less attractive because the decline in the price reduces expectations about future cash flows and prices.[11]

Specifically, an increase in the price has two effects on the expected return: a direct effect stemming from the change in today's price, and an indirect effect stemming from changes in expectations of future cash flows and discount rates, both of which feed into changes in expectations of future prices. The expression (16) for how a change in the price affects the average investor's expected returns, and hence demand for the asset, stems from the decomposition (see proof of Lemma 2):

$$\frac{\partial E\left[q_{i,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)} = A_i\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}+A_i\frac{\partial E\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t-E\left[P_t|\mathcal{H}_t\right]\right)}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} \quad (68)$$

[11] Haddad et al (2025) qualitatively relate a model based on dispersed information to demand elasticities. Davis et al (2025) show that, empirically, changes in asset prices have limited passthroughs to expected returns; this finding is consistent with asset prices conveying information. A contemporaneous working paper of Binsbergen et al (2025) similarly notes that "price-shift" terms may be persistent, and hence affect expectations about future prices; see also He et al (2025).

Both terms in (68) reflect the direct effect of a change in today's price. In addition, the first term captures what a change in the price reveals about future cash flows; and the second term captures what a change in the price reveals about the discount rate term Z_t, which, given persistence, affects futures prices also.

Overall, (68) captures how an investor's holding of the risky asset changes in response to a price shock with a cause that is unknown to the investor, and holding constant the investor's own desire to the hold asset, $Z_t + u_{i,t}$, and own signal about cash flows, $y_{i,t}$. This is the object estimated in demand-based asset pricing.

The estimates derived in this paper can be used to empirically estimate the decomposition (68) for the representative agent. Specifically, using the smallest possible value for the information advantage of an uninformed investor relative to the econometrician, $\left(\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}\right)^{-1}$, viz., a value of 1, delivers

$$
\begin{aligned}
& P_t \frac{\partial E\left[q_{i,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} \\
= \; & \frac{1}{E\left[X_t\right]} \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} \frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} + \frac{1}{E\left[X_t\right]} \frac{1}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}} \left(1 - \frac{\partial P_t}{\partial \epsilon_{D,t}} \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\right) \frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} \\
\approx \; & \overbrace{.05}^{\text{cash flow term}} - \overbrace{.19}^{\text{discount rate term}} = -.14.
\end{aligned}
\tag{69}
$$

Recall that quantities are normalized so that the average investor holds one unit; hence (69) corresponds to a demand elasticity of $-.14$. From the decomposition: an increase in the price communicates a positive innovation to future cash flows and a negative innovation to the discount rate term Z_t. Because prices are decreasing in the discount rate term, this means that a price increase communicates positive news about future cash flows and prices via both channels. This positive news partially offsets the negative direct effect of an increase in today's price, thereby reducing the responsiveness of quantity demanded to a price increase.[12]

Gabaix and Koijen's estimate a price elasticity of $\approx \frac{1}{5}$. As such, estimate (69) suggests that Gabaix and Koijen's estimated price elasticity can be largely explained by the information content of prices.

[12] Using larger values for the information advantage of an uninformed investor relative to the econometrician raises the first term, potentially as high as .19, because a larger information advantage leads to a greater ability to extract information from the price. Numerically, larger values for the information advantage of an uninformed investor relative to the econometrician have only very small effects on the second term. The upper bound of the information advantage corresponds precisely to the two terms offsetting each other, corresponding to a vertical (perfectly inelastic) demand curve.

10 Concluding remarks

I derive and implement a sufficient statistics formula for how well financial prices aggregate dispersed information. The key inputs to the formula are closely related to the outputs of the price-dividend predictability literature. The formula follows from market-clearing. Empirical implementation suggests a low level of aggregation, viz., the information of a *single* representative investor is much more informative than the information conveyed by the price. I further derive formulae for the value of a representative investor's information; empirical implementation suggests the value is small. Finally, I use the formulae derived to produce an estimate of demand elasticity, taking into account the information an investor derives from a price change of unknown origin; this exercise yields a highly inelastic value consistent with Gabaix and Koijen (2023)'s estimate.

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A Appendix: Proofs

A.1 Results omitted from main text

Lemma 4

$$var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1} = var\left[\epsilon_{D,t}\right]^{-1} + \left(\frac{c_D}{c_Z}\right)^2 \left(var\left[\epsilon_{Z,t}\right]^{-1} + var\left[u_{i,t}\right]^{-1}\right).$$

Proof of Lemma 4:

$$\begin{aligned} var\left[\epsilon_{D,t}|\mathcal{J}_t\right] = var\left[\epsilon_{D,t}|\mathcal{H}_t, P_t\right] &= var\left[\epsilon_{D,t}\right] - \frac{cov\left[\epsilon_{D,t}, P_t|\mathcal{H}_t\right]^2}{var\left[P_t|\mathcal{H}_t\right]^2} \\ &= var\left[\epsilon_{D,t}\right] - \frac{c_D^2 var\left[\epsilon_{D,t}\right]^2}{c_D^2 var\left[\epsilon_{D,t}\right] + c_Z^2 var\left[\epsilon_{Z,t}\right]} \\ &= \frac{c_Z^2 var\left[\epsilon_{D,t}\right] var\left[\epsilon_{Z,t}\right]}{c_D^2 var\left[\epsilon_{D,t}\right] + c_Z^2 var\left[\epsilon_{Z,t}\right]} \end{aligned}$$

and hence

$$var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1} = var\left[\epsilon_{D,t}\right]^{-1} + \left(\frac{c_D}{c_Z}\right)^2 var\left[\epsilon_{Z,t}\right]^{-1}.$$

Moreover,

$$\begin{aligned}
var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right] &= var\left[\epsilon_{D,t}|\mathcal{J}_t\right] - \frac{cov\left[\epsilon_{D,t}, Z_t + u_{i,t}|\mathcal{J}_t\right]^2}{var\left[Z_t + u_{i,t}|\mathcal{J}_t\right]} \\
&= var\left[\epsilon_{D,t}|\mathcal{J}_t\right] - \frac{cov\left[\epsilon_{D,t}, \epsilon_{Z,t}|\mathcal{J}_t\right]^2}{var\left[\epsilon_{Z,t}|\mathcal{J}_t\right] + var\left[u_{i,t}\right]} \\
&= var\left[\epsilon_{D,t}|\mathcal{J}_t\right] - \frac{\left(\frac{c_D}{c_Z}\right)^2 var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^2}{\left(\frac{c_D}{c_Z}\right)^2 var\left[\epsilon_{D,t}|\mathcal{J}_t\right] + var\left[u_{i,t}\right]} \\
&= \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right] var\left[u_{i,t}\right]}{\left(\frac{c_D}{c_Z}\right)^2 var\left[\epsilon_{D,t}|\mathcal{J}_t\right] + var\left[u_{i,t}\right]}.
\end{aligned}$$

Hence

$$var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]^{-1} = \left(\frac{c_D}{c_Z}\right)^2 var\left[u_{i,t}\right]^{-1} + var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^{-1},$$

establishing the result.

A.2 Proofs of results stated in the main text

Proof of Lemma 1: Let W denote the row vector of variables in $\mathcal{I}_{i,t}$. Since all random variables are normally distributed,

$$\begin{aligned}
\frac{\partial}{\partial \epsilon_{D,t}} E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right] &= cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} \left(\frac{\partial W}{\partial \epsilon_{D,t}}\right)^{\intercal} = \frac{1}{var\left[\epsilon_{D,t}\right]} cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} cov\left[\epsilon_{D,t}, W\right]^{\intercal} \\
\frac{\partial}{\partial \epsilon_{Z,t}} E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right] &= cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} \left(\frac{\partial W}{\partial \epsilon_{Z,t}}\right)^{\intercal} = \frac{1}{var\left[\epsilon_{Z,t}\right]} cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} cov\left[\epsilon_{Z,t}, W\right]^{\intercal}
\end{aligned}$$

Moreover,

$$var\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] = var\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right] - cov\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}, W\right] var\left[W\right]^{-1} cov\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}, W\right]^{\intercal},$$

and so

$$\begin{aligned}
var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right] &= var\left[\epsilon_{D,t}\right] - cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} cov\left[\epsilon_{D,t}, W\right]^{\intercal} \\
cov\left[\epsilon_{D,t}, \epsilon_{Z,t}|\mathcal{I}_{i,t}\right] &= cov\left[\epsilon_{D,t}, \epsilon_{Z,t}\right] - cov\left[\epsilon_{D,t}, W\right] var\left[W\right]^{-1} cov\left[\epsilon_{Z,t}, W\right]^{\intercal}.
\end{aligned}$$

Hence

$$
\begin{aligned}
\frac{\partial}{\partial \epsilon_{D,t}} E\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\right] &= \frac{1}{var\left[\epsilon_{D,t}\right]}\left(var\left[\epsilon_{D,t}\right] - var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\right]\right) \\
\frac{\partial}{\partial \epsilon_{Z,t}} E\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\right] &= \frac{1}{var\left[\epsilon_{Z,t}\right]}\left(cov\left[\epsilon_{D,t}, \epsilon_{Z,t}\right] - cov\left[\epsilon_{D,t}, \epsilon_{Z,t} | \mathcal{I}_{i,t}\right]\right),
\end{aligned}
$$

and similarly,

$$
\begin{aligned}
\frac{\partial}{\partial \epsilon_{D,t}} E\left[\epsilon_{Z,t} | \mathcal{I}_{i,t}\right] &= \frac{1}{var\left[\epsilon_{D,t}\right]}\left(cov\left[\epsilon_{D,t}, \epsilon_{Z,t}\right] - cov\left[\epsilon_{D,t}, \epsilon_{Z,t} | \mathcal{I}_{i,t}\right]\right) \\
\frac{\partial}{\partial \epsilon_{Z,t}} E\left[\epsilon_{Z,t} | \mathcal{I}_{i,t}\right] &= \frac{1}{var\left[\epsilon_{Z,t}\right]}\left(var\left[\epsilon_{Z,t}\right] - var\left[\epsilon_{Z,t} | \mathcal{I}_{i,t}\right]\right).
\end{aligned}
$$

Since $cov\left[\epsilon_{D,t}, \epsilon_{Z,t}\right] = 0$,

$$
\frac{\partial}{\partial \epsilon_{D,t}} E\left[\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix} | \mathcal{I}_{i,t}\right] - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\frac{1}{var\left[\epsilon_{D,t}\right]}\begin{pmatrix} var\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\right] \\ cov\left[\epsilon_{D,t}, \epsilon_{Z,t} | \mathcal{I}_{i,t}\right] \end{pmatrix} \tag{70}
$$

$$
\frac{\partial}{\partial \epsilon_{Z,t}} E\left[\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix} | \mathcal{I}_{i,t}\right] - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{var\left[\epsilon_{Z,t}\right]}\begin{pmatrix} cov\left[\epsilon_{D,t}, \epsilon_{Z,t} | \mathcal{I}_{i,t}\right] \\ var\left[\epsilon_{Z,t} | \mathcal{I}_{i,t}\right] \end{pmatrix}. \tag{71}
$$

Taking the ratio of (70) with analogous identity for information set $\mathcal{J}_t$ yields (11). Similarly, the ratio of (71) with analogous identity for information set $\mathcal{J}_t$ yields (12), completing the proof.

Proof of Lemma 2: To establish (15), note first that

$$
E\left[\epsilon_{D,t} | \mathcal{I}_{i,t}\right] = E\left[\epsilon_{D,t} | \mathcal{I}_{i,t} \backslash P_t\right] + \frac{cov\left[\epsilon_{D,t}, P_t | \mathcal{I}_{i,t} \backslash P_t\right]}{var\left[P_t | \mathcal{I}_{i,t} \backslash P_t\right]}\left(P_t - E\left[P_t | \mathcal{I}_{i,t} \backslash P_t\right]\right).
$$

Further note that

$$
P_t - E\left[P_t | \mathcal{I}_{i,t} \backslash P_t\right] = P_t - E\left[P_t | \mathcal{H}_t\right] - \left(E\left[P_t | \mathcal{I}_{i,t} \backslash P_t\right] - E\left[P_t | \mathcal{H}_t\right]\right)
$$

and that

$$
\begin{aligned}
E\left[P_t | \mathcal{I}_{i,t} \backslash P_t\right] - E\left[P_t | \mathcal{H}_t\right] &= E\left[c_D \epsilon_{D,t} + c_Z \epsilon_{Z,t} + E\left[P_t | \mathcal{H}_t\right] | \mathcal{I}_{i,t} \backslash P_t\right] - E\left[P_t | \mathcal{H}_t\right] \\
&= \begin{pmatrix} c_D & c_Z \end{pmatrix} E\left[\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix} | \mathcal{I}_{i,t} \backslash P_t\right].
\end{aligned}
$$

From these observations,

$$
\begin{aligned}
& \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} \\
= \quad & \frac{cov\left[\epsilon_{D,t}, P_t|\mathcal{I}_{i,t}\backslash P_t\right]}{var\left[P_t|\mathcal{I}_{i,t}\backslash P_t\right]} \\
= \quad & \frac{c_D var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\backslash P_t\right]}{c_D^2 var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\backslash P_t\right] + c_Z^2 var\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\backslash P_t\right]} \\
= \quad & \frac{1}{c_D}\frac{\left(\frac{c_D}{c_Z}\right)^2\left(var\left[\epsilon_{D,t}\right]^{-1}+\tau_i\right)^{-1}}{\left(\frac{c_D}{c_Z}\right)^2\left(var\left[\epsilon_{D,t}\right]^{-1}+\tau_i\right)^{-1} + \left(var\left[\epsilon_{Z,t}\right]^{-1}+var\left[u_{i,t}\right]^{-1}\right)^{-1}} \\
= \quad & \frac{1}{c_D}\frac{\left(\frac{c_D}{c_Z}\right)^2\left(var\left[\epsilon_{Z,t}\right]^{-1}+var\left[u_{i,t}\right]^{-1}\right)}{var\left[\epsilon_{D,t}\right]^{-1}+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(var\left[\epsilon_{Z,t}\right]^{-1}+var\left[u_{i,t}\right]^{-1}\right)}. \qquad (72)
\end{aligned}
$$

Substituting in Lemma 4, expression (72) equals

$$
\frac{1}{c_D} var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]\left(var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]^{-1} - var\left[\epsilon_{D,t}\right]^{-1}\right).
$$

Substituting of $c_D = \frac{\partial P_t}{\partial \epsilon_{D,t}}$ delivers (15).

To establish (16), note that

$$
\frac{\partial E\left[X_{t+1}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} = \frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} + \frac{\partial E\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}. \qquad (73)
$$

From (5),

$$
c_D\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} + c_Z\frac{\partial E\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)} = 1.
$$

Hence the RHS of (73) equals

$$
\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} + \frac{1}{c_Z}\left(1 - c_D\frac{\partial E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}{\partial\left(P_t - E\left[P_t|\mathcal{H}_t\right]\right)}\right)\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}.
$$

Substitution of $c_D = \frac{\partial P_t}{\partial \epsilon_{D,t}}$ and $c_Z = \frac{\partial P_t}{\partial \epsilon_{Z,t}}$ delivers (16), completing the proof.

Proof of Lemma 3: Using (22) and (23), the market-clearing conditions (18) and (19) rewrite as

$$\begin{aligned}\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \frac{1}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}\frac{\int A_i var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\tau_i)\right]}{\int A_i}\frac{\frac{\partial}{\partial \epsilon_{D,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}(0)\right]\right)}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]} &= 0 \\ \frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} - \frac{\int B_i}{\int A_i} - \frac{\int A_i var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\tau_i)\right]}{\int A_i}\frac{\frac{\partial}{\partial \epsilon_{Z,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}(0)\right]\right)}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]} &= 0.\end{aligned}$$

The result then follows immediately.

Proof of Proposition 3: Note that

$$V = \frac{E\left[E\left[q_{i,\bar{\tau},t}X_{t+1}|\mathcal{H}_t\right]\right]}{E\left[E\left[q_{i,0,t}X_{t+1}|\mathcal{H}_t\right]\right]} = \frac{E\left[E\left[q_{i,\bar{\tau},t}|\mathcal{H}_t\right]E\left[X_{t+1}|\mathcal{H}_t\right] + cov\left[q_{i,\bar{\tau},t}, X_{t+1}|\mathcal{H}_t\right]\right]}{E\left[E\left[q_{i,0,t}|\mathcal{H}_t\right]E\left[X_{t+1}|\mathcal{H}_t\right] + cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right]\right]}.$$

By Corollary 1, $cov\left[q_{i,\bar{\tau},t}, X_{t+1}|\mathcal{H}_t\right] = 0$. By (31) and (32),

$$E\left[q_{i,\bar{\tau},t}|\mathcal{H}_t\right] = E\left[q_{i,0,t}|\mathcal{H}_t\right] = 1.$$

Hence

$$V = \frac{E\left[E\left[X_{t+1}|\mathcal{H}_t\right]\right]}{E\left[E\left[X_{t+1}|\mathcal{H}_t\right] + cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right]\right]} = \frac{E\left[X_{t+1}\right]}{E\left[X_{t+1}\right] + cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right]}.$$

From the market-clearing condition (19) ,

$$\bar{A}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} - \bar{B} - \bar{A}\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}\frac{\partial}{\partial \epsilon_{Z,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}(0)\right]\right) = 0.$$

Noting that

$$\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}} = \bar{A}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}} - \bar{B} - \bar{A}\frac{\partial}{\partial \epsilon_{Z,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}(0)\right]\right),$$

it follows that

$$\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}} + \left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}\right)\bar{A}\frac{\partial}{\partial \epsilon_{Z,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}(0)\right]\right) = 0. \tag{74}$$

Further below, I establish that forecast error sensitivities are related according to

$$\frac{\frac{\partial}{\partial \epsilon_{Z,t}}\left(X_t - E\left[X_t|\mathcal{I}_{i,t}\right]\right)}{\frac{\partial}{\partial \epsilon_{D,t}}\left(X_t - E\left[X_t|\mathcal{I}_{i,t}\right]\right)} = -\frac{var\left[\epsilon_{D,t}\right]}{var\left[\epsilon_{Z,t}\right]}\frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}. \tag{75}$$

Equality (75) follows entirely from updating rules, making use of the fact that all relevant date t information sets include the price P_t. Substitution into (74) yields

$$\frac{\partial q_{i,0,t}}{\partial \epsilon_{Z,t}} = \bar{A}\left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]}\right)\frac{var\left[\epsilon_{D,t}\right]}{var\left[\epsilon_{Z,t}\right]}\frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\frac{\partial}{\partial \epsilon_{D,t}}\left(X_t - E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right]\right). \tag{76}$$

Market-clearing condition (18) and (22) deliver

$$\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]}\frac{\partial}{\partial \epsilon_{D,t}}\left(X_{t+1} - E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right]\right) = 0, \tag{77}$$

and hence

$$\left(1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(\bar{\tau}\right)\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\left(0\right)\right]}\right)\frac{\partial}{\partial \epsilon_{D,t}}\left(X_t - E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right]\right) = -\frac{\partial}{\partial \epsilon_{D,t}}E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right].$$

Noting that $\frac{\partial q_{i,0,t}}{\partial \epsilon_{D,t}} = \bar{A}\frac{\partial}{\partial \epsilon_{D,t}}E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right]$, it follows that

$$\begin{aligned}\frac{cov\left[q_{i,0,t}, X_{t+1}|\mathcal{H}_t\right]}{\bar{A}} &= \left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}var\left[\epsilon_{D,t}\right] - \frac{var\left[\epsilon_{D,t}\right]}{var\left[\epsilon_{Z,t}\right]}\frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}var\left[\epsilon_{Z,t}\right]\right)\frac{\partial}{\partial \epsilon_{D,t}}E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right] \\ &= -\frac{1 - \frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}}{\frac{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(\bar{\tau})\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}(0)\right]}}\left(\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}} - \frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}}\frac{\partial X_{t+1}}{\partial \epsilon_{Z,t}}\right)\frac{\partial X_{t+1}}{\partial \epsilon_{D,t}}var\left[\epsilon_{D,t}\right],\end{aligned}$$

where the second equality following from by substituting for $\frac{\partial}{\partial \epsilon_{D,t}}E\left[X_{t+1}|\mathcal{I}_{i,t}\left(0\right)\right]$ using (77). Finally, substitution of (36) delivers (35).

It remains to establish (75). Since the variance-covariance matrix of $\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}$ conditional on $\mathcal{I}_{i,t}$ is singular, expressions (70) and (71) from the proof of Lemma 1 can be written as

$$\begin{aligned}\frac{\partial}{\partial \epsilon_{D,t}}E\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] - \begin{pmatrix}1\\0\end{pmatrix} &= -\frac{1}{var\left[\epsilon_{D,t}\right]}\begin{pmatrix}var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]\\ cov\left[\epsilon_{D,t}, \epsilon_{Z,t}|\mathcal{I}_{i,t}\right]\end{pmatrix} \\ \frac{\partial}{\partial \epsilon_{Z,t}}E\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] - \begin{pmatrix}0\\1\end{pmatrix} &= -\frac{1}{var\left[\epsilon_{Z,t}\right]}\begin{pmatrix}cov\left[\epsilon_{D,t}, \epsilon_{Z,t}|\mathcal{I}_{i,t}\right]\\ \frac{cov\left[\epsilon_{D,t},\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]^2}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}\end{pmatrix},\end{aligned}$$

and hence

$$\frac{\partial}{\partial \epsilon_{Z,t}}\left(E\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] - \begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right) = \frac{var\left[\epsilon_{D,t}\right]}{var\left[\epsilon_{Z,t}\right]}\frac{cov\left[\epsilon_{D,t}, \epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]}\frac{\partial}{\partial \epsilon_{D,t}}\left(E\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] - \begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right)$$

Moreover, the analogue of (10) for the information set $\mathcal{I}_{i,t}$ implies

$$\frac{cov\left[\epsilon_{D,t},\epsilon_{Z,t}|\mathcal{I}_{i,t}\right]}{var\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right]} = -\frac{c_D}{c_Z} = -\frac{\frac{\partial P_t}{\partial \epsilon_{D,t}}}{\frac{\partial P_t}{\partial \epsilon_{Z,t}}},$$

delivering (75) and completing the proof.

B Appendix: Parameter values in (42)-(44)

B.1 Details for subsection 8.1

The now-standard present value approximation (Campbell and Shiller 1988; for completeness, see Appendix E) is

$$pd_t - \bar{pd} = \frac{g_t - \bar{\Delta d}}{1-\rho\phi_g} - \frac{\mu_t - \bar{\mu}}{1-\rho\phi_\mu}. \tag{78}$$

Substitution of (38) and (39) into (78) yields

$$pd_{t+1} - \bar{pd} = \frac{\phi_g\left(g_t - \bar{\Delta d}\right)}{1-\rho\phi_g} - \frac{\phi_\mu\left(\mu_t - \bar{\mu}\right)}{1-\rho\phi_\mu} + \frac{\nu_{g,t+1}}{1-\rho\phi_g} - \frac{\nu_{\mu,t+1}}{1-\rho\phi_\mu},$$

which after a further substitution of (78) yields

$$pd_{t+1} - \bar{pd} = \frac{\phi_g - \phi_\mu}{1-\rho\phi_g}\left(g_t - \bar{\Delta d}\right) + \phi_\mu\left(pd_t - \bar{pd}\right) + \frac{\nu_{g,t+1}}{1-\rho\phi_g} - \frac{\nu_{\mu,t+1}}{1-\rho\phi_\mu}. \tag{79}$$

Equalities (40) and (44) in the main text directly follow from (78).

B.2 From observable moments to parameter values

The parameters to estimate are: $\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu, a_{pd}, a_d, \sigma^2_{pd}, \sigma^2_d, \sigma_{pd,d}\right\}$.

Of these, $\bar{pd}$ and $\bar{\Delta d}$ are estimated using the sample means of pd_t and Δd_t, and ρ is in turn a function of $\bar{pd}$.

The remaining seven parameters $\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu, a_{pd}, a_d, \sigma^2_{pd}, \sigma^2_d, \sigma_{pd,d}\right\}$ are estimated from the observed variance and covariance of pd_t and Δd_t, including lags.

As preliminaries: Let a_0 denote the coefficient on g_t in the pd_t transition equation,

$$a_0 = \frac{\phi_g - \phi_\mu}{1-\rho\phi_g}.$$

The following variance and covariances, which are not directly observable, enter many expressions below:

$$
\begin{aligned}
var\left[g_t\right] &= \frac{var\left[a_{pd}\nu_{pd,t}+a_d\nu_{d,t}\right]}{1-\phi_g^2} = \frac{a_{pd}^2\sigma_{pd}^2+a_d^2\sigma_d^2+2a_{pd}a_d\sigma_{pd,d}}{1-\phi_g^2} \qquad (80)\\
cov\left[g_t,\Delta d_t\right] &= \phi_g var\left[g_t\right]+cov\left[a_{pd}\nu_{pd,t}+a_d\nu_{d,t},\nu_{d,t}\right]=\phi_g var\left[g_t\right]+a_{pd}\sigma_{pd,d}+a_d\sigma_d^2\\
cov\left[g_t,pd_t\right] &= \frac{a_0\phi_g var\left[g_t\right]+cov\left[a_{pd}\nu_{pd,t}+a_d\nu_{d,t},\nu_{pd,t}\right]}{1-\phi_g\phi_\mu} = \frac{a_0\phi_g var\left[g_t\right]+a_{pd}\sigma_{pd}^2+a_d\sigma_{pd,d}}{1-\phi_g\phi_\mu}.
\end{aligned}
$$

The observable moments are

$$
\begin{aligned}
var\left[pd_t\right] &= \frac{a_0^2 var\left[g_t\right]+2a_0\phi_\mu cov\left[g_t,pd_t\right]+\sigma_{pd}^2}{1-\phi_\mu^2} \qquad (81)\\
var\left[\Delta d_t\right] &= var\left[g_t\right]+\sigma_d^2 \qquad (82)\\
cov\left[\Delta d_t,pd_t\right] &= a_0 var\left[g_t\right]+\phi_\mu cov\left[g_t,pd_t\right]+\sigma_{pd,d}, \qquad (83)
\end{aligned}
$$

and

$$
\begin{aligned}
cov\left[\Delta d_{t+1},\Delta d_t\right] &= cov\left[g_t+\nu_{d,t+1},\Delta d_t\right]=cov\left[g_t,\Delta d_t\right] \qquad (84)\\
cov\left[\Delta d_{t+1},pd_t\right] &= cov\left[g_t+\nu_{d,t+1},pd_t\right]=cov\left[g_t,pd_t\right] \qquad (85)\\
cov\left[pd_{t+1},\Delta d_t\right] &= cov\left[a_0g_t+\phi_\mu pd_t+\nu_{pd,t+1},\Delta d_t\right]\\
&= a_0cov\left[g_t,\Delta d_t\right]+\phi_\mu cov\left[pd_t,\Delta d_t\right]\\
&= a_0cov\left[\Delta d_{t+1},\Delta d_t\right]+\phi_\mu cov\left[pd_t,\Delta d_t\right]\\
cov\left[pd_{t+1},pd_t\right] &= cov\left[a_0g_t+\phi_\mu pd_t+\nu_{pd,t+1},pd_t\right]\\
&= a_0cov\left[g_t,pd_t\right]+\phi_\mu var\left[pd_t\right]\\
&= a_0cov\left[\Delta d_{t+1},pd_t\right]+\phi_\mu var\left[pd_t\right]
\end{aligned}
$$

and

$$
\begin{aligned}
cov\left[\Delta d_{t+2},\Delta d_t\right] &= \phi_g cov\left[g_t,\Delta d_t\right]=\phi_g cov\left[\Delta d_{t+1},\Delta d_t\right]\\
cov[\Delta d_{t+2},pd_t] &= \phi_g cov\left[g_t,pd_t\right]=\phi_g cov\left[\Delta d_{t+1},pd_t\right]\\
cov\left[pd_{t+2},\Delta d_t\right] &= cov\left[a_0g_{t+1}+\phi_\mu pd_{t+1},\Delta d_t\right]\\
&= a_0\phi_g cov\left[g_t,\Delta d_t\right]+\phi_\mu cov\left[pd_{t+1},\Delta d_t\right]\\
&= a_0\phi_g cov\left[\Delta d_{t+1},\Delta d_t\right]+\phi_\mu cov\left[pd_{t+1},\Delta d_t\right]\\
cov\left[pd_{t+2},pd_t\right] &= cov\left[a_0g_{t+1}+\phi_\mu pd_{t+1},pd_t\right]\\
&= a_0\phi_g cov\left[g_t,pd_t\right]+\phi_\mu cov\left[pd_{t+1},pd_t\right]\\
&= a_0\phi_g cov\left[\Delta d_{t+1},pd_t\right]+\phi_\mu cov\left[pd_{t+1},pd_t\right].
\end{aligned}
$$

(One can continue to compute further lag covariances, but doing so does not yield any additional information.)

The parameter ϕ_g is given by

$$\phi_g = \frac{cov\left[\Delta d_{t+2}, \Delta d_t\right]}{cov\left[\Delta d_{t+1}, \Delta d_t\right]}.$$

Given ϕ_g, the parameter ϕ_μ can be inferred from the combination of $cov\left[pd_{t+2}, pd_t\right]$, $cov\left[\Delta d_{t+1}, pd_t\right]$ and $cov\left[pd_{t+1}, pd_t\right]$.

Given ϕ_g and ϕ_μ, the remaining non-redundant moment conditions are (81)-(85).

To solve for $\left\{a_{pd}, a_d, \sigma_{pd}^2, \sigma_d^2, \sigma_{pd,d}\right\}$, first substitute (85) into (81) and (83) and rearrange to yield expressions for $\sigma_{pd}^2, \sigma_d^2, \sigma_{pd,d}$ in terms of observable moments and $var\left[g_t\right]$.

$$\begin{aligned} \sigma_{pd}^2 &= \left(1-\phi_\mu^2\right) var\left[pd_t\right] - a_0^2 var\left[g_t\right] - 2a_0\phi_\mu cov\left[\Delta d_{t+1}, pd_t\right] && (86)\\ \sigma_d^2 &= var\left[\Delta d_t\right] - var\left[g_t\right] && (87)\\ \sigma_{pd,d} &= cov\left[\Delta d_t, pd_t\right] - a_0 var\left[g_t\right] - \phi_\mu cov\left[\Delta d_{t+1}, pd_t\right]. && (88) \end{aligned}$$

Next, substitute in for $cov\left[g_t, \Delta d_t\right]$ and $cov\left[g_t, pd_t\right]$ in (84) and (85) and rearrange to yield

$$\begin{aligned} a_{pd}\sigma_{pd,d} + a_d\sigma_d^2 &= cov\left[\Delta d_{t+1}, \Delta d_t\right] - \phi_g var\left[g_t\right]\\ a_{pd}\sigma_{pd}^2 + a_d\sigma_{pd,d} &= \left(1-\phi_g\phi_\mu\right) cov\left[\Delta d_{t+1}, pd_t\right] - a_0\phi_g var\left[g_t\right], \end{aligned}$$

and hence

$$\begin{aligned} \left(\sigma_{pd,d}^2 - \sigma_{pd}^2\sigma_d^2\right) a_{pd} &= \sigma_{pd,d}\left(cov\left[\Delta d_{t+1}, \Delta d_t\right] - \phi_g var\left[g_t\right]\right)\\ &- \sigma_d^2\left(\left(1-\phi_g\phi_\mu\right) cov\left[\Delta d_{t+1}, pd_t\right] - a_0\phi_g var\left[g_t\right]\right) && (89)\\ \left(\sigma_{pd,d}^2 - \sigma_{pd}^2\sigma_d^2\right) a_d &= \sigma_{pd,d}\left(\left(1-\phi_g\phi_\mu\right) cov\left[\Delta d_{t+1}, pd_t\right] - a_0\phi_g var\left[g_t\right]\right)\\ &- \sigma_{pd}^2\left(cov\left[\Delta d_{t+1}, \Delta d_t\right] - \phi_g var\left[g_t\right]\right). && (90) \end{aligned}$$

Together, equations (86)-(90) give $\left\{a_{pd}, a_d, \sigma_{pd}^2, \sigma_d^2, \sigma_{pd,d}\right\}$ in terms of observable moments and $var\left[g_t\right]$. The term $var\left[g_t\right]$ itself can be solved for using (80).

B.3 Recovering observable moments from the reported estimates in BK

BK estimate the system

$$
\begin{aligned}
g_{t+1} - \bar{\Delta d}^{BK} &= \phi_g^{BK}\left(g_t - \bar{\Delta d}^{BK}\right) + \nu_{g,t+1}^{BK} \\
\Delta d_{t+1} &= g_t + \nu_{d,t+1}^{BK} \\
pd_{t+1} - \bar{pd}^{BK} &= \frac{\phi_g^{BK} - \phi_\mu^{BK}}{1-\rho^{BK}\phi_g^{BK}}\left(g_t - \bar{\Delta d}^{BK}\right) + \phi_\mu^{BK}\left(pd_t - \bar{pd}^{BK}\right) \\
&- \frac{1}{1-\rho^{BK}\phi_\mu^{BK}}\nu_{\mu,t+1}^{BK} + \frac{1}{1-\rho^{BK}\phi_g^{BK}}\nu_{g,t+1}^{BK}
\end{aligned}
$$

under the restriction that $cov\left[\nu_{g,t+1}^{BK}, \nu_{d,t+1}^{BK}\right] = 0$.

First note that the relation between $\left\{\bar{\Delta d}^{BK}, \bar{pd}^{BK}, \rho^{BK}, \phi_g^{BK}, \phi_\mu^{BK}\right\}$ and observable moments is exactly the same as the relation between $\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu\right\}$ and observable moments. So it is immediate that

$$
\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu\right\} = \left\{\bar{\Delta d}^{BK}, \bar{pd}^{BK}, \rho^{BK}, \phi_g^{BK}, \phi_\mu^{BK}\right\}. \tag{91}
$$

As such, it is only necessary to recover the five moments that are used to infer $\left\{a_{pd}, a_d, \sigma_{pd}^2, \sigma_d^2, \sigma_{pd,d}\right\}$, namely $var\left[pd_t\right]$, $var\left[\Delta d_t\right]$, $cov\left[\Delta d_t, pd_t\right]$, $cov\left[\Delta d_{t+1}, \Delta d_t\right]$ and $cov\left[\Delta d_{t+1}, pd_t\right]$. Explicit evaluation implies that these five moments are given by the following expressions (in light of (91), I drop the BK subscripts on $\left\{\bar{\Delta d}, \bar{pd}, \rho, \phi_g, \phi_\mu\right\}$):

$$
\begin{aligned}
var\left[g_t\right] &= \frac{\left(\sigma_g^{BK}\right)^2}{1-\phi_g^2} \\
cov\left[g_t, \Delta d_t\right] &= \phi_g var\left[g_t\right] \\
cov\left[g_t, pd_t\right] &= \frac{a_0\phi_g var\left[g_t\right] - \frac{\sigma_{\mu,g}^{BK}}{1-\rho\phi_\mu} + \frac{\left(\sigma_g^{BK}\right)^2}{1-\rho\phi_g}}{1-\phi_g\phi_\mu} \\
var\left[pd_t\right] &= \frac{a_0^2 var\left[g_t\right] + 2a_0\phi_\mu cov\left[g_t, pd_t\right]}{1-\phi_\mu^2} \\
&+ \frac{\left(\frac{1}{1-\rho\phi_\mu}\right)^2\left(\sigma_\mu^{BK}\right)^2 + \left(\frac{1}{1-\rho\phi_g}\right)^2\left(\sigma_g^{BK}\right)^2 - 2\frac{1}{1-\rho\phi_\mu}\frac{1}{1-\rho\phi_g}\sigma_{\mu,g}^{BK}}{1-\phi_\mu^2} \\
var\left[\Delta d_t\right] &= var\left[g_t\right] + \left(\sigma_d^{BK}\right)^2 \\
cov\left[\Delta d_t, pd_t\right] &= a_0 var\left[g_t\right] + \phi_\mu cov\left[g_t, pd_t\right] - \frac{1}{1-\rho\phi_\mu}\sigma_{\mu,d}^{BK} \\
cov\left[\Delta d_{t+1}, \Delta d_t\right] &= cov\left[g_t, \Delta d_t\right] \\
cov\left[\Delta d_{t+1}, pd_t\right] &= cov\left[g_t, pd_t\right].
\end{aligned}
$$

Because the first three of the moments namely $var\left[pd_t\right]$, $var\left[\Delta d_t\right]$, $cov\left[\Delta d_t, pd_t\right]$, $cov\left[\Delta d_{t+1}, \Delta d_t\right]$ and $cov\left[\Delta d_{t+1}, pd_t\right]$ are used in the analysis, I report the recovered values here:

Moment	Estimated value
$var\left[pd_t\right]$	.1983
$var\left[\Delta d_t\right]$	.0038
$cov\left[\Delta d_t, pd_t\right]$	$-.0034$
$cov\left[\Delta d_{t+1}, \Delta d_t\right]$	.0014
$cov\left[\Delta d_{t+1}, pd_t\right]$	$-.000106$

Table 3: Estimates of key moments, calculated from BK estimates

C Appendix: Details for calculations in subsections 8.2.1 to 8.4.2

C.1 Details for subsection 8.2.1

For use here and elsewhere: From the standard return approximation (113),

$$var\left[r_{t+1}|\mathcal{J}_t\right] = var\left[\rho pd_{t+1} + \Delta d_{t+1}|\mathcal{J}_t\right] = var\left[\rho \nu_{pd,t+1} + \nu_{d,t+1}|\mathcal{J}_t\right].$$

Evaluating,

$$var\left[r_{t+1}|\mathcal{J}_t\right] = .147^2. \tag{92}$$

For any random variable Y,

$$e^Y \approx e^{E[Y]} + \left(Y - E\left[Y\right]\right) e^{E[Y]} + \frac{1}{2}\left(Y - E\left[Y\right]\right)^2 e^{E[Y]},$$

and hence

$$E\left[e^Y\right] \approx e^{E[Y]}\left(1 + \frac{1}{2} var\left[Y\right]\right).$$

Hence (substituting in for b_{pd} using (45))

$$\begin{aligned} dE\left[e^{r_{t+1}}|\mathcal{J}_t\right] &\approx e^{E[r_{t+1}|\mathcal{J}_t]}\left(1 + \frac{1}{2} var\left[r_{t+1}|\mathcal{J}_t\right]\right) dE\left[r_{t+1}|\mathcal{J}_t\right] \\ &= e^{E[r_{t+1}|\mathcal{J}_t]}\left(1 + \frac{1}{2} var\left[r_{t+1}|\mathcal{J}_t\right]\right) b_{pd}\frac{dP_t}{P_t}. \end{aligned}$$

Note that

$$X_{t+1} = \left(e^{r_{t+1}} - R\right) P_t. \tag{93}$$

Hence[13]

$$\begin{aligned} \frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t} &\approx e^{E\left[r_{t+1}|\mathcal{J}_t\right]}\left(1+\frac{1}{2}var\left[r_{t+1}|\mathcal{J}_t\right]\right)b_{pd} \\ &= e^{.09}\left(1+\frac{.147^2}{2}\right)(-.0898) = -.0993. \end{aligned} \tag{94}$$

C.2 Details for subsection 8.2.2

To establish the second equality in (47), note that Evaluating,

$$\begin{aligned} cov\left[pd_t, \Delta d_{t+1}|\Delta d_t, \mathcal{J}_{t-1}\right] &= cov\left[pd_t, g_t|\Delta d_t, \mathcal{J}_{t-1}\right] \\ &= cov\left[\nu_{pd,t}, a_{pd}\nu_{pd,t}|\nu_{d,t}\right] \end{aligned} \tag{95}$$

and

$$\begin{aligned} var\left[\Delta d_{t+1}|\Delta d_t, \mathcal{J}_{t-1}\right] &= var\left[\nu_{d,t+1}\right] + var\left[g_t|\Delta d_t, \mathcal{J}_{t-1}\right] \\ &= var\left[\nu_{d,t+1}\right] + var\left[a_{pd}\nu_{pd,t}|\nu_{d,t}\right], \end{aligned} \tag{96}$$

Evaluation of (48) requires an input for $var\left[\nu_{pd,t}|\nu_{d,t}\right]$:

$$\begin{aligned} var\left[\nu_{pd,t}|\nu_{d,t}\right] &= var\left[\nu_{pd,t}\right] - \left(\frac{cov\left[\nu_{pd,t}, \nu_{d,t}\right]}{var\left[\nu_{d,t}\right]}\right)^2 var\left[\nu_{d,t}\right] \\ &= var\left[\nu_{pd,t}\right]\left(1 - corr\left[\nu_{pd,t}, \nu_{d,t}\right]^2\right) = 0.0230. \end{aligned} \tag{97}$$

[13] Note that in evaluating (94) I have used only the effect of P_t on $E\left[e^{r_{t+1}}|\mathcal{J}_t\right]$, without considering the effect on the "base" P_t in (93). The reason is that the P_t term in (93) corresponds to the market capitalization of the asset in question, here the combined SP500; but the econometric estimate of $b_{pd} = \frac{\partial E[r_{t+1}|\mathcal{J}_t]}{\partial \ln P_t}$ concerns a change in SP500 index value, which isn't proportional to SP500 market capitalization because of, for example, share issues and repurchases.

Evaluation of (50) requires the expected value of $\frac{P_t}{D_{t+1}}$:

$$\begin{aligned}
E\left[\frac{P_t}{D_{t+1}}\right] = E\left[\frac{\frac{P_t}{D_t}}{\frac{D_{t+1}}{D_t}}\right] &= \frac{E\left[\frac{P_t}{D_t}\right]}{E\left[\frac{D_{t+1}}{D_t}\right]} + cov\left[\frac{P_t}{D_t}, \frac{1}{\frac{D_{t+1}}{D_t}}\right] \\
&= \frac{E\left[e^{pd_t}\right]}{E\left[e^{\Delta d_{t+1}}\right]} + cov\left[e^{pd_t}, e^{-\Delta d_{t+1}}\right] \\
&\approx \frac{e^{\bar{pd}}\left(1+\frac{1}{2}var\left[pd_t\right]\right)}{e^{\bar{\Delta d}}\left(1+\frac{1}{2}var\left[\Delta d_{t+1}\right]\right)} - \frac{e^{\bar{pd}}}{e^{\bar{\Delta d}}} cov\left[pd_t, \Delta d_{t+1}\right].
\end{aligned}$$

Evaluating (see Table 3 for values of the variance and covariance terms):

$$E\left[\frac{P_t}{D_{t+1}}\right] \approx e^{3.571-.062}\left(\frac{1+\frac{.1983}{2}}{1+\frac{.0038}{2}} + .0001\right) = 33.4 \times 1.097 = 36.7.$$

C.3 Details for subsection 8.2.3

$$\begin{aligned}
\left.\frac{\partial\left(P_{t+1}+D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t} &= 1 + \frac{P_{t+1}}{D_{t+1}} \left.\frac{\partial \ln P_{t+1}}{\partial \ln D_{t+1}}\right|_{\mathcal{J}_t} \\
&= 1 + \frac{P_{t+1}}{D_{t+1}} \left.\frac{\partial \ln \frac{P_{t+1}}{D_{t+1}}}{\partial \ln \frac{D_{t+1}}{D_t}} + 1\right|_{\mathcal{J}_t} \\
&= 1 + e^{pd_{t+1}}\left(\left.\frac{\partial pd_{t+1}}{\partial \Delta d_{t+1}}\right|_{\mathcal{J}_t} + 1\right).
\end{aligned}$$

To numerically evaluate, I replace $e^{pd_{t+1}}$ with its expected value:

$$E\left[e^{pd_{t+1}}\right] \approx e^{\bar{pd}}\left(1+\frac{1}{2}var\left[pd_{t+1}\right]\right) = e^{3.571}\left(1+\frac{.1983}{2}\right) = 39.1.$$

C.4 Details for subsection 8.3.1

C.4.1 Calculation of $var\left[pd_t|\Delta d_{t+1}, \Delta d_t, \mathcal{J}_{t-1}\right]$

Both here and below it is necessary to evaluate

$$var\left[pd_t|\Delta d_{t+1}, \Delta d_t, \mathcal{J}_{t-1}\right].$$

To do so, note that this equals

$$\begin{aligned}
& var\left[E\left[pd_t|\mathcal{J}_{t-1}\right]+\nu_{pd,t}|g_t+\nu_{d,t+1},g_{t-1}+\nu_{d,t},\mathcal{J}_{t-1}\right] \\
= & \; var\left[\nu_{pd,t}|\phi_g g_{t-1}+a_{pd}\nu_{pd,t}+a_d\nu_{d,t}+\nu_{d,t+1},\nu_{d,t},\mathcal{J}_{t-1}\right] \\
= & \; var\left[\nu_{pd,t}|a_{pd}\nu_{pd,t}+\nu_{d,t+1},\nu_{d,t}\right],
\end{aligned}$$

which expands as

$$\begin{aligned}
& \sigma_{pd}^2-\left(\begin{array}{cc} a_{pd}\sigma_{pd}^2 & \sigma_{pd,d}\end{array}\right)\left(\begin{array}{cc} a_{pd}^2\sigma_{pd}^2+\sigma_d^2 & a_{pd}\sigma_{pd,d} \\ a_{pd}\sigma_{pd,d} & \sigma_d^2\end{array}\right)^{-1}\left(\begin{array}{c} a_{pd}\sigma_{pd}^2 \\ \sigma_{pd,d}\end{array}\right) \\
= & \; \sigma_{pd}^2-\frac{\left(\begin{array}{cc} a_{pd}\sigma_{pd}^2 & \sigma_{pd,d}\end{array}\right)}{a_{pd}^2\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right)+\sigma_d^4}\left(\begin{array}{cc} \sigma_d^2 & -a_{pd}\sigma_{pd,d} \\ -a_{pd}\sigma_{pd,d} & a_{pd}^2\sigma_{pd}^2+\sigma_d^2\end{array}\right)\left(\begin{array}{c} a_{pd}\sigma_{pd}^2 \\ \sigma_{pd,d}\end{array}\right) \\
= & \; \sigma_{pd}^2-\frac{\left(\begin{array}{cc} a_{pd}\sigma_{pd}^2 & \sigma_{pd,d}\end{array}\right)}{a_{pd}^2\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right)+\sigma_d^4}\left(\begin{array}{c} a_{pd}\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right) \\ \sigma_d^2\sigma_{pd,d}\end{array}\right) \\
= & \; \sigma_{pd}^2-\frac{a_{pd}^2\sigma_{pd}^2\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right)+\sigma_d^2\sigma_{pd,d}^2}{a_{pd}^2\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right)+\sigma_d^4} \\
= & \; \frac{\sigma_{pd}^2\sigma_d^4-\sigma_d^2\sigma_{pd,d}^2}{a_{pd}^2\left(\sigma_{pd}^2\sigma_d^2-\sigma_{pd,d}^2\right)+\sigma_d^4} \\
= & \; \frac{1-\left(\frac{\sigma_{pd,d}}{\sigma_{pd}\sigma_d}\right)^2}{a_{pd}^2\left(1-\left(\frac{\sigma_{pd,d}}{\sigma_{pd}\sigma_d}\right)^2\right)\sigma_d^{-2}+\sigma_{pd}^{-2}}.
\end{aligned}$$

Numerically evaluating,

$$var\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\approx .0226=.150^2. \tag{98}$$

C.4.2 Main calculation

The variance of any random variable conditional on information $\{D_{t+1},D_t\}\cup\mathcal{J}_{t-1}$, or equivalently $\{\Delta d_{t+1},\Delta d_t\}\cup\mathcal{J}_{t-1}$, is independent of any additively separable terms measurable with respect to this information set. Below, I write $\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right)$ for such terms.Expanding,

$$\begin{aligned}
E\left[X_{t+1}|D_{t+1},\mathcal{J}_t\right] &= D_tE\left[e^{pd_{t+1}+\Delta d_{t+1}}+e^{\Delta d_{t+1}}-Re^{pd_t}|D_{t+1},\mathcal{J}_t\right] \\
&= D_t\left(E\left[e^{pd_{t+1}+\Delta d_{t+1}}|\Delta d_{t+1},\mathcal{J}_t\right]-Re^{pd_t}\right)+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
\end{aligned}$$

Note that

$$
\begin{aligned}
& E\left[e^{pd_{t+1}+\Delta d_{t+1}}|D_{t+1},\mathcal{J}_t\right] \\
\approx\ & e^{E[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\mathcal{J}_t]}\left(1+\frac{1}{2}var\left[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right]\right) \\
\approx\ & e^{E[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}\left(1+E\left[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right]-E\left[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right) \\
\times\ & \left(1+\frac{1}{2}var\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]\right) \\
=\ & E\left[pd_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right]e^{E[pd_{t+1}+\Delta d_{t+1}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}\left(1+\frac{1}{2}var\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]\right)+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
Re^{pd_t} &\approx Re^{E[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}\left(1+pd_t-E\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right) \\
&= pd_t Re^{E[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
\end{aligned}
$$

Next, note that

$$
\begin{aligned}
& E\left[pd_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right] \\
=\ & E\left[\nu_{pd,t+1}|\Delta d_{t+1},\mathcal{J}_t\right]+E\left[pd_{t+1}|\mathcal{J}_t\right] \\
=\ & E\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]+\frac{\phi_g-\phi_\mu}{1-\rho\phi_g}g_t+\phi_\mu pd_t+\text{constant terms} \\
=\ & \frac{cov\left[\nu_{pd,t+1},\nu_{d,t+1}\right]}{var\left[\nu_{d,t+1}\right]}\nu_{d,t+1}+\frac{\phi_g-\phi_\mu}{1-\rho\phi_g}\left(a_{pd}\nu_{pd,t}+a_d\nu_{d,t}\right)+\phi_\mu\nu_{pd,t}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
\end{aligned}
$$

Note that $\nu_{d,t}=\Delta d_t-g_{t-1}=\Delta d_t-E\left[\Delta d_t|\mathcal{J}_{t-1}\right]$ is measurable with respect to $\{\Delta d_{t+1},\Delta d_t\}\cup\mathcal{J}_{t-1}$, while

$$
\begin{aligned}
\nu_{d,t+1}=\Delta d_{t+1}-g_t &= \Delta d_{t+1}-\nu_{g,t}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right) \\
&= -a_{pd}\nu_{pd,t}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
\end{aligned}
$$

Hence

$$
E\left[pd_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right]=\left(\left(\frac{\phi_g-\phi_\mu}{1-\rho\phi_g}-\frac{cov\left[\nu_{pd,t+1},\nu_{d,t+1}\right]}{var\left[\nu_{d,t+1}\right]}\right)a_{pd}+\phi_\mu\right)\nu_{pd,t}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
$$

Similarly,

$$
pd_t = \nu_{pd,t}+E\left[pd_t|\mathcal{J}_{t-1}\right]=\nu_{pd,t}+\mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right).
$$

Putting everything together,

$$\begin{aligned} E\left[X_{t+1}|D_{t+1},\mathcal{J}_t\right] &\approx D_t K \nu_{pd,t} + \mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right) \\ &= P_{t-1}e^{\Delta d_t - pd_{t-1}} K \nu_{pd,t} + \mathcal{M}\left(\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right) \end{aligned} \tag{99}$$

where

$$\begin{aligned} K &= e^{E[pd_{t+1}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}e^{\Delta d_{t+1}} \\ &\times \left(1+\frac{1}{2}var\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]\right)\left(\left(\frac{\phi_g-\phi_\mu}{1-\rho\phi_g}-\frac{cov\left[\nu_{pd,t+1},\nu_{d,t+1}\right]}{var\left[\nu_{d,t+1}\right]}\right)a_{pd}+\phi_\mu\right) \\ &- Re^{E[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}. \end{aligned}$$

Note that $var\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]$ is evaluated in (97).

Evaluating at steady-state values,

$$\begin{aligned} e^{\Delta d_t - pd_{t-1}}K &\approx e^{2\bar{\Delta d}}\left(1+\frac{1}{2}var\left[\nu_{pd,t+1}|\nu_{d,t+1}\right]\right)\left(\left(\frac{\phi_g-\phi_\mu}{1-\rho\phi_g}-\frac{cov\left[\nu_{pd,t+1},\nu_{d,t+1}\right]}{var\left[\nu_{d,t+1}\right]}\right)a_{pd}+\phi_\mu\right) \\ &- Re^{\bar{\Delta d}} \\ &= -.0189, \end{aligned} \tag{100}$$

and hence (using (98))

$$var\left[E\left[X_{t+1}|D_{t+1},\mathcal{J}_t\right]|D_{t+1},D_t,\mathcal{J}_{t-1}\right] \approx P_{t-1}^2 \times (-.0189)^2 \times .150^2 = P_{t-1}^2 \times .0028^2.$$

C.5 Details for subsection 8.3.2

Note that

$$\begin{aligned} P_t &= P_{t-1}e^{\Delta d_t - pd_{t-1}}e^{pd_t} \\ &\approx P_{t-1}e^{\Delta d_t - pd_{t-1}}e^{E[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}]}\left(1+pd_t - E\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right), \end{aligned}$$

and hence

$$var\left[P_t|D_{t+1},D_t,\mathcal{J}_{t-1}\right] \approx \left(P_{t-1}e^{\bar{\Delta d}}\right)^2 var\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right],$$

and so (using (98))

$$var\left[P_t|D_{t+1},D_t,\mathcal{J}_{t-1}\right] \approx P_{t-1}^2 \times .160^2.$$

C.6 Details for subsection 8.4.1

C.6.1 Evaluation of first term in (57)

By the law of total variance,[14]

$$\begin{aligned} var\left[P_t|D_t,\mathcal{J}_{t-1}\right] &\approx var\left[P_t|D_{t+1},D_t,\mathcal{J}_{t-1}\right]+var\left[E\left[P_t|D_{t+1},D_t,\mathcal{J}_{t-1}\right]|D_t,\mathcal{J}_{t-1}\right] \\ &= var\left[P_t|D_{t+1},D_t,\mathcal{J}_{t-1}\right]+var\left[\frac{\partial P_t}{\partial \epsilon_{D,t}}\epsilon_{D,t}\right]. \end{aligned} \tag{101}$$

That is: the variance of P_t given $D_t, \mathcal{J}_{t-1}$ stems from $\epsilon_{D,t}$ and $\epsilon_{Z,t}$. The term $var\left[P_t|D_{t+1},D_t,\mathcal{J}_t\right]$ isolates the effect stemming from $\epsilon_{Z,t}$. Evaluating:

$$\begin{aligned} var\left[\frac{\partial P_t}{\partial \epsilon_{D,t}}\epsilon_{D,t}\right] &= D_t^2\left(var\left[e^{pd_t}|\Delta d_t,\mathcal{J}_{t-1}\right]-var\left[e^{pd_t}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right) \\ &= \left(P_{t-1}e^{\Delta d_t-pd_{t-1}}\right)^2\left(var\left[e^{pd_t}|\Delta d_t,\mathcal{J}_{t-1}\right]-var\left[e^{pd_t}|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right) \\ &\approx P_{t-1}^2e^{2\bar{\Delta}d-2\bar{pd}}\left(e^{\bar{pd}}\right)^2\left(var\left[pd_t|\Delta d_t,\mathcal{J}_{t-1}\right]-var\left[pd_t|\Delta d_{t+1},\Delta d_t,\mathcal{J}_{t-1}\right]\right) \\ &= P_{t-1}^2e^{2\bar{\Delta}d}\left(var\left[\nu_{pd,t}|\nu_{d,t}\right]-var\left[\nu_{pd,t}|\nu_{d,t},a_{pd}\nu_{pd,t}+a_d\nu_{d,t}+\nu_{d,t+1}\right]\right), \end{aligned}$$

where

$$\begin{aligned} & var\left[\nu_{pd,t}|\nu_{d,t},a_{pd}\nu_{pd,t}+a_d\nu_{d,t}+\nu_{d,t+1}\right] \\ =\; & var\left[\nu_{pd,t}|\nu_{d,t},a_{pd}\nu_{pd,t}+\nu_{d,t+1}\right] \\ =\; & var\left[\nu_{pd,t}|\nu_{d,t}\right]-var\left[E\left[\nu_{pd,t}|\nu_{d,t},a_{pd}\nu_{pd,t}+\nu_{d,t+1}\right]|\nu_{d,t}\right] \\ =\; & var\left[\nu_{pd,t}|\nu_{d,t}\right]-var\left[\frac{a_{pd}var\left[\nu_{pd,t}|\nu_{d,t}\right]}{var\left[a_{pd}\nu_{pd,t}+\nu_{d,t+1}|\nu_{d,t}\right]}\left(a_{pd}\nu_{pd,t}+\nu_{d,t+1}\right)|\nu_{d,t}\right] \\ =\; & var\left[\nu_{pd,t}|\nu_{d,t}\right]-\frac{a_{pd}^2var\left[v_{pd,t}|\nu_{d,t}\right]^2}{var\left[\nu_{d,t+1}\right]+a_{pd}^2var\left[v_{pd,t}|\nu_{d,t}\right]}. \end{aligned}$$

Hence

$$var\left[\frac{\partial P_t}{\partial \epsilon_{D,t}}\epsilon_{D,t}\right]\approx P_{t-1}^2e^{2\bar{\Delta}d}\frac{a_{pd}^2var\left[v_{pd,t}|\nu_{d,t}\right]^2}{var\left[\nu_{d,t+1}\right]+a_{pd}^2var\left[v_{pd,t}|\nu_{d,t}\right]}. \tag{102}$$

Hence the date $t+1$ return variance attributable to date $t+1$ dividend innovations incorporated into the date t price is (using (46), (97), (48))

$$\left(\frac{\partial E\left[X_{t+1}|\mathcal{J}_t\right]}{\partial P_t}\right)^2 var\left[\frac{\partial P_t}{\partial \epsilon_{D,t}}\epsilon_{D,t}\right]\approx .0993^2\times P_{t-1}^2e^{2\times .062}\times .0230\times .0159=P_{t-1}^2\times .002^2.$$

[14] Equation (101) holds exactly under joint normality.

C.6.2 Evaluation of second term in (57)

Note that (52) implies

$$
\begin{aligned}
var\left[\frac{\partial\left(\epsilon_{D,t}-E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial\epsilon_{D,t}}\epsilon_{D,t}\right] = \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]^2}{var\left[\epsilon_{D,t}\right]} &= \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{var\left[\epsilon_{D,t}\right]}D_t^2 var\left[e^{\Delta d_{t+1}}|\mathcal{J}_t\right] \\
&\approx \frac{var\left[\epsilon_{D,t}|\mathcal{J}_t\right]}{var\left[\epsilon_{D,t}\right]}\left(P_{t-1}e^{\Delta d_t - pd_{t-1}}\right)^2\left(e^{\bar{\Delta d}}\right)^2 var\left[\nu_{d,t+1}\right].
\end{aligned}
$$

Hence the variance attributable to date $t+1$ dividend innovations unincorporated into the date t price is (using (53), (52))

$$
\begin{aligned}
\left(\left.\frac{\partial\left(P_{t+1}+D_{t+1}\right)}{\partial D_{t+1}}\right|_{\mathcal{J}_t}\right)^2 var\left[\frac{\partial\left(\epsilon_{D,t}-E\left[\epsilon_{D,t}|\mathcal{J}_t\right]\right)}{\partial\epsilon_{D,t}}\epsilon_{D,t}\right] &\approx 6.31^2\times(1-0.139)\times P_{t-1}^2 e^{2\times(.062-3.571)}e^{2\times .062} \times \\
&= P_{t-1}^2\times .011^2.
\end{aligned}
$$

C.6.3 Evaluation of (60)

The evaluation of (60) is very similar to the evaluation of (55). Note that

$$
\left(\frac{\partial X_{t+1}}{\partial\epsilon_{Z,t+1}}\right)^2 var\left[\epsilon_{Z,t+1}\right] = var\left[X_{t+1}|D_{t+2},D_{t+1},\mathcal{J}_t\right] = var\left[P_{t+1}|D_{t+2},D_{t+1},\mathcal{J}_t\right]
$$

and

$$
\begin{aligned}
P_{t+1} &= P_{t-1}e^{\Delta d_{t+1}+\Delta d_t - pd_{t-1}}e^{pd_{t+1}} \\
&\approx P_{t-1}e^{\Delta d_{t+1}+\Delta d_t - pd_{t-1}}e^{E[pd_{t+1}|\Delta d_{t+1},\mathcal{J}_t]}\left(1+pd_{t+1}-E\left[pd_{t+1}|\Delta d_{t+1},\mathcal{J}_t\right]\right),
\end{aligned}
$$

and hence

$$
var\left[P_{t+1}|D_{t+2},D_{t+1},\mathcal{J}_t\right]\approx\left(P_{t-1}e^{2\bar{\Delta d}}\right)^2 var\left[pd_{t+1}|\Delta d_{t+2},\Delta d_{t+1},\mathcal{J}_t\right].
$$

This is differs from the calculation above only by a factor of $\left(e^{\bar{\Delta d}}\right)^2$, and so

$$
var\left[P_{t+1}|D_{t+2},D_{t+1},\mathcal{J}_t\right]\approx P_{t-1}\times\left(e^{\bar{\Delta d}}\times .160\right)^2 = P_{t-1}\times .170^2.
$$

C.7 Details for subsection 8.4.2

$$\begin{aligned} E\left[X_{t+1}\right] &= E\left[E\left[D_t\left(e^{r_{t+1}}-R\right)\frac{P_t}{D_t}|\mathcal{J}_t\right]\right] \\ &= E\left[E\left[P_{t-1}e^{\Delta d_t - pd_{t-1}}\left(e^{r_{t+1}}-R\right)e^{pd_t}|\mathcal{J}_t\right]\right] \\ &\approx P_{t-1}e^{\bar{\Delta d}}\left(e^{\bar{\mu}}\left(1+\frac{1}{2}var\left[r_{t+1}|\mathcal{J}_t\right]\right)-R\right). \end{aligned}$$

Evaluating using (92),

$$E\left[X_{t+1}\right] \approx P_{t-1} \times 0.0915.$$

D Appendix: A VAR(1) process for (D_{t+1}, Z_t) implies linear prices (5)

Suppose that

$$\begin{pmatrix} D_{t+1} \\ Z_t \end{pmatrix} = \Lambda \begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix} + K + \begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}, \tag{103}$$

where Λ is a 2×2 matrix and K is column vector. Assume that Λ satisfies the following pair of mild assumptions, both of which hold generically:

$$\begin{aligned} \left|\Lambda - RI\right| &\neq 0 \qquad (104) \\ \left(\Lambda\left(\Lambda - IR\right)^{-1}\right)_{21} &\neq -1. \qquad (105) \end{aligned}$$

This appendix establishes that there is an equilibrium in which the price takes the form

$$P_t = c\begin{pmatrix} D_{t+1} \\ Z_t \end{pmatrix} + d\begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix} + \kappa_P \tag{106}$$

for some pair of row vectors c and d, and scalar κ_P.

The proof is conjecture-then-verification. Suppose that the price indeed takes form (106). Together with (103), it follows that

$$P_t = \left(c\Lambda + d\right)\begin{pmatrix} D_t \\ Z_{t-1} \end{pmatrix} + cK + \kappa_P + c\begin{pmatrix} \epsilon_{D,t} \\ \epsilon_{Z,t} \end{pmatrix}.$$

Investor i's information set includes P_t, D_t, Z_{t-1}, $D_{t+1}+\epsilon_{i,t}$, $Z_t+u_{i,t}$. Hence

$$\begin{aligned}
E\left[\epsilon_{D,t}|\mathcal{I}_{i,t}\right] &= E\left[\epsilon_{D,t}|\epsilon_{D,t}+\epsilon_{i,t}, c_D\epsilon_{D,t}+c_Z\epsilon_{Z,t}, \epsilon_{Z,t}+u_{i,t}\right] \\
&= E\left[\epsilon_{D,t}|\epsilon_{D,t}+\epsilon_{i,t}, \epsilon_{D,t}+\frac{c_Z}{c_D}\epsilon_{Z,t}, \epsilon_{D,t}-\frac{c_Z}{c_D}u_{i,t}\right] \\
&= \frac{\tau_i\left(\epsilon_{D,t}+\epsilon_{i,t}\right)+\left(\frac{c_D}{c_Z}\right)^2\tau_Z\left(\epsilon_{D,t}+\frac{c_Z}{c_D}\epsilon_{Z,t}\right)+\left(\frac{c_D}{c_Z}\right)^2\tau_u\left(\epsilon_{D,t}-\frac{c_Z}{c_D}u_{i,t}\right)}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right)},
\end{aligned}$$

and similarly,

$$\begin{aligned}
E\left[\epsilon_{Z,t}|\mathcal{I}_{i,t}\right] &= \frac{\tau_u\left(\epsilon_{Z,t}+u_{i,t}\right)+\left(\frac{c_Z}{c_D}\right)^2\tau_D\left(\epsilon_{Z,t}+\frac{c_D}{c_Z}\epsilon_{D,t}\right)+\left(\frac{c_Z}{c_D}\right)^2\tau_i\left(\epsilon_{Z,t}-\frac{c_D}{c_Z}\epsilon_{i,t}\right)}{\tau_Z+\tau_u+\left(\frac{c_Z}{c_D}\right)^2\left(\tau_D+\tau_i\right)} \\
&= \frac{\tau_D\left(\epsilon_{Z,t}+\frac{c_D}{c_Z}\epsilon_{D,t}\right)+\tau_i\left(\epsilon_{Z,t}-\frac{c_D}{c_Z}\epsilon_{i,t}\right)+\left(\frac{c_D}{c_Z}\right)^2\tau_u\left(\epsilon_{Z,t}+u_{i,t}\right)}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right)}.
\end{aligned}$$

Define

$$\mathcal{E}_i \equiv \frac{1}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right)}\begin{pmatrix} \tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right) & \frac{c_D}{c_Z}\tau_Z \\ \frac{c_D}{c_Z}\tau_D & \tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\tau_u \end{pmatrix}.$$

Hence

$$E\left[\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}|\mathcal{I}_{i,t}\right] = \mathcal{E}_i\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}+\frac{1}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right)}\begin{pmatrix}\tau_i & -\frac{c_D}{c_Z}\tau_u \\ -\frac{c_D}{c_Z}\tau_i & \left(\frac{c_D}{c_Z}\right)^2\tau_u\end{pmatrix}\begin{pmatrix}\epsilon_{i,t}\\ u_{i,t}\end{pmatrix}.$$

Evaluating investor i's demand,

$$\begin{aligned}
q_{i,t} &= A_iE\left[P_{t+1}+D_{t+1}-RP_t|\mathcal{I}_{i,t}\right]-B_i\left(Z_t+u_{i,t}\right) \\
&= A_iE\left[(c\Lambda+d)\begin{pmatrix}D_{t+1}\\ Z_t\end{pmatrix}+cK+\kappa_P+c\begin{pmatrix}\epsilon_{D,t+1}\\ \epsilon_{Z,t+1}\end{pmatrix}+\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}D_{t+1}\\ Z_t\end{pmatrix}|\mathcal{I}_{i,t}\right] \\
&- A_iR\left((c\Lambda+d)\begin{pmatrix}D_t\\ Z_{t-1}\end{pmatrix}+cK+\kappa_P+c\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right)-B_i\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}D_{t+1}\\ Z_t\end{pmatrix}-B_iu_{i,t}
\end{aligned}$$

and hence

$$
\begin{aligned}
q_{i.t} &= A_i\left(c\Lambda+d+\begin{pmatrix}1 & 0\end{pmatrix}\right)\left(\Lambda\begin{pmatrix}D_t\\ Z_{t-1}\end{pmatrix}+K+\mathcal{E}_i\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right)+A_i\left(cK+\kappa_P\right)\\
&- A_iR\left(\left(c\Lambda+d\right)\begin{pmatrix}D_t\\ Z_{t-1}\end{pmatrix}+cK+\kappa_P+c\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right)-B_i\begin{pmatrix}0 & 1\end{pmatrix}\left(\Lambda\begin{pmatrix}D_t\\ Z_{t-1}\end{pmatrix}+K+\begin{pmatrix}\epsilon_{D,t}\\ \epsilon_{Z,t}\end{pmatrix}\right.\\
&+ \text{ idiosyncratic terms.}
\end{aligned}
$$

From market-clearing, the equilibrium conditions are

$$
\begin{aligned}
\int\left(A_i\left(c\Lambda+d+\begin{pmatrix}1 & 0\end{pmatrix}\right)\mathcal{E}_i-A_iRc-B_i\begin{pmatrix}0 & 1\end{pmatrix}\right)di &= 0 \quad (107)\\
\int\left(A_i\left(c\Lambda+d+\begin{pmatrix}1 & 0\end{pmatrix}\right)\Lambda-A_iR\left(c\Lambda+d\right)-B_i\begin{pmatrix}0 & 1\end{pmatrix}\Lambda\right)di &= 0 \quad (108)\\
\int\left(A_i\left(c\Lambda+d+\begin{pmatrix}1 & 0\end{pmatrix}\right)K+A_i\left(1-R\right)\left(cK+\kappa_P\right)-B_i\begin{pmatrix}0 & 1\end{pmatrix}K\right)di &= 1 \quad (109)
\end{aligned}
$$

To confirm the conjecture, I show that there is a solution c, d, κ_P that solve (107), (108), (109).

Condition (108) rewrites as

$$
\int\left(A_i\left(c\Lambda+d\right)\left(\Lambda-IR\right)+\left(A_i\begin{pmatrix}1 & 0\end{pmatrix}-B_i\begin{pmatrix}0 & 1\end{pmatrix}\right)\Lambda\right)di=0. \qquad (110)
$$

Substitution of (110) into (107) yields (using (104))

$$
\int\left(A_i\begin{pmatrix}1 & 0\end{pmatrix}\mathcal{E}_i-A_iRc-B_i\begin{pmatrix}0 & 1\end{pmatrix}-\left(A_i\begin{pmatrix}1 & 0\end{pmatrix}-B_i\begin{pmatrix}0 & 1\end{pmatrix}\right)\Lambda\left(\Lambda-IR\right)^{-1}\right)di=0. \qquad (111)
$$

Post-multiplying (111) by the column vector

$$
\begin{pmatrix}1 & -\frac{c_D}{c_Z}\end{pmatrix}^T
$$

and noting that

$$
\begin{pmatrix}1 & 0\end{pmatrix}\mathcal{E}_i\begin{pmatrix}1\\ -\frac{c_D}{c_Z}\end{pmatrix}=\frac{\tau_i+\left(\frac{c_D}{c_Z}\right)^2\tau_u}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2\left(\tau_Z+\tau_u\right)}
$$

yields

$$\int\left(A_i\frac{\tau_i+\left(\frac{c_D}{c_Z}\right)^2\tau_u}{\tau_D+\tau_i+\left(\frac{c_D}{c_Z}\right)^2(\tau_Z+\tau_u)}+B_i\frac{c_D}{c_Z}-\left(A_i\begin{pmatrix}1&0\end{pmatrix}-B_i\begin{pmatrix}0&1\end{pmatrix}\right)\Lambda\left(\Lambda-IR\right)^{-1}\begin{pmatrix}\frac{c_Z}{c_D}&-1\end{pmatrix}^T\right)di \tag{112}$$

By (105), equation (112) has a solution in $\frac{c_D}{c_Z}$. The value of c_Z is straightforwardly implied by (111) post-multiplied by the column vector $\begin{pmatrix}0&1\end{pmatrix}^T$, hence delivering the vector c. Given c, the vector d is directly implied by (110). Given c and d, the constant term κ_P is directly implied by (109). This completes the verification that there is an equilibrium in which the price takes form (106).

E Appendix: Standard present value approximation

$$\begin{aligned}
r_{t+1} &= \ln\frac{P_{t+1}+D_{t+1}}{P_t}\\
&= \ln\frac{P_{t+1}+D_{t+1}}{D_{t+1}}\frac{D_{t+1}}{D_t}\frac{D_t}{P_t}\\
&= \ln\left(1+\exp\left(\log\frac{P_{t+1}}{D_{t+1}}\right)\right)+\ln\frac{D_{t+1}}{D_t}-\ln\frac{P_t}{D_t}\\
&= \ln\left(1+\exp\left(pd_{t+1}\right)\right)+\Delta d_{t+1}-pd_t\\
&\approx \ln\left(1+\exp\left(\bar{pd}\right)\right)+\frac{\exp\left(\bar{pd}\right)}{1+\exp\left(\bar{pd}\right)}\left(pd_{t+1}-\bar{pd}\right)+\Delta d_{t+1}-pd_t.
\end{aligned} \tag{113}$$

Recalling $\rho=\frac{\exp\left(\bar{pd}\right)}{1+\exp\left(\bar{pd}\right)}$ and defining $K_{\bar{pd}}=\log\left(1+\exp\left(\bar{pd}\right)\right)-\rho\bar{pd}$

$$pd_t\approx\rho pd_{t+1}+\Delta d_{t+1}-r_{t+1}+K_{\bar{pd}}.$$

Iterating forwards

$$pd_t\approx\sum_{s=0}^{\infty}\rho^{s-1}\left(\Delta d_{t+s}-r_{t+s}\right)+\frac{K_{\bar{pd}}}{1-\rho}.$$

Taking expectations of both sides, the AR1 assumption implies

$$pd_t\approx\frac{g_t-\bar{\Delta d}}{1-\rho\phi_g}-\frac{\mu_t-\bar{\mu}}{1-\rho\phi_\mu}+\frac{\bar{\Delta d}-\bar{\mu}}{1-\rho}+\frac{K_{\bar{pd}}}{1-\rho},$$

and hence

$$pd_t - \bar{pd} \approx \frac{g_t - \bar{\Delta d}}{1 - \rho\phi_g} - \frac{\mu_t - \bar{\mu}}{1 - \rho\phi_\mu}.$$