Keyword Management Costs and "Broad Match" in Sponsored Search Advertising

Wilfred Amaldoss
wilfred.amaldoss@duke.edu
Duke University

Kinshuk Jerath
jerath@columbia.edu
Columbia University

Amin Sayedi
sayedi@unc.edu
UNC – Chapel Hill

June 2014

Working paper; Comments welcome

Keyword Management Costs and "Broad Match" in Sponsored Search Advertising

Abstract

In sponsored search advertising, advertisers choose the keywords to bid on, decide how much to bid on them, customize the ads for the chosen keywords by geographic region, time of day and season, and then measure and manage campaign effectiveness. We call the costs that arise from such activities *keyword management costs*. To reduce these costs and increase advertisers' participation in keyword auctions, search engines offer a tool called *broad match* which automates bidding on keywords. These automated bids are based on the search engine's estimates of the advertisers' valuations and, therefore, may be less accurate than the bids the advertisers would have turned in themselves. Using a game-theoretic model, we examine the strategic role of keyword management costs, and of broad match, in sponsored search advertising. We show that because these costs inhibit participation by advertisers in keyword auctions, the search engine has to reduce the reserve price, which reduces the search engine's profits. This motivates the search engine to offer broad match as a tool to reduce keyword management costs. If the accuracy of broad match bids is high enough, advertisers adopt broad match and benefit from the cost reduction, whereas if the accuracy is very low, advertisers do not use it. Interestingly, at moderate levels of bid accuracy, advertisers individually find it attractive to reduce costs by using broad match, but competing advertisers also adopt broad match and the increased competition hurts all advertisers' profits, and thus a "prisoner's dilemma" arises. Adoption of broad match by advertisers increases search engine profits, and it therefore seems natural to expect that the search engine will be motivated to improve broad match accuracy. Our analysis shows that the search engine will increase broad match accuracy upto the point where advertisers choose broad match, but increasing the accuracy any further reduces the search engine's profits. We consider a number of extensions to the basic model to show the robustness of our insights.

1 Introduction

Search advertising is emerging as an indispensable part of a firm's advertising strategy. In the US, over 17 billion dollars were spent on search advertising in 2012, accounting for nearly half of total digital advertising expenditure (eMarketer 2012). In sponsored search advertising, advertisers bid in an auction run by a search engine to be displayed in response to specific keywords searched by consumers. If successful, their advertisements are displayed when consumers search those keywords.

However, running an effective search advertising campaign is an effort intensive and costly task for advertisers. First, choosing the keywords to bid on, and deciding how much to bid on them, is a complex task. It is extremely difficult to predict the keywords that consumers will search in the future. Indeed, roughly 20% of the searches Google receives in a day are ones not seen in the previous 90 days.[1] Furthermore, consumers might use any variation of the chosen keywords, corresponding plural and singular forms, synonyms, and misspellings. This unpredictable search behavior of consumers makes it very costly, if not impossible, for advertisers to continuously select keywords to bid on. In addition, it is costly to continuously track clicks, measure conversion rates, choose the right bid amount, and create ad copies to be displayed on the search results page. We call such costs as *keyword management costs*.

As the costs of participating in keyword search auctions are nontrivial, search engines provide campaign management tools to advertisers to reduce the costs in an effort to further spur the growth of search advertising. A popular campaign management tool that search engines offer is called *broad match*, which is an alternative keyword matching process pioneered by Google in 2003 and subsequently adopted by Microsoft and Yahoo (who call it "advanced match"). Under broad match, the search engine runs an advertiser's ads when consumers search for not only the exact keywords specified by the advertiser but also variations of the keywords, such as synonyms, singular and plural forms, and misspellings. Under broad match, the search engine also automatically bids on behalf of an advertiser after assessing the valuation of potentially related keywords. Broad match, thus, reduces the keyword management costs for advertisers.

To appreciate how broad match may affect keyword auction, consider the following example. If an advertiser chooses the keyword "chocolate" and adopts broad match, then its advertisements may be shown on related searches such as "dark chocolate," "bitter dark chocolate," "white chocolate," and possibly even "cocoa."[2] But if the same advertiser adopts the traditional "exact match," its advertisement will be displayed only when consumers search exactly for "chocolate." Suppose that Advertiser 1 chooses broad match for its keyword "dark chocolate," Advertiser 2 specifies that it should be exact matched on "chocolate," and Advertiser 3 specifies that it should be broad matched on "chocolate." Further suppose that a user enters the keyword "chocolate." In this case, all three advertisers will be included in the auction to be run in response to the keyword search. The search

[1] `https://support.google.com/adwords/answer/2497828?hl=en`

[2] For more detailed descriptions of "broad match" from Google and "advanced match" from Yahoo, please see `https://support.google.com/adwords/answer/2497828?hl=en` and `http://help.yahoo.com/l/h1/yahoo/ysm/sps/start/overview_matchtypes.html`, respectively.

engine will place a bid on behalf of Advertisers 1 and 3, while Advertiser 2 will place its own bid. Next suppose that another user enters the keyword "dark chocolate." In this case, Advertisers 1 and 3 will be included in the auction and the search engine will place bids on their behalf.

Many advertisers seek the services of search engine marketing firms to develop and manage their advertising campaigns. Several of these firms have developed algorithms to generate lists of keywords to bid. Still the challenge of developing an exhaustive list of all the relevant keywords that will be searched by online users is so daunting that search engine marketing firms also resort to using tools such as broad match. Recent industry studies attest to the popularity of broad match in comparison to exact match — on Google, 56% of clicks are through broad-matched keywords compared to only 33% through exact-matched keywords, and on Bing, these numbers are 70% and 20%, respectively (Ballard 2013).[3]

Note that in broad match the search engine bids on related keywords based on its own heuristics for imputing advertisers' valuations for the keywords. On the other hand, despite the complexities in its execution, exact match offers advertisers greater control over their search advertising campaigns. In particular, the bids in exact match are based on advertisers' valuations of the keywords rather than the search engine's estimates of valuations. Therefore, the accuracy of the bids placed in broad match may be lower. Herein lies a challenge for an advertiser: How should an advertiser make a trade-off between reduced keyword management cost and reduced bid accuracy? When should an advertiser adopt broad match instead of exact match? It is clear that broad match directly reduces advertisers' costs. Yet, will broad match improve the overall profits of advertisers given that competing advertisers are strategic players? Also, will broad match improve the search engine's profits? An important limitation of broad match is that the search engine's bids may not be accurate. With better optimization technology, however, the search engine could potentially improve the accuracy of its bids. How far will the search engine go to make investments in improving bid accuracy?

[3]The remaining clicks can be attributed to "phrase match" which is a matching technique similar to broad match, but produces narrower matches. Specifically, when an advertiser uses phrase match, a match is triggered only when the search query contains all the keywords in the phrase specified by the advertiser, i.e., other variations are not considered. As an example, if the bidding string is "dark chocolate" and there are three search queries, "bitter chocolate," "bitter dark chocolate" and "dark chocolate," then broad match will match all three queries, phrase match will match only the second and third queries, and exact match will match only the third query. Phrase match is conceptually similar to broad match (only narrower). Therefore, our results on broad match can be expected to directionally extend to phrase match, and we do not explicitly consider phrase match in our paper.

The increasing importance of sponsored search advertising has motivated a growing body of theoretical and empirical academic work on this topic.[4] This body of work has made the simplifying assumption that keyword management costs of advertisers are zero and has not examined broad match. However, as we have argued earlier, in reality advertisers incur nontrivial costs for designing advertisement campaigns, measuring the effectiveness of the campaigns and adjusting their bids over time. To understand the role of keyword management costs in search advertising, we first consider a model in which an advertiser incurs a cost for participating in a keyword auction. Consistent with our intuition, as keyword management costs increase, advertisers participate less frequently in a keyword auction; in response, the search engine reduces the reserve price for participation in the auction. Overall, the search engine earns less profits with higher advertisers' costs. This analysis thus offers a plausible explanation for why the search engine may be motivated to reduce advertisers' cost by offering tools such as broad match.

Next, as broad match bids made by the search engine on behalf of an advertiser may not be accurate, advertisers need to make a careful tradeoff between keyword management cost and bid accuracy in deciding whether to adopt exact match (high keyword management cost with high bid accuracy) or broad match (negligible keyword management cost with lower bid accuracy). To study this tradeoff, we incorporate broad match into our model with keyword management costs. We find that advertisers adopt broad match as long as bid inaccuracy is not too high. A counter-intuitive insight we obtain is that the seemingly helpful broad match could actually hurt advertisers' profits because of a "prisoners' dilemma" situation among advertisers. Even though each advertiser finds it attractive to use broad match to reduce its costs, the resulting increased competition among advertisers, coupled with a higher reserve price set by the search engine (because of reduced participation costs), hurts advertisers' profits. This situation, however, arises only for moderate levels of broad match bid accuracy and keyword management cost. For high levels of broad match accuracy and high keyword management cost, it is indeed profitable for competing advertisers to pursue broad match because the direct positive effect of a reasonably accurate bid at reduced cost dominates the indirect strategic effect of increased competition.

As broad match raises the search engine's profits, one may wonder whether the search engine's

[4] Edelman et al. 2007, Varian 2007, Katona and Sarvary 2009, Yang and Ghose 2010, Athey and Elison 2011, Jerath et al. 2011, Rutz and Bucklin 2011, Yao and Mela 2011, Zhu and Wilbur 2011, Amaldoss et al. *forthcoming*, Hu et al. 2013, Sayedi et al. 2014, Jerath et al. 2014, Desai et al. *forthcoming*.

profits will increase as broad match bid accuracy improves. Interestingly, we find that the search engine will increase broad match accuracy to the point that advertisers choose broad match, but not any further. This occurs because, given that advertisers use broad match, higher bid variability can improve the search engine's profits.

After establishing these key insights, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings. In the main model, we assume the valuation of an advertiser for a keyword changes frequently, such that the advertiser has to make its match strategy choice before its valuation is realized. In an extension, we consider the case where an advertiser's valuation is stable enough that, from a modeling standpoint, match strategy can be chosen after the valuation is realized. We find that this variation in the model does not change any of the fundamental forces operating in the model, and advertisers continue to use broad match when keyword management cost is high enough for any given level of search engine's error in assessing advertiser's valuation, and a prisoners' dilemma arises for medium cost.

Second, search engines often use advertiser-specific quality scores to calculate the effective bid for an advertiser (typically obtained by multiplying the actual bid with the quality score). We extend our main model to incorporate such a quality score and find that the insights of the main model are qualitatively unchanged.

Third, we allow competing advertisers to have different keyword management costs because of differences in operational efficiency and experience with search advertising. In such a situation, the benefit accruing from broad match may be asymmetric across advertisers. We find that broad match takes away the competitive advantage of the low-cost advertiser and, if the cost asymmetry is sufficiently high, the high-cost firm will adopt broad match whereas the low-cost firm will use exact match, and this hurts the low-cost firm only.

Fourth, we increase the number of competing advertisers in the market. If the number of competing advertisers is large enough (in our model, more than five), then the negative effect of heightened competition becomes so strong that even a perfectly accurate broad match bid cannot improve advertisers' payoffs. Still, because of the prisoners' dilemma situation, advertisers will use broad match and this will improve the search engine's profits. Fifth, in a similar vein as above, we consider a situation with multiple advertising slots and find that our basic insights are unchanged.

Sixth, note that broad match permits the search engine to bid on behalf of advertisers. Although there is no evidence that the search engine is abusing this authority, clearly these is scope for the search engine to systematically overbid on behalf of advertisers. This raises the theoretical question of what will happen if the search engine consistently overbids to increase its revenues. To study this, we embed our basic model in a repeated game in which advertisers can periodically observe the degree of overbidding by the search engine. We find that, due to cost savings in broad match, advertisers will put up with systematic overbidding by the search engine as long as the extent of overbidding is not too high.

Seventh, in our main analysis we model broad match in its most general form, which is broad match with automatic and flexible bidding. Search engines also allow advertisers to specify a maximum bid that they can use on the advertisers' behalf in broad match. We model broad match with this feature and show that our basic insights and results hold qualitatively.

Finally, we assume that the search engine adjusts its estimate of the advertisers' valuation using Bayesian updating. We find that our key results on advertiser match strategies continue to hold under this variation.

Related Literature. In addition to the keyword advertising literature noted earlier, our work is related to auction theory that studies bidding costs (also known as participation costs or entry costs). Samuelson (1985) and Stegeman (1996) study the effect of bidding costs on market efficiency. Samuleson (1985) shows that excluding some bidders ex ante could improve the efficiency of the first-price auction. Stegeman (1996) shows a similar result for asymmetric equilibria of the second-price auction. The most relevant paper to our work is Tan and Yilankaya (2006). Assuming that the bidders are symmetric and the cumulative distribution of bidders' valuations is concave, they show that a "cutoff strategy" is the unique equilibrium of a second price auction with bidding cost. This result is applicable in our model. Milgrom (2008) models bidding costs in position auctions to justify conflation through a restrictive bidding language. None of these papers, however, studies bidding cost reduction tools such as broad match, which is our primary focus.

A small body of literature from the search engine industry has begun investigating broad match. Researchers at Google, consider two bidding languages, *query language* and *keyword language*, under broad match (Even-Dar et al. 2009). Given the complexity of broad match, they present an approximate algorithm for determining advertisers' bids and calculating the search engine's revenue in each language. Researchers at Yahoo! propose algorithmic techniques to find relevant keywords for advertisers' campaigns (Broder et al. 2008, Radlinsky et al. 2008). Singh and Roychowdhury (2013) investigate how an advertising budget could be split among keywords matched when using broad match. In this research, we view broad match as a tool that facilitates the bidding process and reduces the cost of bidding. Unlike this literature which examines optimization methods and algorithmic techniques, we focus on equilibrium analysis and the managerial implications of broad match.

The rest of this paper is structured as follows. In the next section, we develop our basic model and derive preliminary results highlighting the effect of keyword management cost on advertisers' payoffs and the search engine's revenue. In Section 3, we incorporate broad match into our model. We identify the conditions under which advertisers will adopt broad match instead of exact match and show how broad match affects advertisers' payoffs and search engine revenue. In Section 4, we consider various extensions to the model. In Section 5, we summarize the results, discuss their managerial implications, and present directions for future research. We provide proofs and supplementary analyses in the Appendix.

2 Role of Keyword Management Costs in Search Advertising

We consider a search advertising market with two risk-neutral advertisers, namely Advertiser 1 and Advertiser 2, and one keyword.[5] There is one advertising slot available for the keyword. A search engine sells the slot in a second-price auction with reserve price R. In particular, the advertiser with the highest bid wins the slot and pays the maximum of the second-highest bid and the reserve price. If the highest bid is smaller than the reserve price, the slot remains unsold.

We assume that Advertiser i has private value v_i for the slot. Values v_1 and v_2 are independently drawn from the uniform distribution $U[0, 1]$. It costs an advertiser $c > 0$ to manage its keyword. The cost captures the efforts involved in tailoring ads for different related keywords and for different geographical regions, adjusting the bids over time, measuring the outcome, and maintaining the effectiveness of the advertising campaign. We assume that c is common knowledge, and that the clicks volume of the keyword is one unit. If an advertiser participates in the auction and wins the auction, then its utility is $v_i - p - c$ where p is maximum of R and the second-highest bid. If the

[5]The results are easily extendable to $n > 1$ independent keywords.

advertiser loses the auction, its utility is $-c$ as it still incurs the cost of participating in the auction. If the advertiser does not participate in the auction, then its utility is zero.

The advertisers and the search engine play the following game. In Stage 1, the search engine sets the reserve price R. In Stage 2, each advertiser learns its private value v_i for the slot. Then, advertisers simultaneously decide if they want to participate in the auction. If they choose to participate, they incur cost c and place a bid for the slot. Finally, in Stage 3, the search engine runs a second-price auction with reserve price R, and collects the payment from the winner of the auction. It is important to note that the cost c is not part of the search engine's revenue.

We solve for the subgame-perfect equilibrium of this game to understand strategic behavior. Below we present two lemmas and then show how keyword management cost can affect keyword search advertising.

Lemma 1 *For an advertiser i who decides to participate in the auction, it is weakly dominant to truthfully bid its value v_i.*

Lemma 2 *There exists a threshold value τ such that Advertiser i participates in the auction if and only if its private value v_i is at least τ.*

The intuition driving Lemma 1 is that the keyword management cost c does not directly affect an advertiser's bid amount, but only influences the decision regarding whether or not to participate in the auction. Therefore, if an advertiser decides to participate in the auction, it bids truthfully because the auction is a second-price auction. Lemma 2 states that an advertiser participates in the auction if and only if its value is high enough.

From Lemma 1 and Lemma 2, we can reason that finding an advertiser's bidding strategy reduces to solving for the optimum threshold τ (as a function of c and R) — below this threshold the advertiser will not participate, and above this threshold the advertiser will participate and bid truthfully. After obtaining the threshold value of τ, we can calculate the expected revenue of the search engine (as a function of R and c), and maximize this revenue with respect to the reserve price R to obtain the optimal value of the reserve price (as a function of c). The proposition below characterizes these quantities.

Proposition 1

(a) An advertiser participates in the auction if its valuation is greater than or equal to the threshold τ given by:

$$\tau = \frac{R + \sqrt{R^2 + 4c}}{2}. \tag{1}$$

(b) The optimum reserve price of the search engine is given by:

$$R^* = \frac{1}{4}\left(3 - \sqrt{1 + 8c}\right). \tag{2}$$

From the above proposition, we obtain the following corollary.

Corollary 1 *As the keyword management cost, c, increases, we observe the following:*

(a) The optimum reserve price decreases.

(b) The probability of an advertiser participating in the auction decreases. However, conditional on an advertiser winning the auction, its expected payment decreases. Overall, the advertiser's expected utility decreases.

(c) The search engine's expected revenue decreases.

We can see from equation (2) that as keyword management cost increases, the search engine reduces the optimum reserve price to facilitate more competition. After substituting for the optimal reserve price in equation (1), we can show that τ is an increasing function of c. This suggests that the probability of participating in the auction decreases when cost c increases. This finding, though intuitive, has an interesting implication: As c increases, the competition between the advertisers decreases. Thus, the expected payment of an advertiser, conditional on the advertiser winning the auction, is a decreasing function of c. In other words, as the keyword management cost increases, an advertiser participates in the auction less frequently, which decreases its expected utility, but when it does participate, it wins the auction for a lower price, which increases its expected utility. Overall, an advertiser's expected utility decreases with c. We also note that the search engine's expected revenue decreases in keyword management cost c.

The above analysis suggests that if the search engine could reduce keyword management cost, not only would the advertisers' surplus increase, but the search engine's own revenue could also increase. Moreover, the search engine could also set a higher reserve price. This offers an explanation for why search engines are developing a wide range of campaign optimization tools for advertisers.[6] In the next section, we study broad match, which is one such widely used tool.

3 Broad Match in Search Advertising

One of the tools most commonly used by advertisers to reduce keyword management costs is "broad match." Under broad match, the search engine automatically finds new relevant keywords for an advertiser, estimates the advertiser's valuation for those keywords, and accordingly bids on behalf of the advertiser. Therefore, broad match essentially reduces keyword management costs to zero. However, the major issue with broad match is that it is not always accurate, because the search engine may under-estimate or over-estimate advertisers' valuations, and advertisers would come to know about this only after they pay for the keyword.

In this section, we model broad match as a tool that reduces the advertiser's keyword management cost to zero, but the search engine's bids on behalf of the advertiser may not be accurate. Given these two conflicting effects of broad match, we examine how it affects the advertisers' equilibrium strategies and the search engine's revenue. We allow advertisers to decide whether or not they want to use broad match. An advertiser who does not use broad match will, by default, use "exact match," implying the advertiser submits its own bid and the search engine uses this exact bid in the auction.

In the presence of broad match, the decision sequence is as follows. In Stage 1, the search engine decides and announces its reserve price R. In Stage 2, before realizing their values, advertisers simultaneously decide whether to use broad match or exact match. If an advertiser uses broad match, its keyword management cost reduces to zero. (Our results hold if the keyword management cost under broad match is positive, as long as it is sufficiently smaller than the corresponding cost under exact match.) However, the value that the search engine bids on behalf of the advertiser may not be accurate. In particular, for an advertiser with valuation v, the search engine could bid $v + \varepsilon$ where $\varepsilon \sim U[-E, E]$. This is an abstraction of the idea that the search engine estimates a valuation on behalf of the advertiser and, given that it is a second-price auction, submits this valuation as the advertiser's bid. However, the technology that the search engine uses to estimate the valuation

[6] For example, see `http://adwords.blogspot.com/2007/07/campaign-optimizer-now-available.html`.

is such that the support of the distribution from which the valuation is picked is from $-E$ to $1+E$ rather than from 0 to 1.[7] The wider support reflects the inaccuracy in valuation estimation by the search engine. The accuracy of the search engine's broad match algorithm decreases as the "error" E increases. Depending on broad match accuracy and keyword management cost, advertisers decide if they want to use broad match or not. In Stage 3, each advertiser learns its own private value v_i. An advertiser who chose exact match in Stage 2 decides whether it wants to participate in the auction, and if so, how much to bid for the keyword. On the other hand, an advertiser who chose broad match in Stage 2 does not have to do anything in Stage 3 as the search engine bids on behalf of the advertiser. Finally, in Stage 4, the search engine runs a second-price auction with reserve price R. The timeline of the game is summarized in Figure 1. We solve for the subgame-perfect equilibrium of the game. To facilitate exposition, we assume $E \leq 0.5$.

We note a couple of points about our modeling choices. First, when a keyword is searched by a user, the search engine determines the set of competing bidders for the auction related to the keyword (based on their exact match/broad match choices). Next, the auction is run. When the auction is run, the search engine determines the bids for bidders who are included through broad match, while for the bidders who are included through exact match the search engine uses their pre-specified own bids (by this stage, they have realized their valuations). The assumption that an advertiser learns its valuation after making its match strategy decision faithfully reflects a common reality. Often advertisers do not even know all the keywords that may be searched and will be relevant to their ads; these are revealed only after a user keys in the search keyword. In other words, the keywords for which an advertiser's ad gets displayed through broad match are not even known to the advertiser (and this is a major advantage of using broad match) — in such a case an advertiser clearly cannot choose exact match. Even for a keyword known to the advertiser, valuation can change with time because of idiosyncratic factors, related to external events or geographic location, that are extremely difficult to predict. Moreover, sponsored search auctions are run continuously and within milliseconds. In other words, valuations change much

[7] For simplicity, we allow negative valuation estimates by the search engine. However, this will make no difference to the analysis because negative valuations are rendered irrelevant due to the reserve price. Furthermore, the assumption that advertisers' valuations and the error in search engine's broad match bids are both distributed uniformly is made for analytical tractability. We have also conducted the analysis of our model using other distributions, such as normal distribution, for the above quantities. This analysis is presented in Section **??**. Even in this situation, we obtain qualitatively the same results.

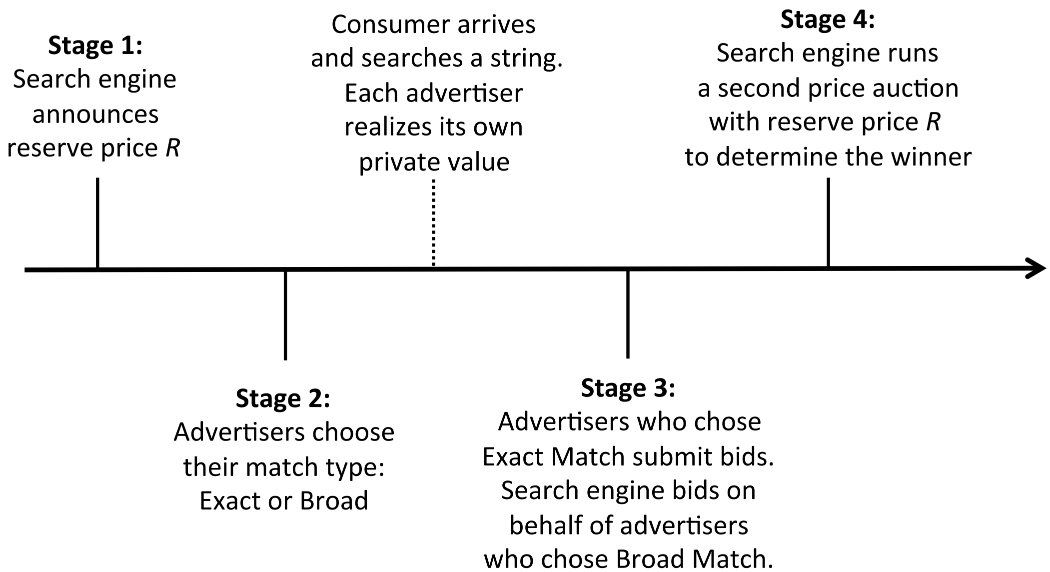


Figure 1: Timeline of the game

too frequently for the advertiser to always be able to make match strategy choice after knowing the valuations (note that determining match strategy and communicating it to the search engine is expected to be a slower, time-consuming process, even if it can be automated). Therefore, in our main analysis here, we assume that an advertiser makes its match strategy decision before learning its valuation. Nevertheless, there may be cases in which valuations are stable and do not change very frequently. In such cases, it is reasonable to assume that match strategy choice is made after valuations are known. In Section 4.1, we analyze this scenario and find that our key insights are unchanged.

Second, for analytical tractability, we assume uniform distrbutions for valuations and error. We have also conducted our analysis assuming normal distributions for these quantities, for instance, $v \sim N(\mu, \sigma)$ and $\varepsilon \sim N(0, E)$. In this case, we do not have closed-form solutions but the model is amenable for a numerical analysis. We find that our key insights are robust to this variation.

We note that our main model is based on broad match in its most general form, i.e., *broad* *match with automatic and flexible bidding* in which the advertisers allow the search engine to bid any amount.[8] Search engines also give advertisers the option to specify a maximum bid under automatic bidding, such that the search engine's bid cannot be higher than this value. We analyze this case in Section 4.7 and show that our results stay qualitatively the same.

We now continue with our analysis. Let $T_i \in \{X, B\}$ denote whether Advertiser i chooses exact match (X) or broad match (B) in Stage 2. Furthermore, let EU_{T_1,T_2} denote the expected utility of Advertiser 1 when Advertiser 1 uses T_1-type match and Advertiser 2 uses T_2-type match. For example, $EU_{B,X}$ is the expected utility of Advertiser 1 if Advertiser 1 uses broad match and Advertiser 2 uses exact match. Depending on whether Advertiser i, $i \in \{1, 2\}$, uses exact match or broad match, we have four possible scenarios. Because the two advertisers are symmetric, calculating the expected utility of Advertiser 1 for each of the four cases is enough for our analyses. When an advertiser uses broad match, the probability distribution of its bid is obtained by the convolution of the distributions of v and ε, and is given by

$$p(x) = \begin{cases} 0 & \text{if } x < -E, \\ \frac{E+x}{2E} & \text{if } -E \leq x < E, \\ 1 & \text{if } E \leq x < 1-E, \\ \frac{1+E-x}{2E} & \text{if } 1-E \leq x < 1+E, \\ 0 & \text{if } x \geq 1+E. \end{cases} \tag{3}$$

Using the above probability distribution function, we calculate advertisers' expected utilities for each of the four possible cases.

Case 1: Both advertisers use broad match. In this case, the expected utility of Advertiser 1 with valuation v is

$$EU_{B,B}(v,E) = \int_{\max(R,v-E)}^{v+E} \left(\int_{-E}^{R} (v-R)p(y)dy + \int_{R}^{x} (v-y)p(y)dy + \int_{x}^{1+E} 0p(y)dy \right) \frac{1}{2E}dx. \tag{4}$$

Therefore, the expected utility of Advertiser 1 using broad match, conditional on Advertiser 2

[8]For a description of automatic bidding see https://support.google.com/adwords/answer/2390311

using broad match as well, is

$$EU_{B,B}(E) = \int_0^1 EU_{B,B}(v,E)dv = \frac{1}{30}\left((-5+E)E^2 + 5(-1+R)^2(1+2R)\right), \tag{5}$$

where the expression on the right hand side assumes that $R \in [E, 1-E]$.

From the search engine's point of view, this case is equivalent to a second-price auction in which advertisers' valuations come from the probability distribution function $p(x)$. The optimum reserve price in this case is $R^* = 1/2$, confirming our earlier assumption of $R \in [E, 1-E]$.

Case 2: Advertiser 1 uses exact match but Advertiser 2 uses broad match. In this case, the expected utility derived by Advertiser 1 with valuation v is

$$EU_{X,B}(v,E,c) = \int_{-E}^{R}(v-R)p(y)dy + \int_{R}^{v}(v-y)p(y)dy + \int_{v}^{1+E} 0p(y)dy - c \tag{6}$$

Let τ_B be the value of v at which this expected utility is zero. Upon assuming $\tau_B \leq 1-E$, the critical value of τ_B simplifies to

$$\tau_B = \sqrt{2c+R^2}. \tag{7}$$

Therefore, Advertiser 1 participates in the auction if and only if $v \geq \tau_B$, and its expected utility is

$$\begin{aligned} EU_{X,B}(E,c) &= \int_0^{\tau_B} 0dv + \int_{\tau_B}^1 EU_{X,B}(v,E,c)dv \\ &= \frac{1}{48}\left(-E^3 + 16c\left(-3+2\sqrt{2c+R^2}\right) + 8\left(1+R^2\left(-3+2\sqrt{2c+R^2}\right)\right)\right). \end{aligned} \tag{8}$$

Case 3: Advertiser 1 uses broad match whereas Advertiser 2 uses exact match. In this case, the expected utility derived by Advertiser 1 with valuation v is

$$EU_{B,X}(v,E,c) = \int_{\max(R,v-E)}^{v+E}\left(\int_0^{\tau_B}(v-R)dy + \int_{\tau_B}^{x}(v-y)dy + \int_x^1 0dy\right)\frac{1}{2E}dx \tag{9}$$

The corresponding expected utility of Advertiser 1 is

$$\begin{aligned} EU_{B,X}(E,c) &= \int_0^1 EU_{B,X}(v,E,c)dv \\ &= \frac{1}{6}\left(1-E^2+\sqrt{2c+R^2}\left(-2c+2(-3+R)R+3\sqrt{2c+R^2}\right)\right). \end{aligned} \quad (10)$$

Case 4: Both advertisers use exact match. We have already studied this case in Section 2.

Recall that the threshold value, τ_X, under which advertisers do not participate is

$$\tau_X = \frac{1}{2}\left(R+\sqrt{4c+R^2}\right). \quad (11)$$

Then the expected utility derived by an advertiser with valuation v is

$$EU_{X,X}(v,c) = \frac{1}{4}\left(-2c-R\left(R+\sqrt{4c+R^2}\right)+2v^2\right), \quad (12)$$

and the corresponding expected utility of the advertiser is given by

$$\begin{aligned} EU_{X,X}(c) &= \int_{\tau_X}^1 EU_{X,X}(v,c)dv \quad (13) \\ &= \frac{1}{12}\left(2+R(-3+2R)\left(R+\sqrt{4c+R^2}\right)+2c\left(-3+3R+\sqrt{4c+R^2}\right)\right). \end{aligned}$$

As discussed earlier, the optimum reserve price of the search engine in this case is $R^* = \frac{1}{4}\left(3-\sqrt{1+8c}\right)$.

Given the reserve price set by the search engine and the expected utilities above, advertisers decide if they want to adopt broad match or exact match. The game in Stage 2 can be modeled as the normal-form game in Table 1. We find that only (exact match, exact match) and (broad match, broad match) can emerge as equilibria for any reserve price. Interestingly, the advertisers may be helped or hurt by their choice of broad match. The equilibrium that emerges and the implications for the advertisers depend on the keyword management cost, c, and the broad match error, E. Figure 2 shows advertisers' equilibrium strategies as functions of keyword management cost and broad match error, and Table 2 describes the regions in Figure 2. In Regions A and B, both advertisers use broad match in equilibrium. In Region D, both advertisers use exact match.

In Region C, we have multiple equilibria: either both advertisers use broad match or both use exact match. Region A is the only region in which broad match improves the utility of both advertisers. In Region B, advertisers face a "prisoners' dilemma" situation — in equilibrium, both advertisers use broad match even though broad match reduces the utility of each individual advertiser. In Region C, as in Region B, broad match hurts the advertisers (if the (broad match, broad match) equilibrium emerges).

We note that the optimal reserve price is different in each of the four regions. Given that only the symmetric equilibria exist, the only relevant optimal reserve prices are 1/2 (as per Case 1) and $\frac{1}{4}\left(3 - \sqrt{1+8c}\right)$ (as per Case 4). In Regions A and B, the search engine sets a reserve price equal to 1/2; at this reserve price, only the (broad match, broad match) equilibrium exists in these two regions. In Region D, at any reserve price, only the (exact match, exact match) equilibrium exists, and the search engine sets the reserve price equal to $\frac{1}{4}\left(3 - \sqrt{1+8c}\right)$. In Region C both the (broad match, broad match) and the (exact match, exact match) equilibria exist at both the relevant reserve prices.

We see that if broad match is highly accurate, both advertisers benefit from using it if keyword management costs are high enough (Region A). This is because the advertisers save on keyword management costs with only small distortions in their bids. Interestingly, however, we also find that both advertisers may be worse off in the equilibrium where both use broad match (Regions B and C). In other words, under certain conditions, broad match creates a prisoners' dilemma situation: advertisers use broad match in equilibrium even though their utilities decrease because of broad match. Each advertiser uses broad match to avoid the keyword management cost and to participate more often in the auction. However, when using broad match, two forces are at play that reduce the advertiser's payoff. First, broad match increases competition because it eliminates the keyword management cost, which motivates advertisers to participate more in the auction. Second, broad match increases the optimum reserve price. In other words, in the absence of broad match, as discussed in Section 2, the search engine has to reduce the reserve price to compensate for the cost. But there is no need for such an adjustment when both advertisers use broad match. Therefore, in the equilibrium where both advertisers use broad match, their payoffs could actually decrease because of broad match (Regions B and C). Yet, a high level of accuracy in broad match could

		Advertiser 2	
		broad match (B)	exact match (X)
Advertiser 1	broad match (B)	$(EU_{B,B},\ EU_{B,B})$	$(EU_{B,X},\ EU_{X,B})$
	exact match (X)	$(EU_{X,B},\ EU_{B,X})$	$(EU_{X,X},\ EU_{X,X})$

Table 1: Advertisers' choice of broad match and exact match as a normal-form game.

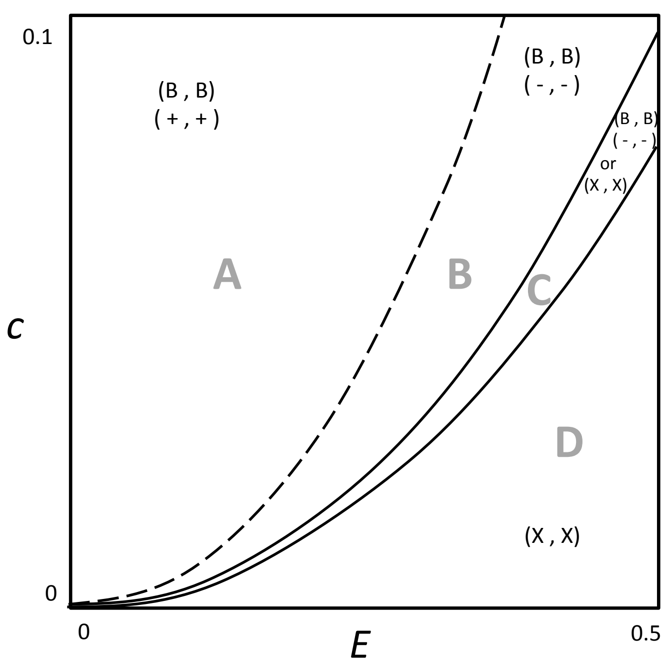


Figure 2: Advertisers' equilibrium strategies as functions of keyword management cost and broad match error

Region	Equilibrium	Broad Match for Advertisers	Broad Match for Search Engine
A	(B, B)	Helps	Helps
B	(B, B)	Hurts	Helps
C	(B, B) and (X, X)	Hurts	Helps
D	(X, X)	NA	NA

Table 2: Description of regions in Figure 2

potentially compensate for these negative forces (Region A).[9]

Finally, we note that if keyword management cost is small enough or the broad match error is large enough (Region D), both advertisers adopt exact match. This makes intuitive sense; given the lack of accuracy of broad match bids, it is a better strategy to avoid the errors in bids even though the management cost has to be incurred for exact match. We obtain the following proposition.

Proposition 2 *For a low level of broad match accuracy, both advertisers use exact match (Region D in Figure 2). For a high level of broad match accuracy, both advertisers use broad match and broad match increases the advertisers' expected utilities (Region A in Figure 2). For a medium level of broad match accuracy, both advertisers use broad match in equilibrium even though broad match reduces the expected utility of both advertisers (Regions B and C in Figure 2). For any level of broad match accuracy, the advertisers use broad match if keyword management cost is high enough.*

Next, we study how the search engine's revenue is affected by broad match accuracy. Recall that the advertisers' incentive to use broad match increases as broad match accuracy increases. Furthermore, broad match increases the search engine's revenue as it increases competition in the keyword auction by removing the keyword management cost. This leads to the question: Would the search engine's revenue increase as broad match accuracy increases? The answer to this question, surprisingly, is no. More specifically, it is in the search engine's best interest to make broad match just accurate enough so that advertisers use broad match, but not make it overly accurate. The intuition for this result is as follows. A larger broad match error increases the variability in the advertisers' bids. Given this, draws from the right side of the advertisers' bid distribution $p(.)$ increase the search engine's revenue, whereas draws from the left side of the distribution cannot decrease the search engine's revenue when the bids are already lower than the reserve price. In other words, if the variability of the advertisers' bids increases (that is, broad match accuracy decreases), the search engine benefits from higher bids but is partially shielded from reduction in revenue from lower bids. Therefore, in the broad match equilibrium, we could observe an increase in the search engine's revenue as broad match accuracy decreases. We state this in the following proposition.

[9]In Section 4.4, we show that if there are more advertisers in the market, even a completely accurate broad match cannot compensate for these negative forces, i.e., advertisers use broad match in equilibrium, but it always reduces their payoffs.

Proposition 3 *Broad match increases the optimum reserve price and the search engine's revenue. However, in an equilibrium in which advertisers use broad match, the search engine's revenue is a decreasing function of broad match accuracy.*

Proposition 3 shows that even if the search engine could make broad match very accurate, it will not do so. In other words, the search engine makes broad match accurate enough so that advertisers use broad match in equilibrium. However, improving broad match accuracy any further hurts the search engine's revenue. This result has the flavor of results found in Ganuza (2004) and Coleff and Garcia (2014).

4 Model Extensions

In the preceding analysis, we made simplifying assumptions to facilitate the exposition of our key results. We now extend the model in multiple ways to relax some of our assumptions and in the process capture a few additional features of the keyword advertising market.

4.1 Stable Advertiser Valuations

In our main model, we assume that advertisers' valuations vary over time and that they choose their match strategies before valuations are revealed. In this section, we assume that advertisers know their valuations before choosing their match strategies. This reflects situations in which advertisers' valuations for keywords remain stable over time. In terms of the stages in the model, first, the search engine announces the reserve price. Second, each advertiser realizes its valuation v. Third, given its valuation v, each advertiser decides whether to use broad match and thereby delegate the bidding authority to the search engine but save on keyword management cost, use exact match and specify a bid but incur the cost, or not participate in the auction. Fourth, the auction is run.

Our analysis shows that an advertiser's choice of broad match versus exact match depends on the relative values of the error E versus the keyword management cost c and, for this model, does not depend on the advertiser's valuation. In particular, when participating in the auction both advertisers use broad match in equilibrium, if and only if $c \geq \frac{E^2}{6}$. If $c < \frac{E^2}{6}$, advertisers prefer exact match, but participate only if their value is at least τ where $\tau = \frac{R+\sqrt{R^2+4c}}{2}$ as shown in Section 2. If $c \geq \frac{E^2}{6}$, advertisers prefer broad match, but participate only if their valuation is above

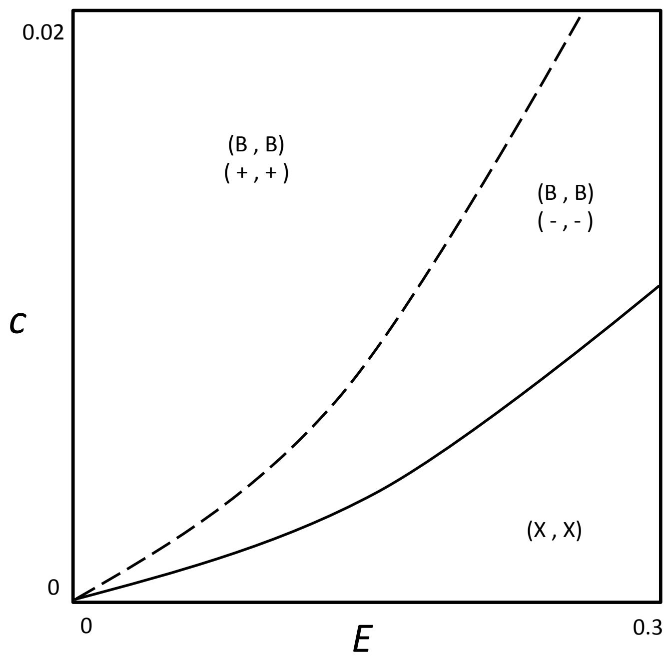


Figure 3: Equilibrium bidding strategies of advertisers as a function of keyword management cost c and broad match error E.

the threshold t; the analysis to derive this threshold (and other associated analysis) is presented in the Appendix.

Figure 3 presents the optimal match strategy of an advertiser as a function of c and E. Note that this figure is qualitatively the same as Figure 2. In particular, if c is small enough and E is large enough, both advertisers use exact match. If c is large enough and E is small enough, both advertisers use broad match, and benefit from broad match. For medium values of E and c, we have a prisoners' dilemma situation in which both advertisers use broad match, though it hurts their profits.

4.2 Different Bid Multipliers

In practice, search engines use advertiser-specific quality scores to calculate the "effective bid" for an advertiser, typically obtained by multiplying the actual submitted bid with the quality score. The quality score is often calculated based on factors such as the past click-through rate on the advertiser's ads and the advertiser's general reputation in the market. For example, if an advertiser's ads have not been performing well in the past, or the advertiser's firm is a lower-quality firm, then

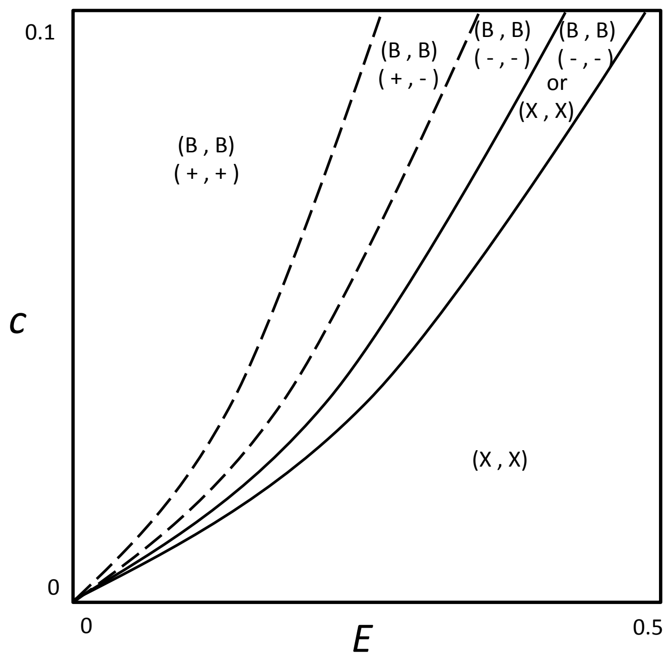


Figure 4: Equilibrium strategy of advertisers when $\mu = 2/3$. The first component in each pair corresponds to Advertiser 1 and the second component to Advertiser 2.

the bid multiplier for the advertiser may be less than 1, which effectively shades the bid of the advertiser.

In our main model, we implicitly assume that the bid multiplier is 1 for all advertisers. In this section, we let advertisers have different bid multipliers. Without loss of generality, we assume that the bid multiplier of Advertiser 1 is 1, and the bid multiplier of Advertiser 2 is $\mu < 1$. Note that from Advertiser 1's perspective, Advertiser 2's valuation comes from the uniform distribution $U[0, \mu]$.

If both advertisers use exact match, an advertiser participates in the auction if and only if its valuation is above a threshold. Let τ_1 and τ_μ be the thresholds for Advertisers 1 and 2, respectively. Using the same techniques as discussed in Sections 2 and 3, we can calculate the thresholds τ_1 and τ_μ, and the utilities of the advertisers corresponding to different match strategy choices, namely $U_{X,X}, U_{B,X}, U_{X,B}$ and $U_{B,B}$. While the details of this analysis are provided in the Appendix, below we discuss the main results.

Figure 4 shows advertisers' strategies for the representative case of $\mu = 2/3$. It can be seen that the results are very similar to those discussed in Section 3 (see Figure 2). Specifically, advertisers use broad match if c is large enough or if E is small enough. Furthermore, they both benefit from the broad match option if c is sufficiently large or if E is sufficiently small. Because of the asymmetry in advertisers' bid multipliers, we notice that for medium values of c and E, when both advertisers use broad match, broad match helps Advertiser 1 (the advertiser with larger bid multiplier) but hurts Advertiser 2 (the advertiser with smaller bid multiplier). This is because when the valuation of Advertiser 1 is high enough, its effective bid will be higher than any effective bid from Advertiser 2. In such cases, the error in broad match bids does not affect Advertiser 1's payment or its utility. Therefore, Advertiser 1 is hurt less by broad match inaccuracy than Advertiser 2. Furthermore, the optimum reserve price when both advertisers use broad match is $\frac{\mu+1}{4}$ and it becomes $\frac{1}{8}\left(3\mu+3-\sqrt{\mu(32c+\mu+2)+1}\right)$ when both advertisers use exact match.

4.3 Asymmetric Keyword Management Costs

In our main model, we assume that advertisers' keyword management costs are the same. In reality, advertisers could have asymmetric costs because of differences in their bid making process and level of automation. Now we allow for this possibility by assuming that one advertiser is of the high (H) type and the other is of the low (L) type. The high-type advertiser has automated its bidding process with its keyword management cost being c_H, whereas the low-type advertisers uses a less sophisticated bidding process with its cost being c_L. We assume that $c_H < c_L$, implying that the high-type advertiser is more efficient than the low-type advertiser. In this setup, let τ_L and τ_H be the threshold values at which the low-type advertiser and the high-type advertiser participate in the auction. We compute these threshold values through an analysis similar to that in Section 2. After obtaining the advertisers' bidding strategies, we compute the search engine's expected revenue and the optimal reserve price R.

Figure 5 presents the optimum value of R for different values of c_L and c_H. Note that the optimum reserve price is a decreasing function of c_H. However, it is a non-monotone function of c_L. Furthermore, there is a discontinuity in the optimum reserve price when the L-type advertiser drops out of the auction. If c_L is sufficiently high and c_H is sufficiently low, the L-type advertiser does not participate in the auction any more. In this case, the search engine sets the reserve price independent of c_L, and the optimum reserve price, at the point of discontinuity, jumps to a higher value (given by $R^* = \frac{1-c_H}{2}$). Moreover, after the discontinuity, the optimum reserve price

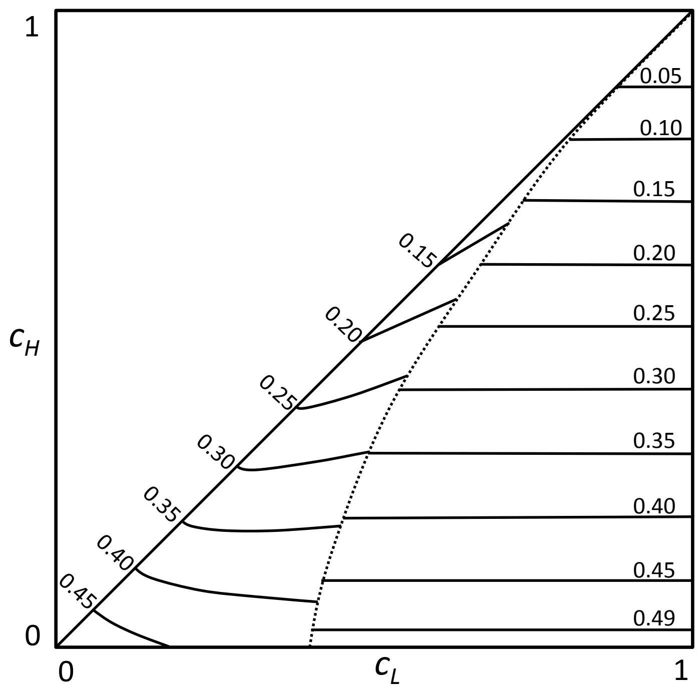


Figure 5: Optimum value of R (represented by the contours) for different values of c_L and c_H, where $c_H < c_L$

is constant in c_L; but before the discontinuity, the optimum reserve price could be increasing or decreasing in c_L (depending on value of c_H).

As mentioned above, the search engine may *increase* the reserve price as the cost of the L-type advertiser increases. The intuition is as follows. Existence of the L-type advertiser forces the optimum reserve price to be lower than what it would be for the H-type advertiser only. However, as the L-type advertiser's cost increases, the search engine may benefit from "sacrificing" the L-type advertiser in order to extract higher revenue from the H-type advertiser. In other words, from the search engine's perspective, as the keyword management cost of the L-type advertiser increases, the advertiser becomes "less valuable" for improving the search engine's profits. Therefore, for large enough c_L, the search engine increases the reserve price, which induces the L-type advertiser to exit, but extracts more revenue from the H-type advertiser.

It is interesting that as c_H increases, the L-type advertiser becomes more likely to be sacrificed. For example, when $c_H = 0$, a change in c_L from 0.30 to 0.31 decreases the optimum reserve price. However, if $c_H = 0.25$, the same change in c_L increases the optimum reserve price. The reason is as follows: When c_H is small (and the reserve price is large), the marginal effect of reserve price on

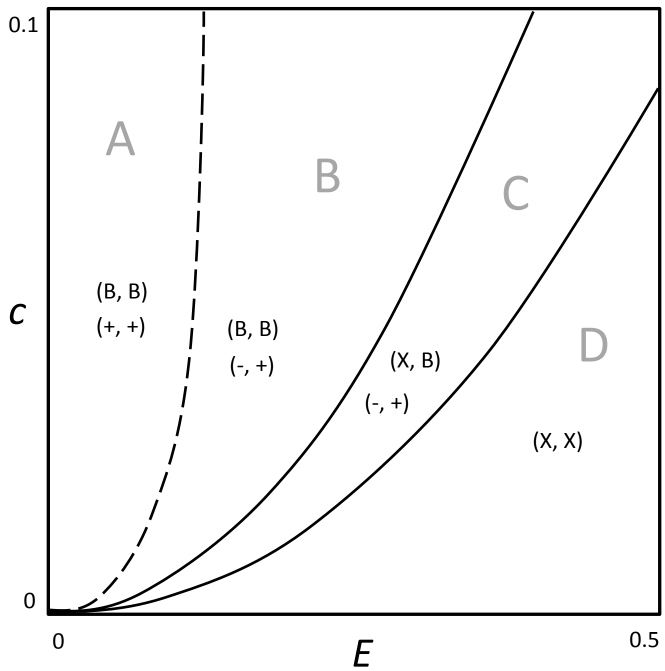


Figure 6: Equilibrium strategies of asymmetric advertisers as functions of keyword management cost and broad match error (Advertisers 1 and 2 have keyword management costs $c/2$ and c, respectively; pairs of letters X/B denote equilibrium strategies with B and X standing for broad match and exact match, respectively; pairs of symbols +/- show how presence of broad match affects advertisers' utilities.)

the search engine's revenue from the H-type advertiser is low. Therefore, the search engine could more easily lower the reserve price to accommodate the change in L-type advertiser's keyword management cost. However, as c_H increases, the marginal effect of the reserve price on the search engine's revenue from the H-type advertiser increases. Therefore, for a large enough c_H, an increase in c_L leads to an increase in the optimum reserve price (sacrificing the L-type advertiser).

Next, we analyze the effect of broad match on the strategies of the asymmetric advertisers. To help see the effect of broad match error and keyword management cost on advertisers' behavior, in Figure 6 we plot the firms' strategies when the level of cost asymmetry is constant. Specifically, we assume that $c_L = c$ and $c_H = c/2$. The results are qualitatively the same for other levels of asymmetry in keyword management costs. The following proposition characterizes the range of possibilities.

Proposition 4 *If broad match accuracy or keyword management cost is low enough, both advertisers use exact match (Region D in Figure 6). If broad match accuracy or keyword management cost* *is high enough, both advertisers use broad match and benefit from using broad match (Region A in Figure 6). For medium-low values of broad match accuracy and keyword management cost, only the L-type (high cost) advertiser uses broad match (Region C in Figure 6). However, for medium-high values of broad match accuracy and keyword management cost, both advertisers use broad match (Region B in Figure 6). In Regions B and C, the L-type advertiser benefits from broad match while the H-type advertiser is hurt by broad match.*

Note that if $\frac{c_H}{c_L}$ becomes sufficiently small, then Region A in Figure 6 shrinks to zero. This implies that, although the H-type advertiser uses broad match if broad match accuracy is high enough, it does not benefit from broad match for any level of broad match accuracy. This is because broad match eliminates the H-type advertiser's cost advantage over the L-type advertiser. Therefore, as the asymmetry between advertisers increases, the H-type advertiser is hurt more by broad match. For a large-enough cost asymmetry, even a broad match with zero error cannot compensate the H-type advertiser's loss due to elimination of the cost advantage.

Next, we turn to the search engine's revenue. We obtain the following proposition.

Proposition 5 *In an equilibrium in which one or both advertisers use broad match, the search engine's revenue is a decreasing function of broad match accuracy. If c_H is sufficiently small and c_L is sufficiently large, the search engine's revenue is maximized at an accuracy level where only the L-type advertiser uses broad match. Otherwise, the search engine's revenue is maximized at an accuracy level where both advertisers use broad match.*

Consistent with Proposition 3, the above proposition shows that, conditional on advertisers' using broad match, the search engine benefits from a larger broad match error. However, unlike Proposition 3, Proposition 5 shows that the search engine does not always try to get both advertisers to use broad match. In particular, if c_H is small enough and c_L is large enough, the search engine only targets the L-type advertiser to use broad match. Intuitively, when c_H is small, broad-match accuracy must be high for the H-type advertiser to use broad match. On the other hand, when c_L is large, even at a low broad-match accuracy, the L-type advertiser would use broad match. Therefore, when c_H is low enough and c_L is high enough, the search engine gives up on the H-type advertiser and sets a relatively low broad match accuracy to extract more profit form the L-type advertiser. Note that this also increases competition for the H-type advertiser.

4.4 Multiple Advertisers

Now we generalize the basic model in Section 2 to allow for $N > 2$ symmetric advertisers. As the number of advertisers increases, τ, the threshold valuation below which an advertiser does not participate in the auction, increases. In other words, some advertisers do not participate in the auction even for a small keyword management cost c, because they know that their probability of winning is low. Only advertisers with very high valuations participate. Because only advertisers with very high valuations participate, even the winner may have a negative payoff if two or more advertisers participate. In particular, when many advertisers compete and their valuations are all very high and close to each other, the winner's payoff $v_i - p$ may not be enough to cover the cost c. Therefore, for a large enough N, an advertiser will only participate if it is almost sure that no other advertiser participates.

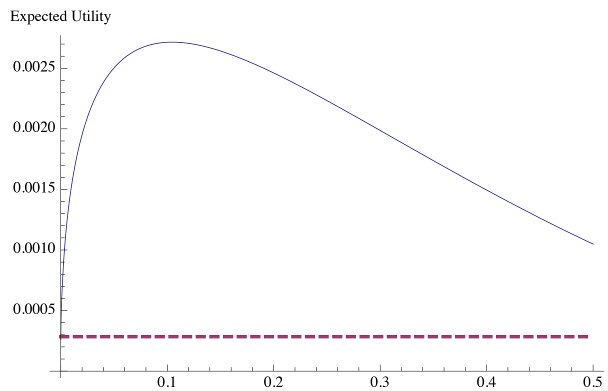
The valuation threshold for participation, τ, is defined by the following equation:

$$\tau^{N-1}(\tau - R) - c = 0. \tag{14}$$

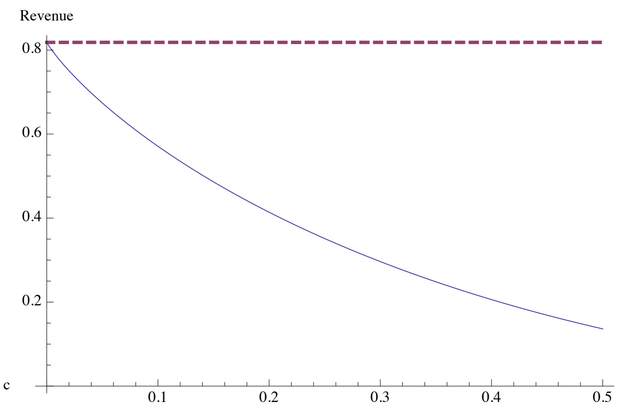
This equation does not have an analytical solution; therefore, to analyze the advertisers' strategies, we numerically calculate the solution. The rest of the analysis proceeds as before.

In Proposition 1 we discussed two forces, created by keyword management cost c, that affect the advertisers' utilities. On the one hand, as the cost increases, an advertiser has to pay a higher cost for participation. Therefore, increasing c could negatively affect the advertisers' utilities. On the other hand, if the cost increases, fewer advertisers participate in the auction. This softens the competition and could positively affect advertisers' utilities. Interestingly, if N is large enough (specifically, if $N > 5$), the "softening competition" effect of keyword management cost may become the dominant force. In particular, when N is large enough, advertisers' expected utilities may increase as the cost increases. As the solid line in Figure 7(a) shows, advertisers' utility increases in c when c is small enough.

Next, we incorporate broad match into the model to understand its impact. Figure 7 shows advertisers' expected utility and search engine's revenue under broad match and exact match for case of ten advertisers ($N = 10$)and completely accurate broad match ($E = 0$). It is easy to see that broad match decreases advertisers' expected utility even though there is no inaccuracy in bids



(a) Advertisers' Expected Utility



(b) Search Engine's Revenue

Figure 7: Advertisers' expected utility and search engine's revenue as functions keyword management cost c (for the figure, we assume $N = 10$ and $E = 0$; the dashed line represents broad match, and the solid line represents exact match)

through broad match. This is because, in the presence of many competitors, entry costs typically soften competition. But because broad match eliminates keyword management costs, it increases competition and reduces profits. Yet all the advertisers choose broad match in this case due to a prisoners' dilemma situation. Thus, as shown in Figure 7(b), broad match increases the search engine's revenue relative to exact match.

4.5 Multiple Advertising Slots

Now we extend the basic model to consider two advertising slots. Following the previous literature, we assume that the second slot has a lower click-through rate than the first slot. Without loss of generality, through scaling, we assume that the click-through rate of the first slot is 1 and the click-through rate of the second slot is $h < 1$.

Note that when there are two advertising slots available, bidding truthfully is not the optimum strategy anymore. In general, as shown in Edelman et al. (2007), there are multiple equilibria and different bidding strategies. In the case of two advertisers and two slots, however, each advertiser has a weakly dominant strategy. This gives us a unique equilibrium.

Lemma 3 *An advertiser with valuation v bids $v - h(v - R)$.*

Lemma 3 provides us advertisers' bids if they decide to participate. Knowing this bidding strategy, on the lines of the analysis in Sections 2 and 3, we can calculate the threshold valuations

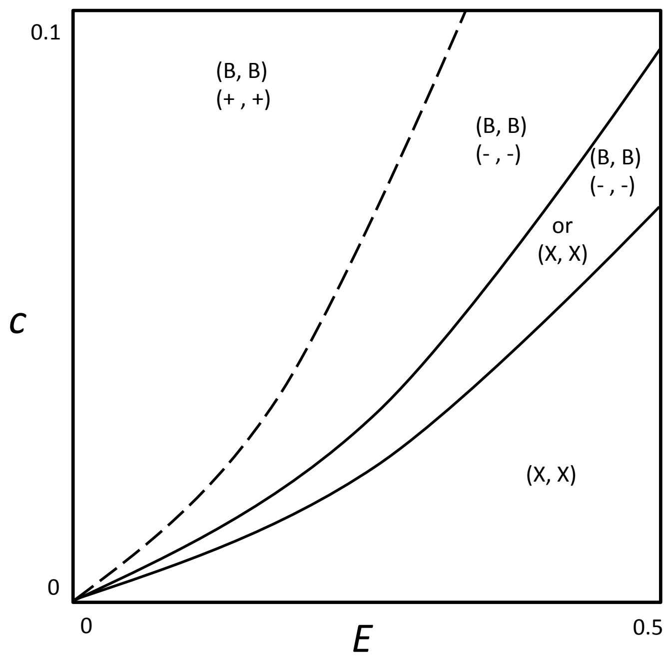


Figure 8: Advertisers' equilibrium strategies with two advertising slots.

above which the advertisers will participate in auction, and their utilities under exact match and broad match. The details of this analysis are provided in the Appendix.

Figure 8 shows advertisers' strategies and the effect of broad match on advertisers' utilities for different values of c and E, when $h = 0.5$. Not surprisingly, as the basic forces at play do not change, the results are very similar to the case with one advertising slot.

4.6 Bid Manipulation

If an advertiser uses broad match and gives the search engine the ability to bid on its behalf, the search engine could potentially over-bid on behalf of the advertiser to improve the search engine's revenue. We note that there is no evidence that search engines are actually doing this. Nevertheless, we examine this issue for its theoretical interest.

In this extension, we assume (for simplicity) that the search engine knows advertisers' exact valuations and manipulates advertisers' bids to increase its own revenues. Specifically, the search engine bids γv on behalf of an advertiser whose valuation is v where $\gamma > 1$. We examine advertisers' decisions in an infinitely repeated game with a per-period discount factor $0 < \delta < 1$. In each round

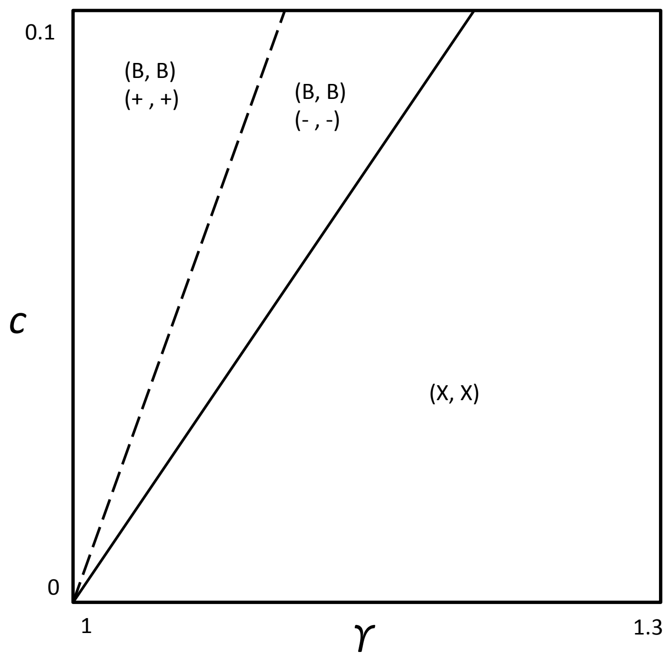


Figure 9: Advertisers' strategies as functions of keyword management cost, c, and bid manipulation parameter, γ

.

i, advertisers decide whether to use exact match or broad match. If they choose broad match, the search engine decides how much to overbid in that round, namely γ_i. Then, the auction takes place, and the advertisers observe γ_i.[10]

Using the folk theorem of infinitely repeated games, we can show that advertisers' response to search engine's overbidding depends on the value of γ_i. In particular, if γ_i is below a certain threshold $\gamma^*(c)$, advertisers "tolerate" search engine's overbidding. However, if $\gamma_i > \gamma^*(c)$, for any i, advertisers use the punishment strategy of switching to exact match and using exact match to infinity. If the discount factor δ is sufficiently large, in equilibrium, search engine never sets γ_i more than $\gamma^*(c)$. The optimum value of $\gamma^*(c)$ is given by the following equation which is derived from folk theorem's minmax condition:

$$\gamma\left(\sqrt{\gamma(8c+\gamma)}+2\right)+\frac{4}{\gamma}=4c\sqrt{\gamma(8c+\gamma)}+7.$$

[10]Note that, in practice, although advertisers are not necessarily able to detect a manipulation after every single auction, they can still detect it by looking at the aggregate daily payments.

The solution $\gamma^*(c)$ for this equation does not have an analytical closed form; however, we can show that $\gamma^*(c)$ is an increasing function of c. In other words, as the cost c increases, advertisers' tolerance for search engine's overbidding also increases. This is because when advertisers' expected utility from exact match decreases, they have a lower incentive to switch to exact match. Moreover, in equilibrium the search engine's revenue from broad match is an increasing function of γ, and hence we find that the optimum value of γ is $\gamma^*(c)$. In other words, the search engine's optimum strategy is to overbid to the highest amount at which the advertisers continue to use broad match.

Figure 9 shows advertisers' strategies as a function of γ and c. As discussed, if γ_i is too large, at any round i, advertisers switch to exact match and keep playing exact match to infinity. However, if γ is smaller than $\gamma^*(c)$, depicted by the solid line in Figure 9, advertisers keep using broad match. Note that because of a prisoners' dilemma situation, broad match may hurt advertisers in this case. Finally, if γ is sufficiently small, similar to our results in earlier sections, advertisers use broad match and benefit from using it.

4.7 Setting an Upper-Bound for Broad Match Bid

In our main analysis, we consider broad match with automatic and flexible bidding, where the search engine has the ability to bid any amount on behalf of the advertisers. In Section 4.6, we show that advertisers will continue to use broad match even if the search engine systematically overbids for them to some extent. Search engines also give advertisers the option to specify a maximum bid under automatic bidding, which implies that the search engine cannot bid higher than this value on the advertiser's behalf (although lower bid values are allowed). This is an option that advertisers sometimes choose to limit their downside from bids that may be too high.

In this section, we consider the situation where an advertiser can set an upper-bound Ω on how much the search engine can bid on its behalf under automatic bidding in broad match. If the search engine's estimate for advertiser's valuation is larger than Ω, the search engine bids Ω on behalf of the advertiser. We solve for the symmetric pure strategy Nash equilibria of the game. If both bids are the same, we assume that the ties are broken randomly with probability half for each advertiser.

We first calculate the optimum upper-bound that the advertisers set in equilibrium. Then, we calculate advertisers' choice of match type in equilibrium. We find that, although search en-

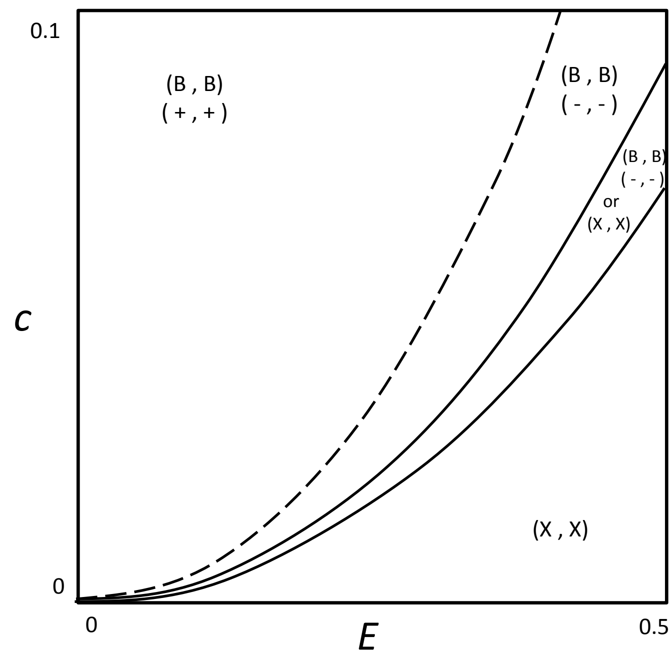


Figure 10: Advertisers' strategies as functions of keyword management cost, c, and bid error, E, when the advertisers can specify an upper bound for bids under broad match

.

gine's revenue decreases when advertisers set an upper-bound for broad match bids, broad match equilibrium still exists and it increases search engine's revenue.

When broad match bids have an upper-bound Ω, probability distribution function of broad match bids, assuming $1 - E \leq \Omega \leq 1$, becomes

$$p(x) = \begin{cases} 0 & \text{if } x < -E, \\ \frac{E+x}{2E} & \text{if } -E \leq x < E, \\ 1 & \text{if } E \leq x < 1-E, \\ \frac{1+E-x}{2E} & \text{if } 1-E \leq x < \Omega, \\ \frac{(1+E-\Omega)^2}{4E}\delta & \text{if } x = \Omega. \end{cases} \tag{15}$$

where δ is Dirac's pulse function.

In the Appendix, we show that the equilibrium value of the upper bound is given by $\Omega = 1 - E/2$. On computing advertisers' expected utilities under broad match, we find that an advertiser's net utility is higher in this case than in the basic case presented in Section 2. Figure 10 shows advertisers' equilibrium strategies as function of E and c. On comparing Figures 2 and 10, we can readily see that our results stay qualitatively the same when an advertiser can specify a maximum bid under broad match.

5 Conclusions

In this paper, we study the strategic implications of keyword management costs and of broad match, a tool offered by search engines to reduce advertisers' costs, in sponsored search advertising. Our theoretical analysis offers useful insights on several issues of managerial significance.

Key Insights. We find that due to keyword management costs, fewer advertisers participate in the search advertising auction. This reduces competition among advertisers, which in turn reduces the amount an advertiser pays to the search engine conditional on winning. Therefore, search engines have an incentive to reduce keyword management costs of advertisers and, to this end, they offer tools such as broad match. While broad match reduces advertisers' costs, the downside for them is that the bids that the search engine places on the advertisers' behalf may be inaccurate. Interestingly, for moderate levels of broad match error, a prisoners' dilemma arises, and competing advertisers choose broad match even though this hurts their profits. However, advertisers find using broad match to be more profitable than using exact match when broad match accuracy is high and keyword management cost is also high. Of course, if broad match accuracy is very low, advertisers will not use broad match.

This leads to the question of whether the search engine will be motivated to eliminate the inaccuracy in broad match bids. Interestingly, we find that it is in the interest of the search engine to sufficiently increase the level of accuracy of broad match so that advertisers adopt broad match. However, when advertisers already adopt broad match in equilibrium, there is no incentive to improve broad match bid accuracy any further. This is because the inaccuracy of broad match induces variation in advertisers' bids, with some bids being lower and others higher than advertisers' own valuations. The search engine can protect itself from the lower bids by stipulating a reserve price, and yet profit from the higher bids.

Even though broad match reduces an advertiser's keyword management cost, it causes an interesting effect: it could raise the equilibrium prices for keywords. We observe this because broad match makes participation in keyword auctions easier, increasing the competition among advertisers. If the number of competing advertisers is sufficiently large, then the negative effect of competition becomes so strong that even a completely accurate broad match will hurt advertisers' equilibrium payoffs. On examining advertisers with asymmetric costs, we note that broad match may have a stronger negative effect on advertisers who are cost efficient. This is because broad match essentially removes their competitive cost advantage over the less cost-efficient low-type advertisers.

Extensions and Robustness. We have relaxed several of the simplifying assumptions of the basic model to assess the robustness of our findings and also better reflect reality. Specifically, we extended the basic model to permit advertisers to choose their match strategies after learning their valuations, allow for different quality scores (bid multipliers) for different advertisers, accommodate multiple advertising slots, create scope for the search engine to systematically overbid on the advertisers' behalf when they choose broad match, and let advertisers stipulate an upper bound for broad match bids. We find that our key insights regarding broad match are robust under these relaxations.

An interesting question is that if the search engine could endogenously set the value of broad match error, E, to maximize its revenue, what would be the value of E? On analyzing this issue, we find that, for any value of keyword management cost c, the search engine sets the optimal value of E such that the broad match equilibrium becomes the only equilibrium of the game and, in this equilibrium, both advertisers are hurt on choosing broad match (i.e., a prisoners' dilemma arises). More specifically, with respect to Figure 2, for a given c, the value of E is chosen at the boundary of Regions B and C, on the side of B. More details of this analysis are available on request. Next, because the search engine knows that its estimate of the advertiser's valuation has an error, and because the search engine knows the error distribution as well as the advertisers' valuation distribution, it can use an updated Bayes estimate to bid on behalf of the advertiser. Specifically, the search engine can use the estimate $\hat{x} = E[v|x]$ for making the broad match bid. We conduct this analysis and find that our original insights can be obtained under this variation as well.

The beneficial effect of broad could raise another interesting question. Instead of providing the broad match option for free to advertisers, if the search engine were to charge a fee for this service, what fee would it charge and what would be the implications for the equilibrium outcomes? We find that the search engine sets the optimum broad match fee to extract all the surplus that advertisers make from using broad match. With respect to Figure 2, Region A disappears (gets replaced by Region B), while Regions C and D, and their boundaries, remain unchanged. More details of this analysis are available on request.

Limitations and Directions for Further Research. Our work is a first step towards studying the impact of keyword management costs and the widely-used cost-reduction tool, broad match, on search advertising auctions. The are several opportunities for further research. For example, we do not model budget constraints in this paper. Modeling budget constraints and understanding their effects on advertisers' adoption of broad match could lead to valuable insights. Next, as we mentioned earlier, firms often use marketing agencies to execute search engine advertising on their behalf. It would be interesting to investigate how the insertion of an intermediary, which may introduce agency considerations, affects our insights. Some marketing agencies also generate exhaustive lists of keywords to bid on, given a starting seed set of keywords, and also estimate advertisers' valuations for these keywords. One of our key results is that the search engine, when estimating valuations, does not want to improve the accuracy of the estimation beyond a point. It would be interesting to understand whether intermediaries have the same or different incentives, and why. Future work can also study other tools provided by search engines to advertisers. One such tool is "bid throttling," where the search engine ensures that the limited budget of an advertiser lasts a specified period of time (determined, say, by the campaign duration) by adjusting the bids and smoothing the spend over time. Finally, another promising avenue for future research would be an empirical study of the effect of keyword management costs, and other cost reduction tools, on advertisers' strategies.

References

Amaldoss, W., P. Desai, and W. Shin, "Keyword Search Advertising and First-Page Bid Estimates: A Strategic Analysis," forthcoming *Management Science*.

Athey, S. and G. Ellison (2011), "Position auctions with consumer search," *Quarterly Journal of Economics*, 126 (3), 1213–1270

Ballard, M. (2013), "RKG digital marketing report: Q2 2013," July 10, 2013.

http://www.rimmkaufman.com/blog/rkg-digital-marketing-report-q2-2013-released/10072013.

Broder, A.Z., P. Ciccolo, M. Fontoura, E. Gabrilovich, V. Josifovski, and L. Riedel (2008), "Search advertising using web relevance feedback," In *Proceedings of the 17th ACM Conference on Information and Knowledge Management*, 1013–1022.

Coleff, J. and D. Garcia (2014), "Information Procurement in Provision Auctions," Working paper, University of Vienna.

Chan, T. and Y.-H. Park (2010), "Position competition in sponsored search advertising," Working paper, Cornell University.

Desai, P. S., W. Shin, and R. Staelin, "The Company that You Keep: When to Buy a Competitor's Keyword," forthcoming, *Marketing Science*.

Edelman, B., M. Ostrovsky and M. Schwarz (2007), "Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords," *American Economic Review*, 97 (1), 242–259.

eMarketer (2012), "US digital ad spending to top $37 billion in 2012 as market consolidates," September 20, 2012. http://www.emarketer.com/newsroom/index.php/digital-ad-spending-top-37-billion-2012-market-consolidates.

Even Dar, E., V.S. Mirrokni, S. Muthukrishnan, Y. Mansour, and U. Nadav (2009), "Bid optimization for broad match ad auctions," *Proceedings of the 18th International Conference on World Wide Web*, 231–240.

Ganuza, J.-J. (2004), "Ignorance promotes competition: An auction model with endogenous private valuations," *RAND Journal of Economics*, 35 (3), 583–598.

Ghose, A. and S. Yang (2009), "An empirical analysis of search engine advertising: Sponsored search in electronic markets," *Management Science*, 55 (10), 1605–1622.

Hu, Y., Shin, J. and Z. Tang (2013), "Performance-based pricing models in online advertising: cost-per-click vs. cost-per-action," Working paper, Yale University.

Jerath, K., L. Ma and Y. Park (2014), "Consumer click behavior at a search engine: The role of keyword popularity," *Journal of Marketing Research*, forthcoming.

Jerath, K., L. Ma, Y. Park and K. Srinivasan (2011), "A "position paradox" in sponsored search auctions," *Marketing Science*, 30 (4), 612–627.

Katona, Z. and M. Sarvary (2010), "The race for sponsored links: bidding patterns for search advertising," *Marketing Science*, 29 (2), 199–215.

Milgrom, P. (2008), "Simplified mechanisms with an application to sponsored-search auctions," *Games and Economic Behavior*, 70 (1), 62–70.

Radlinski, F., A. Broder, P. Ciccolo, E. Gabrilovich, V. Josifovski, and L. Riedel (2008), "Optimizing relevance and revenue in ad search: a query substitution approach," In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, 403–410..

Rutz, O. and R. Bucklin (2011), "From generic to branded: A model of spillover dynamics in paid search advertising," *Journal of Marketing Research*, 48 (1), 87–102.

Samuelson, W. F. (1985), "Competitive bidding with entry costs," *Economics Letters*, 17 (1), 53–57.

Sayedi, A., K. Jerath and K. Srinivasan (2014), "Competitive poaching in sponsored search advertising and strategic impact on traditional advertising," *Marketing Science*, forthcoming.

Singh, S.K. and V.P. Roychowdhury (2013), "To broad-match or not to broad-match: An auctioneer's dilemma," Working paper, University of California – Los Angeles.

Stegeman, M. (1996), "Participation costs and efficient auctions," *Journal of Economic Theory*, 71 (1), 228–259.

Tan, G. and O. Yilankaya (2006), "Equilibria in second price auctions with participation costs," *Journal of Economic Theory*, 130 (1), 205–219.

Varian, Hal R. (2007), "Position auctions," *International Journal of Industrial Organization*, 25 (6), 1163–1178.

Yang, S. and A. Ghose (2010), "Analyzing the relationship between organic and sponsored search advertising: Positive, negative, or zero interdependence?" *Marketing Science*, 29 (4), 602–623.

Yao, S. and C. Mela (2011), "A dynamic model of sponsored search advertising," *Marketing Science*, 30 (3), 447–468.

Zhu, Y. and K.C. Wilbur (2011), "Hybrid advertising auctions," *Marketing Science*, 30 (2), 249–273.

Appendix

A1 Analysis for Section 2

Proofs of Lemmas 1 and 2

Note that an advertiser who chooses to participate in the auction has to incur the keyword management cost regardless of whether he wins the auction or not. Hence, the auction becomes equivalent to a second-price auction in which every bidder's utility is reduced by a fixed amount. Therefore, for a bidder who chooses to participate, it is weakly dominant to bid truthfully.

In any symmetric equilibrium of the auction, the expected utility of a bidder with value v from participation is greater than or equal to the expected utility of a bidder with value $v' < v$. Therefore, in equilibrium, if a bidder with value v' participates, a bidder with value v also participates. This shows that, in equilibrium, the bidders use a cutoff strategy. They participate in the auction if and only if their value is higher than a threshold τ. Tan and Yilankaya (2006) provide a proof of uniqueness of this equilibrium.

Proof of Proposition 1

To compute the threshold value τ, we focus on Advertiser 1. In equilibrium, Advertiser 2 would participate in the auction if and only if its valuation is at least τ. Note that because there is one keyword in the auction, $1-\tau$ is the probability of participating in the auction. Furthermore, Advertiser 1 participates in the auction if and only if its expected utility from participating in the auction is greater than or equal to 0. Therefore, it follows that the expected utility of Advertiser 1 from participating in the auction is 0 when its valuation is τ. We will use this observation to calculate the equilibrium value of τ. Note that we solve under the assumption that $\tau \geq R$ because, otherwise, it is not worthwhile for an advertiser to participate in the auction.

The expected utility (ignoring the keyword management cost) of an advertiser with valuation $v \geq \tau$ who participates in the auction can be calculated as follows. Let the competing advertiser's valuation be denoted by u. For u between 0 and τ, which happens with probability τ, the competing advertiser will stay out of the auction and the focal advertiser will obtain the slot and obtain surplus $v-R$. The expected utility of the focal advertiser in this case is $\tau \times (v-R)$. For u between τ and v, the competing advertiser will bid u, such that the focal advertiser will win the auction and make a payment of u. The expected utility of the focal advertiser in this case is $\int_\tau^v (v-u)du = (v-\tau)^2/2$. For u between v and 1, which happens with probability $1-v$, the competing advertiser will bid higher such that the focal advertiser will lose the auction and obtain 0. The expected utility of the focal advertiser in this case is $(1-v) \times 0 = 0$. In all of the above cases, the advertiser will incur cost c. Therefore, the expected utility, after incorporating the keyword management cost, is given by

$$\tau \times (v - R) + \frac{(v-\tau)^2}{2} + (1-v) \times 0 - c. \tag{16}$$

As noted before, if $v = \tau$, the expected utility is zero. Therefore, we have $\tau(\tau - R) - c = 0$, and obtain

$$\tau = \frac{R + \sqrt{R^2 + 4c}}{2}. \tag{17}$$

Given τ, the search engine's expected revenue is given by

$$(1-\tau)^2 \times (\tau + \frac{1-\tau}{3}) + 2\tau(1-\tau) \times R. \tag{18}$$

The first term in the above expression corresponds to the case where both advertisers participate, and the second term corresponds to the case where only one advertiser participates. Substituting for τ, the search engine's revenue as a function of c and R is

$$-\frac{1}{12}\left(-2 + R + \sqrt{4c + R^2}\right)\left(2 - 4c + R + 4R^2 + \sqrt{4c + R^2} + 4R\sqrt{4c + R^2}\right). \tag{19}$$

This revenue, as a function of c is maximized at

$$R^*(c) = \frac{1}{4}\left(3 - \sqrt{1+8c}\right), \tag{20}$$

which is a decreasing function of c. Search engine's revenue, with endogenous reserve price R, as a function of c is

$$\frac{1}{96}\left(26 + 10\sqrt{1+8c} - \sqrt{10 + 72c - 6\sqrt{1+8c}} + 3\sqrt{2+16c}\sqrt{5 + 36c - 3\sqrt{1+8c}} + 8c\left(-27 + 4\sqrt{1+8c}\right)\right), \tag{21}$$

which is a decreasing function of c.

Conditional on winning, Advertiser i's expected payment is

$$\frac{10 - 24c - 6\sqrt{1+8c} + 3\sqrt{10 + 72c - 6\sqrt{1+8c}} - \sqrt{2+16c}\sqrt{5 + 36c - 3\sqrt{1+8c}} + 32v^2}{64v}, \tag{22}$$

which is a decreasing function of c for any $v > 0$. However, Advertiser i's expected utility (not conditional) is

$$\frac{1}{64}\left(-10-40c+6\sqrt{1+8c}-3\sqrt{10+72c-6\sqrt{1+8c}}+\sqrt{2+16c}\sqrt{5+36c-3\sqrt{1+8c}+32v^2}\right), \tag{23}$$

which is a decreasing function of c for any $v > 0$. Finally, note that if either of the advertisers participates in the auction, because of a second-price auction, the allocation is efficient. Inefficiency happens only if both advertisers decide not to participate in the auction. For endogenous reserve price R^* we have

$$\tau = \frac{1}{8}\left(3-\sqrt{1+8c}+\sqrt{10+72c-6\sqrt{1+8c}}\right), \tag{24}$$

which is an increasing function of c. In other words, advertisers participate less frequently as c increases.

A2 Analysis for Section 3

Proof of Proposition 2

Note that for any reserve price, $EU_{X,B}$ and $EU_{X,X}$ are both decreasing functions of c. On the other hand, $EU_{B,B}$ is constant in c and $EU_{B,X}$ is increasing in c. Therefore, as c grows, independent of the opponent's strategy, each advertiser prefers to use broad match. For large enough c, both advertisers use broad match in equilibrium and benefit from using it. The boundary between Region A and Region B in Figure 2 is defined by the equality

$$EU_{X,X} = EU_{B,B}. \tag{25}$$

Note that $EU_{X,X}$ is constant in error E while $EU_{B,B}$ is a decreasing function of E. Therefore, advertisers benefit from broad only if E is small enough.

Similarly, if c is small enough, both advertisers use exact match. Both advertisers using exact match is the unique equilibrium if each advertiser prefers exact match even if its opponent is using broad match. In other words, the boundary between Regions C and D in Figure 2 is defined by

$$EU_{B,B} = EU_{X,B}. \tag{26}$$

Finally, both firms using broad match is the unique equilibrium if each advertiser prefers broad match even if the opponent uses exact match. In other words,

$$EU_{B,X} = EU_{X,X} \tag{27}$$

defines the boundary between Regions B and C in Figure 2. Note that $EU_{X,X}$ is constant in E while $EU_{B,X}$ is decreasing in E. Therefore, broad match is the unique equilibrium only if E is small enough.

A prisoners' dilemma situation arises if $EU_{B,X} > EU_{X,X}$, both advertisers use broad match in (unique) equilibrium, and $EU_{X,X} > EU_{B,B}$, broad match decreases advertisers expected utilities. These two inequalities hold in Region B in Figure 2.

Proof of Proposition 3

Using Myerson's optimal auction framework,[11] we know that optimum reserve price is the value of R such that $R - \frac{1-F(R)}{f(R)} = 0$, where $F(.)$ and $f(.)$ are the CDF and PDF of bidders' valuation distribution, respectively. Therefore, optimum reserve price in a broad match equilibrium is $1/2$, whereas optimum reserve price in an exact match equilibrium is $\frac{1}{4}\left(3 - \sqrt{1+8c}\right)$ which is less than $1/2$ for any $c > 0$. Therefore, broad match increases optimum reserve price.

Search engine's revenue in broad match equilibrium is

$$\frac{1}{60}\left(25 + 4E^3\right) \tag{28}$$

which is an increasing function of E. The revenue in an exact match equilibrium is

$$\frac{1}{96}\left(26 + 10\sqrt{1+8c} - \sqrt{10 + 72c - 6\sqrt{1+8c}} + 3\sqrt{2+16c}\sqrt{5 + 36c - 3\sqrt{1+8c}} + 8c\left(-27 + 4\sqrt{1+8c}\right)\right). \tag{29}$$

The revenue in exact match is a decreasing function of c and is maximized at $\frac{5}{12}$ when $c = 0$. On the other hand, revenue in broad match is an increasing function of E and is minimized at $\frac{5}{12}$ when $E = 0$. Therefore, for $E > 0$ or $c > 0$, search engine revenue in broad match equilibrium is always

[11] Myerson (1981) shows that if $\frac{f(z)}{1-F(z)}$ is a monotone non-decreasing function, the optimal reserve price is the value of R where $R - \frac{1-F(R)}{f(R)} = 0$.

greater than the revenue of exact match equilibrium.

A3 Analysis for Section 4

A3.1 Analysis with Stable Valuations

In this section, we first show that, for our model, an advertiser's choice of broad match or exact match does not depend on his valuation. Suppose that there are two thresholds t and t_b ($R < t \leq t_b \leq 1$) where an advertiser with valuation less than t does not participate, an advertiser with valuation between t and t_b uses broad match, and an advertiser with valuation larger than t_b uses exact match. Let $f(.)$ be the probability distribution function of an advertiser's bid in equilibrium. We have

$$f(x) = t\delta + \begin{cases} 0 & \text{if } x \leq t - E \\ \frac{E-t+x}{2E} & \text{if } x \leq t + E \\ 1 & \text{if } x \leq t_b - E \\ \frac{E+t_b-x}{2E} & \text{if } x \leq t_b + E \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 1 & \text{if } t_b \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

where δ is the Dirac delta function, a pulse at zero. The first term corresponds to an advertiser with valuation less than t, who does not participate (has bid zero). The second term corresponds to an advertiser with valuation between t and t_b who uses broad match. Finally, the third term corresponds to an advertiser with valuation at least t_b who uses exact match.

In equilibrium, an advertiser with valuation t is indifferent between using broad match and not participating. Similarly, an advertiser with valuation t_b is indifferent between using broad match and exact match. Consider an advertiser with valuation $v = t$. For this advertiser to be indifferent between not participating and using broad match, his expected utility when using broad match must be zero. The expected utility when using broad match is:

$$U_B(t) = \frac{1}{2E} \int_R^{t+E} \left(t(t-R) + \int_{t-E}^{R} f(x)(t-R)dx + \int_R^b f(x)(t-x)dx \right) \mathrm{d}b$$

where variable x represents the competitor's bid, and variable b represents the broad match bid (submitted on behalf of the advertiser). This expression simplifies to

$$U_B(t) = -\frac{E^4 - 2E^2(R-5t)(R-t) - 8Et(R-t)^2 + (R-t)^4}{16E^2}.$$

By letting $U_B(t) = 0$ and solving for t we get

$$t = \frac{1}{36}\left(-12\sqrt[3]{-3}2^{2/3}\sqrt[3]{9e^2(7e+3R)+\sqrt{3}\sqrt{-e^3\left(49e^3+42e^2R+93eR^2+32R^3\right)}} + \frac{24(-3)^{2/3}\sqrt[3]{2}e(7e+2R)}{\sqrt[3]{9e^2(7e+3R)+\sqrt{3}\sqrt{-e^3\left(49e^3+42e^2R+93eR^2+32R^3\right)}}} + 108e + 36R\right)$$

An advertiser with valuation t_b who uses exact match has utility

$$U_X(t_b) = t(t_b - R) + \int_{t-E}^{R} f(x)(t_b - R)dx + \int_{R}^{t_b} f(x)(t_b - x)dx - c.$$

An advertiser with valuation t_b who uses broad match has utility:

$$U_B(t_b) = t(t_b - R) + \int_{t-E}^{R} f(x)(t_b - R)dx + \frac{1}{2E}\int_{t_b-E}^{t_b+E}\left(\int_{R}^{b} f(x)(t_b - x)dx\right)\mathrm{d}b.$$

Advertisers use broad match if and only if $U_B(t_b) > U_X(t_b)$. This inequality simplifies to

$$\frac{E^2}{4} > \frac{5E^2}{12} - c$$

which is independent of t_b. In other words, regardless of their valuation, advertisers prefer broad match to exact match if and only if $c > \frac{E^2}{6}$.

Next, we calculate search engine's revenue in the broad match equilibrium. Expected payment of an advertiser whose bid is b is

$$\phi(b) = tR + \int_{t-E}^{R} f(x)R\mathrm{d}x + \int_{R}^{b} f(x)x\mathrm{d}x$$

where x corresponds to the other advertiser's bid. The first term represents the case where the other advertiser does not participate. The second term represents the case where the other advertiser participates, but search engine bids below R on behalf of the advertiser. Finally, the third term corresponds to the case where the other advertiser's bid is $x \geq R$. An advertiser's expected payment is

$$\int_{R}^{1+E} f(b)\phi(b)\mathrm{d}b$$

which evaluates to

$$\frac{1}{480E^2}\begin{pmatrix} 29E^5 - 5E^4(6R - 11t + 8) - 30E^3\left(3R^2 + 4R(t-1) + t(4-3t)\right) - 10E^2\left(12(R-1)R^2 + 6(3R+2)t^2 + 3R(3R-8)t - 7t^3 - 8\right) \\ -5E(R-t)^2\left(15R^2 + 2R(7t-4) + t(8-5t)\right) - 3(R-t)^4(6R-t) \end{pmatrix}$$

A3.2 Analysis with Different Bid Multipliers

First, note that if μ is too small, the search engine's optimum strategy is to sacrifice Advertiser 2 in order to extract higher revenue from Advertiser 1. In this case, the reserve price will be set higher than μ, and Advertiser 2 can never win the auction. Since there is no competition in this case, we exclude that from our analysis. In other words, we assume that μ is sufficiently large ($\mu \geq 1/2$) so that Advertiser 2 has a positive chance of winning the auction.

For the analysis to follow, we assume that $\mu\tau_\mu \leq \tau_1$. (It is easy to see that the case of $\mu\tau_\mu > \tau_1$ has no solution.) If the value of Advertiser 1 is τ_1, it is indifferent between participating and not participating:

$$\mu\tau_\mu\frac{\tau_1 - R}{\mu} + \frac{(\tau_1 - \mu\tau_\mu)^2}{2\mu} - c = 0$$

Similarly, if value of Advertiser 2 is τ_μ, it is indifferent between participating and not participating:

$$\tau_1\frac{\mu\tau_\mu - R}{\mu} - c = 0$$

By solving these two equations we get

$$\tau_1 = \mu\tau_\mu = \frac{1}{2}\left(\sqrt{4c\mu + R^2} + R\right)$$

Next, we analyze Advertiser 1's incentive to use broad match. Advertiser 1's expected utility when using exact match, given that Advertiser 2 also uses exact match, is:

$$U_{X,X} = \int_{\tau_1}^{1}\left(\tau_\mu(v - R) + \frac{(v - \mu\tau_\mu)^2}{2\mu} - c\right)\mathrm{d}v = -\frac{(\tau_1 - 1)\left(-6c\mu + 3\mu\tau_\mu(\mu\tau_\mu - 2R) + \tau_1^2 + \tau_1 + 1\right)}{6\mu}$$

If Advertiser 1 uses broad match, assuming that Advertiser 2 uses exact match, its utility is:

$$U_{B,X} = \int_0^1 \int_{v-E}^{v+E} \int_{\tau_\mu}^1 \mathbb{1}_{b>\mu z}(v-\mu z)\mathrm{d}z\mathrm{d}b\mathrm{d}v + \int_0^1 \int_{v-E}^{v+E} \mathbb{1}_{b>R}(v-R)\tau_\mu \mathrm{d}b\mathrm{d}v$$

which simplifies to

$$U_{B,X} = \frac{-E^2 + \mu\tau_\mu(3(R-2)R - (\mu\tau_\mu - 3)\mu\tau_\mu) + 1}{6\mu}.$$

If Advertiser 2 is using exact match, Advertiser 1 uses broad match if and only if $U_{B,X} \geq U_{X,X}$. Using the same calculation for Advertiser 2 (and similar to our result in Section 3) we can show that if Advertiser 1 uses broad match, Advertiser 2's best response is also to use broad match. Details of the analysis are available upon request.

A3.3 Analysis with Asymmetric Keyword Management Costs

An advertiser of type T, where $T \in \{L, H\}$, will participate in the auction if and only if its valuation is more than τ_T. We are interested in calculating τ_L and τ_H. Note that $\tau_H < \tau_L$ because, at any value v, the expected utility of the H-type advertiser from participating in the auction is more than that of the L-type advertiser.

For a given τ_L, we can calculate τ_H. To do this, we compute the value v at which the H-type advertiser is indifferent between bidding and not bidding for the keyword. The expected utility of this advertiser, upon participating in the auction, is

$$(1-\tau_L) \times 0 + \tau_L(v-R) - c_H. \tag{30}$$

Setting this expression equal to zero and solving for v, we obtain

$$\tau_H = R + \frac{c_H}{\tau_L} \tag{31}$$

Similarly, for a given τ_H we can calculate τ_L. Specifically, we can compute the v at which the L-type advertiser becomes indifferent between bidding and not bidding. The expected utility of the

L-type advertiser on participating in the auction is

$$(1-v)\times 0+(v-\tau_H)\frac{v-\tau_H}{2}+\tau_H(v-R)-c_L. \tag{32}$$

Setting this expression equal to zero and solving for v, we obtain

$$\tau_L=\sqrt{2c_L+(2R-\tau_H)\tau_H}. \tag{33}$$

Using the above equations to simultaneously solve for τ_L and τ_H, we obtain

$$\tau_H = R+\sqrt{\left(2c_L+R^2-\sqrt{(2c_L+R^2)^2-4c_H^2}\right)/2} \tag{34}$$

$$\tau_L = \sqrt{\left(2c_L+R^2+\sqrt{(2c_L+R^2)^2-4c_H^2}\right)/2}. \tag{35}$$

Notice that if $\tau_L > 1$, an L-type advertiser does not participate in the auction for any valuation. In other words, if $c_L > \frac{1+c_H^2-R^2}{2}$, the L-type advertiser stays out of the auction. In this case, we abuse the notation to define $\tau_L = 1$; when $\tau_L = 1$, we have $\tau_H = R + c_H$.

The search engine's expected revenue is given by

$$(1-\tau_L)\tau_H R+(1-\tau_H)\tau_L R+(\tau_L-\tau_H)(1-\tau_L)\frac{\tau_L+\tau_H}{2}+(1-\tau_L)^2(\frac{2\tau_L+1}{3}). \tag{36}$$

We can optimize the search engine revenue to obtain the optimum value of R. Due to tractability issues, we numerically obtain the optimum value of R.

Proof of Proposition 4

Note that cost asymmetry does not affect the expressions calculated for $EU_{X,B}$, $EU_{B,X}$ and $EU_{B,B}$ calculated in Section 3. Furthermore, as in the proof of Proposition 2, $EU_{B,B}$ is constant in c_H and c_L. $EU_{X,B}$ is a decreasing function of Advertiser 1's cost and is constant in Advertiser 2's cost. $EU_{B,X}$ is increasing in Advertiser 2's cost and is constant in Advertiser 1's cost. Finally, $EU_{X,X}$ is decreasing in Advertiser 1's cost and is increasing in Advertiser 2's cost.

Let superscript $T \in \{H, L\}$ denote Advertiser 1's type (in terms of low cost or high cost) in

EU^T. We have $EU^L_{X,B} < EU^H_{X,B}$, because, $EU_{X,B}$ is a decreasing function of keyword management cost. Also, we have $EU^L_{B,B} = EU^H_{B,B}$. Therefore, broad match is the unique equilibrium if $EU^H_{X,B} < EU^H_{B,B}$. The boundary between Regions B and C in Figure 6 is defined by $EU^H_{X,B} = EU^H_{B,B}$. High-type advertiser benefits from broad match only if $EU^H_{B,B} > EU^H_{X,X}$. This condition may not be satisfied if the difference between c_L and c_H becomes large enough, as $EU^H_{X,X}$ is decreasing in c_L but increasing in c_H whereas $EU_{B,B}$ is constant in both. The boundary between Regions A and B in Figure 6 is defined by $EU^H_{B,B} = EU^H_{X,X}$.

Since we know that $EU^H_{X,X} > EU^L_{X,X}$, exact match is the unique equilibrium if $EU^L_{B,X} < EU^L_{X,X}$. As cost c_L increases, $EU^L_{X,X}$ decreases whereas $EU^L_{B,X}$ remains constant. Therefore, for large enough c_L, L-type advertiser uses broad match. The boundary between Regions C and D in Figure 6 is defined by $EU^L_{B,X} = EU^L_{X,X}$.

A prisoners' dilemma situation happens when $EU^H_{X,B} < EU^H_{B,B}$ and $EU^H_{B,B} < EU^H_{X,X}$. Note that $EU^H_{X,B} < EU^H_{X,X}$ is satisfied for any value of R, c_L, c_H and E. Also, $EU^H_{B,B}$ is a decreasing function of E and constant in c_L and c_H. Therefore, for a medium value of E, prisoners' dilemma arises.

Proof of Proposition 5

Note the if both advertisers use broad match, cost asymmetry does not matter. Therefore, using Proposition 3, we know that the search engine's revenue is a decreasing function of broad match accuracy.

If c_H is sufficiently small, using Proposition 4, we know that broad match accuracy has to be sufficiently high in order to have an equilibrium in which both advertisers use broad match. On the other hand, if c_L is sufficiently large, even for low values of broad match accuracy the L-type advertiser uses broad match. Therefore, as c_L increases, the revenue that the search engine can extract from an equilibrium in which only the L-type advertiser uses broad match increases. Similarly, as c_H decreases, the revenue that the search engine can extract from an equilibrium in which both advertisers use broad match decreases. Therefore, if c_H is sufficiently small and c_L is sufficiently large, search engine's revenue is maximized at an accuracy level in which only the L-type advertiser uses broad match. Otherwise, the search engine's revenue is maximized at an accuracy level where both advertisers use broad match.

A3.4 Analysis with Multiple Advertising Slots

Proof of Lemma 3

Consider an advertiser who decides to participate. If the advertiser wins the second slot, its price will be R, and its utility will be $h(v-R)$. If the price of the first slot is $p = v - h(v-R)$, the advertiser will be indifferent between the first slot and the second slot. Therefore, bidding $b = v - h(v-R)$ is a weakly dominant strategy for the advertiser: If the other bid is lower than b, the advertiser prefers the first slot and wins the first slot. If the other bid is higher than b, the advertiser prefers the second slot and wins the second slot. We note that the result of this lemma is not new. We are only including a proof for completeness.

Analysis of Strategies

First, assume that both advertisers use exact match. In this case, the threshold t for participation is when an advertiser with valuation t is indifferent between paying the cost c and participating or not participating:

$$h(1-t)(t-R) + t(t-R) = c$$

which gives us

$$t = \frac{\sqrt{(h(-R)+h+R)^2 - 4c(h-1)} + hR + h - R}{2(1-h)}.$$

Expected payment of an advertiser with valuation v is

$$\text{Payment}^{X,X}(v) = \int_0^t R\mathrm{d}x + \int_t^v b(x)\mathrm{d}x + \int_v^1 hR\mathrm{d}x$$

where $b(x) = x - h(x-R)$ is the bid of an advertiser with valuation x. Variable x in the above expression represents the value of the other advertiser. Similarly, we can write the expected utility of an advertiser with valuation v:

$$\text{Utility}^{X,X}(v) = \int_0^t (v-R)\mathrm{d}x + \int_t^v (v-b(x))\mathrm{d}x + \int_v^1 h(v-R)\mathrm{d}x$$

Search engine's revenue is twice of the expected payment of each advertiser which is

$$\mathrm{SER}^{X,X} = 2\int_t^1 \mathrm{Payment}^{X,X}(v)\mathrm{d}v = \frac{1}{3}(t-1)\left((h-1)(6R+1)t - 6hR - 2(h-1)t^2 + h - 1\right),$$

and the optimum reserve price, the value of R that maximizes the above expression, is

$$R^*_{X,X} = \frac{\sqrt{(h+1)^2 - 8c(h-1)} + h - 3}{4(h-1)}.$$

The expected utility of each advertiser is

$$EU_{X,X} = \int_t^1 \mathrm{Utility}^{X,X}(v)\mathrm{d}v = \frac{1}{6}(t-1)(6c - 2h(t-1)(3R-2t-1) + t(6R-4t-1) - 1).$$

Similarly, we can calculate the advertiser's expected utility in the other three cases: $EU_{B,X}, EU_{X,B}$ and $EU_{B,B}$. We have

$$EU_{B,X} = \frac{1}{24}\left(E^3(-(h-1)) - 4E^2 + 4(h(t-1)(-3(R-2)R + (t-2)t - 2) + t(3(R-2)R - (t-3)t) + 1)\right)$$

$$EU_{X,B} = \frac{1}{48}\left(48c(t-1) + E^3(h-1) - 8(t-1)\left(2h(t-1)(3R-2t-1) - 6R^2 + 3R(t+1) + (t-1)^2\right)\right)$$

$$EU_{B,B} = \frac{1}{30}\left(E^3(-(h-1)) - 5E^2 - 5(R-1)^2(2h(R-1) - 2R - 1)\right).$$

The search engine's revenue in the broad match equilibrium is given by $\mathrm{SER}_{BB} = \frac{1}{60}\left(5(h+5) + 4E^3(1-h)\right)$.

A3.5 Analysis with an Upper-Bound for Broad Match Bid

Consider an advertiser whose competitor uses broad match with upper-bound Ω. Note that if this advertiser sets upper-bound $\Omega' > \Omega$, it wins all the cases in which the search engine's estimate for both advertisers is larger than Ω' (for any $\Omega' > \Omega$). Similarly, if $\Omega' < \Omega$, this advertiser loses all the cases in which the search engine's estimate for both advertisers is larger than Ω. In other words, if the advertiser's expected utility from winning the cases in which the search engine's estimate for both advertisers is larger than Ω is not zero, the optimum value of Ω' has to be either strictly larger than Ω or strictly smaller than Ω. Therefore, for Ω to be the symmetric equilibrium upper-bound, in expectation, advertisers must be indifferent between winning or losing in the situation where the search engine's estimate for both advertisers is larger than Ω. This translates to the following equation

$$\int_{\Omega-E}^{1} \frac{v+E-\Omega}{2E}\left(\frac{(1+E-\Omega)^2}{8E}(v-\Omega)\right)\mathrm{d}v = 0$$

The solution Ω to this equation is $\Omega = 1 - \frac{E}{2}$, which is the optimum upper-bound that both advertisers set in equilibrium.

References

Myerson, R. (1981), "Optimal auction design," *Mathematics of Operations Research*, 6 (1), 58–73.