Real-Time Bidding in Online Display Advertising

Amin Sayedi*

August 2017

*Amin Sayedi (aminsa@uw.edu) is an Assistant Professor of Marketing at the Foster School of Business, University of Washington, Seattle. The author is very grateful to Bita Hajihashemi and Mehrdad Tahvilian for outstanding research assistantship, and to Kinshuk Jerath, Wilfred Amaldoss, Ron Berman, Upender Subramanian, Woochoel Shin, Jeff Shulman, Hema Yoganarasimhan, Mengze Shi, Yi Zhu, Shijie Lu, Tony Ke, Ken Wilbur, Sanjay Jain, Ram Rao, Baojun Jiang and Jiwoong Shin for invaluable discussions and their feedback on the paper.

Real-Time Bidding in Online Display Advertising

Abstract

Display advertising is a major source of revenue for many of the online publishers and content providers. Historically, display advertising impressions have been sold through pre-negotiated contracts, known as *reservation contracts*, between publishers and advertisers. In recent years, a growing number of impressions are being sold in real-time bidding (RTB), where advertisers bid for impressions in real-time, as consumers visit publishers' websites. RTB allows advertisers to target consumers at an individual level using browser cookie information, and enables them to customize their ads for each individual. The rapid growth of RTB has created new challenges for advertisers and publishers on how much budget and ad inventory to allocate to RTB. In this paper, we use a game theory model with two advertisers and a publisher to study the effects of RTB on advertisers' and publishers' strategies and their profits. We show that symmetric advertisers use asymmetric strategies where one advertiser buys all of his impressions in RTB whereas the other advertiser focuses on reservation contracts. Interestingly, we find that while both advertisers benefit from existence of RTB, the advertiser that focuses on reservation contracts benefits more than the advertiser that focuses on RTB. We show that while RTB lowers the equilibrium price of impressions in reservation contracts, it increases the publisher's total revenue. Despite many analysts' belief that, because of being more efficient, RTB will replace reservation contracts in the future, we show that publishers have to sell a sufficiently large fraction of their impressions in reservation contracts in order to maximize their revenue. We extend our model to consider premium consumers, publisher's uncertainty about the number of future visitors, and effectiveness of ad customization.

1 Introduction

Display advertising is a major source of revenue for many of the online publishers and content providers. In 2017, the revenue of this market in the US alone is expected to exceed $40 billion

(Liu 2016). Historically, display ads have been sold in pre-negotiated contracts, known as *Reservation Contracts*, between publishers and advertisers. In a reservation contract, also referred to as a *Direct Buy*, an advertiser and a publisher agree on the price, targeting criteria, and the number of impressions to be delivered in a certain period of time (e.g, 2 million impressions to male visitors from the state of New York in the month of July for $8,000). In recent years, a growing number of display advertising impressions are being sold in a new model known as *Real-Time Bidding* (RTB).

In the RTB model, any time a consumer visits a publisher's website, the publisher sends the consumer's information to a central exchange platform (e.g., Google's DoubleClick or OpenX). The exchange platform runs an auction, in real-time, in which the advertisers bid for their ads to be shown to the consumer. The winning ad is sent back to the publisher to be displayed to the consumer along with the content of the website. The entire process, from the time that the consumer clicks on a link, or types in a URL, until the content and the ad are shown to the the consumer generally takes less than 100 milliseconds.[1] RTB, introduced in 2009, has quickly become one of the main methods of buying and selling display ads. It currently accounts for over a third of the display advertising market in the US, and is estimated to exceed $14 billion in 2017 (Fisher and Liu 2016).

A major change in RTB market has been the adoption of *header bidding* technology by publishers over the past couple of years.[2] Prior to adoption of header bidding, RTB was a remnant auction that enabled publishers to sell their leftover inventory in real-time. Publishers allocated all of their impressions to reservation contracts until those contracts were fulfilled, and then made the leftover available on RTB. Using the header bidding technology, however, publishers make all of their inventory available on RTB. For each impression, after receiving RTB bids, the publisher decides whether to allocate the impression to the highest bidder in RTB or to an advertiser with a reservation contract. In other words, RTB bids compete with reservation contracts, and only if the clearing price in RTB is not sufficiently high, the impression is assigned to a reservation contract.[3]

An important advantage of RTB for advertisers is that they can target consumers at an indi-

[1]https://static.googleusercontent.com/media/74.125.224.88/en/88/doubleclick/pdfs/what-is-rtb.pdf

[2]https://adexchanger.com/publishers/year-header-bidding-went-mainstream/

[3]In header bidding, a publisher places some Java Script code in its website's header. When a page on the website is requested by a consumer, the code reaches out to the supported exchange platforms and advertisers (asking for bids) before the publisher's own inventory of reservation contracts is reached. For more information on header bidding, see https://martechtoday.com/martech-landscape-what-is-header-bidding-and-why-should-publishers-care-157065

vidual level using browser cookie information. In other words, advertisers can set their bids, and customize their ads, *after* they observe each consumer's relevant information. Thus, advertisers typically have higher valuation for impressions in RTB than those in reservation contracts. Furthermore, the fine-grained targeting in RTB results in *unbundling of impressions*, which could improve the efficiency of allocation and increase the social surplus.

On the other hand, the fine-grained targeting in RTB leads to "cherry-picking" of impressions in RTB and negatively affects the average quality of impressions assigned to reservation contracts. As such, RTB lowers the advertisers' willingness to pay for impressions in reservation contracts, which in turn can lower the publisher's revenue. The fine-grained targeting may also hurt the publisher's revenue by *dispersing competition*, i.e., reducing the number of advertisers interested in each impression and making ad auctions thinner (e.g., see Levin and Milgrom 2010; Rafieian and Yoganarasimhan 2017).

The overall impact of RTB on publisher's revenue and advertisers' profits is still unclear and has been a topic of discussion among practitioners.[4] In this paper, we use a game theory model with two advertisers and a publisher to analyze how RTB affects publishers' and advertisers' strategies and their profits. We are interested in answering the following research questions:

1. What are the effects of RTB on publishers' revenue and advertisers' profits?
2. How should a publisher allocate his ad inventory between reservation contracts and RTB?
3. How should an advertiser allocate his budget between reservation contracts and RTB?

RTB has three main effects on a publisher's revenue: First, by improving advertisers' ability to target consumers and customize ads, RTB increases advertisers' willingness to pay for impressions. For example, an advertiser is willing to pay a higher price for an impression to a consumer that has visited his website before and has shown some interest in some of his products, compared to an average visitor of the same publisher. On the other hand, RTB allows the advertisers to cherry-pick impressions, and therefore, could lower the average quality of impressions assigned to reservation contracts. As such, RTB could lower the per-impression price of reservation contracts, which would

[4]For some examples, see https://www.exchangewire.com/blog/2012/01/31/to-rtb-or-not-rtb/ and http://blog.retargeter.com/general/why-publishers-shouldnt-be-afraid-real-time-bidding

hurt the publisher's revenue. Finally, RTB disperses competition by making ad auctions thinner, which could, again, decrease the publisher's revenue.[5]

We find that the publisher could mitigate the competition dispersion effect of RTB by adopting a sufficiently high reserve price. In particular, we show that when an optimal reserve price is used, offering RTB always increases the publisher's revenue.

Interestingly, we find that the importance of setting a high reserve price in RTB motivates the publisher to lower the price of impressions in reservation contracts. By lowering the price, the publisher could sell more impressions in reservation contracts. Therefore, the publisher can ensure that even if the majority of impressions in RTB are left unsold due to the high reserve price, they can still be allocated to reservation contracts. This result also indicates that the publisher's total revenue is maximized when a sufficiently large fraction of the impressions are sold in reservation contracts.

Finally, in addressing the third research question, we show that symmetric advertisers follow asymmetric strategies regarding the use of RTB. We find that, in equilibrium, one advertiser spends all of his budget in RTB whereas the other advertiser focuses on reservation contracts. We show that both advertisers have higher equilibrium profits when RTB is offered than when it is not. However, interestingly, the advertiser that focuses on reservation contracts benefits more from existence of RTB than the advertiser that focuses on RTB.

After establishing these key insights, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings. First, we consider a segment of *premium consumers* for which both advertisers have high valuation. We show that, in equilibrium, both advertisers bid for premium consumers in RTB, and that all such impressions are sold in RTB. This moderates the asymmetry that we find in advertisers' strategies in the main model. However, advertisers still use asymmetric strategies, and our key insights regarding the publisher's and advertisers' strategies and their profits stay the same. Second, we model *publisher's supply uncertainty* by assuming that the publisher does not know how many visitors it would have in the future, at the time of selling impressions in reservation contracts. We show that under supply uncertainty, RTB increases social surplus, and that both advertisers and the

[5]For example, Chen et al. (2014) show that in a platform with 374 actively participating advertisers, each RTB impression receives only 5 bids on average.

publisher benefit from the increase in social surplus. Third, we consider *ad customization* in RTB. Since advertisers can customize their ads for each individual consumer in RTB, they can increase the probability of conversion, and thus, have higher valuation for the same impression in RTB than in reservation contracts. Interestingly, we show that the effectiveness of ad customization could have a non-monotonic effect on advertisers' profits. In particular, as the probability of conversion due to ad customization increases, the advertisers' equilibrium profits may decrease. We show that our key insights from the main model, regarding the advertisers' and the publisher's strategies and their profits continue to hold in all these extensions.

Findings Inline with Recent Trends in RTB Market

Our findings can explain some of the recent trends in this market:

Importance of reserve price optimization. Our results highlight the importance of reserve price, also referred to as "price floor," in determining the clearing price of an RTB auction. In particular, we show that the publisher can mitigate the competition dispersion effect of RTB by adopting a sufficiently high reserve price. This finding is consistent with industry reports showing that in the early years, publishers were unsatisfied with RTB due to low clearing prices. However, after reserve prices were increased, publishers achieved revenues that were higher than before RTB.[6] This has led to development and adoption of advanced technologies, such as dynamic price floors, for reserve price optimization.[7]

Adoption of header bidding. Originally, RTB was a remnant auction. Publishers allocated all of their impressions to reservation contracts until those contracts were fulfilled, and then made the leftover available on RTB. Adoption of *header bidding* affected this order and changed the nature of the competition. Using header bidding, publishers make all of their inventory available for sale on RTB. For each impression, after receiving RTB bids, the publisher decides whether to allocate the impression to RTB or to reservation contracts. Our results indicate that the new model leads to a higher equilibrium revenue for the publisher than the original model. This finding is consistent with industry reports showing that publishers who adopt header bidding sell their impressions at higher prices,[8] and also explains the fast growth in the percentage of publishers who use header

[6] http://digiday.com/platforms/what-is-real-time-bidding/
[7] http://rubiconproject.com/blog/using-dynamic-price-floors-to-protect-publisher-value/
[8] https://adexchanger.com/publishers/year-header-bidding-went-mainstream/

bidding (from virtually 0% in 2014 to over 70% in 2016).[9]

Persistence of reservation contracts. RTB is a new technology that allocates the impressions more efficiently, i.e., each advertiser gets the impressions that are most valuable to him. As such, many analysts believed that RTB would replace reservation contracts in the future.[10] In contrast, we show that existence of reservation contracts allows the publisher to set a higher reserve price in RTB, and thus extract more revenue. In other words, even when publishers use optimal reserve prices in RTB, their equilibrium revenue when they sell a fraction of their inventory in reservation contracts (and the rest in RTB) is higher than when they sell their entire inventory in RTB. Therefore, our results indicate that reservation contracts will be preserved by publishers. Consistent with our prediction, and despite its rapid growth in the past several years, industry reports show that the growth of RTB relative to total display advertising is slowing down. The spending on RTB, as a fraction of total display advertising in the US, is estimated to be 31%, 34%, 36% and 36% in 2015, 2016, 2017 and 2018, respectively (Fisher and Liu 2016; Liu 2016).

Related Literature

Our work is related to the following streams of research in the literature:

Online Advertising Auctions. The increasing prevalence of online advertising auctions has motivated a growing body of theoretical papers in the marketing academic literature. Katona and Sarvary (2010) and Jerath et al. (2011) study advertisers' incentives in obtaining lower versus higher positions in search advertising auctions. Sayedi et al. (2014) investigate advertisers' poaching behavior on trademarked keywords, and their budget allocation across traditional media and search advertising. Desai et al. (2014) analyze the competition between brand owners and their competitors on brand keywords. Lu et al. (2015) and Shin (2015) study budget constraints, and budget allocation across keywords. Zia and Rao (2016) look at the budget allocation problem across search engines. Zhu and Wilbur (2011) and Hu et al. (2015) study the trade-offs involved in choosing between two types of contracts, namely "cost-per-click" and "cost-per-action" contracts. Wilbur and Zhu (2009) find the conditions under which it is in a search engine's interest to allow some click fraud. Cao and Ke (2016) model a manufacturer and retailers' cooperation in search advertising

[9]http://www.businessinsider.com/header-bidding-gains-momentum-drives-up-publisher-ad-revenue-2016-5
[10]http://www.slideshare.net/SearchLaboratory/the-future-of-display-advertising-31126811

and show how it affects intra-brand and inter-brand competition. Amaldoss et al. (2015a) show how a search engine can increase its profits and also improve advertisers' welfare by providing them first-page bid estimates. Berman and Katona (2015) study the impact of search engine optimization, and Amaldoss et al. (2015b) analyze the effect of keyword management costs on advertisers' strategies. Berman (2016) explores the effects of advertisers' attribution models on their bidding behavior and their profits. There are also several empirical papers that study search advertising auctions (e.g., see Yang and Ghose 2010, Rutz and Bucklin 2011, and Yao and Mela 2011). An important difference between search advertising and display advertising markets, which is also the focus of this paper, is that in search advertising the entire inventory is sold in auctions whereas display advertising impressions are sold in both auctions and reservation contracts.

Online Display Advertising. Several papers in marketing literature study various aspects of display advertising. Anand and Shachar (2011), Tucker (2014), Urban et al. (2013), and Hoban and Bucklin (2015) show that ad customization can improve the effectiveness of display advertising. Goldfarb and Tucker (2011) and Lambrecht and Tucker (2013) look at negative effects of ad customization. Lambrecht and Tucker (2013) show that too much ad customization can backfire at the time when consumers begin a product search and their preferences are construed at a high level. Goldfarb and Tucker (2011) show that targeting can negatively affect display ad effectiveness when paired with highly visible creative. Bleier and Eisenbeiss (2015) study the relation between timing and effectiveness of ad customization and show that customization works best when ads are shown on motive-congruent websites. Manchanda et al. (2006) analyze the effect of display advertising on customers' purchase behavior. They show that display advertising has a positive effect on purchase frequency for existing customers, and that this effect is greatest when consumers view a large number of web pages across a variety of websites. Bart et al. (2014) establish the effectiveness of mobile display advertising for high involvement and utilitarian products. Hoban and Bucklin (2015) analyze display ad effects for users at different stages of the purchase funnel and find that display advertising positively affects visitation to the advertiser's website for users in most stages of the purchase funnel, but not for those who previously visited the site without creating an account. Lu and Yang (2016) study the impact of the targeting breadth, which measures the percentage of a consumer's identified interests, on the behaviors and profits of consumers, advertisers, and the platform in behaviorally targeted display advertising. In the context of real-time bidding auctions, Johnson (2013) estimates the financial impact of privacy policies on publishers' revenue and advertisers' surplus. Zeithammer (2017) shows that introducing a soft reserve price, a bid level below which a winning bidder pays his own bid instead of the second-highest bid, does cannot increase publishers' revenue in RTB auctions.

Targeting in Advertising. Our paper also relates to the analytical work on targeting in advertising. Chen et al. (2001) and Iyer et al. (2005) show that improved targeting can increase advertisers' profits by softening competition. From a publisher's perspective, Levin and Milgrom (2010) argue that advertisers' improved targeting can lower the publisher's revenue by dispersing competition and making ad auctions thinner. Bergemann and Bonatti (2011) show that the equilibrium price of advertisements first increases and then decreases as advertisers' targeting capacity increases. In the context of online advertising auctions, Hummel and McAfee (2015) analyze the conditions under which improved targeting increases or decreases a publisher's revenue. Our results also indicate that improved targeting in RTB could potentially lower the publisher's revenue. However, we show that when a sufficiently large fraction of impressions are sold in reservation contracts, the overall effect of improved targeting in RTB on the publisher's revenue is positive.

Real-Time Bidding in Computer Science and Operations Management Literature. There is a growing body of papers in computer science and operations management literature that studies real-time bidding. Muthukrishnan (2009) is the first paper to discuss several research issues in ad exchange markets. Korula et al. (2015) provide a survey of some of the main display advertising problems from a mechanism design point of view. Several papers in operations management and computer science literature study various aspects of bid optimization problem in RTB from an advertiser's point of view (e.g., Chen et al. (2011), Lee et al. (2013), Yuan et al. (2013), and Zhang et al. (2014)). Another set of papers (e.g., Bharadwaj et al. 2010, Radovanovic and Heavlin 2012, Chen et al. 2014 and Balseiro et al. 2014) solve the allocation and revenue maximization problems of a publisher. In contrast, the focus of our paper is on advertisers' and publishers' equilibrium strategies and their equilibrium profits.

Mixed Bundling Literature. The combination of reservation contracts and RTB resembles the theoretical literature on mixed bundling. Previous literature (e.g., Adams and Yellen 1976 and McAfee et al. 1989) shows that when a seller that sells multiple products does not know customers' valuations for each product, offering a mix of bundles and individual products allows the seller to increase its profit. Similarly, in our model, we show that the publisher could increase its revenue by selling some of the impressions in reservation contracts (akin to a bundle) and the rest in RTB (akin to selling the products individually). We should, however, note that there are two important differences between the mixed bundling models in the literature and our model. First, supply is not limited in the mixed bundling literature, i.e., the seller can produce as many units as it wants at a given marginal cost of production; therefore, buyers do not *compete* with each other. Second, in the mixed bundling models, the buyer knows what is included in the bundle at the time of buying; in particular, what one buyer gets in his bundle is not affected by individual products that other buyers buy. In contrast, cherry-picking of one advertiser in RTB could affect the average quality of the "bundle" that another advertiser has already purchased in our setup.

The rest of the paper is structured as follows. In Section 2, we present the model. In Section 3, we analyze the game and discuss the results. Section 4 includes several extensions of the main model in which show the robustness of our results and provide new insights. Finally, we conclude the paper in Section 5. All proofs are relegated to the appendix.

2 Model

The model consists of a publisher, two advertisers and a unit of consumers. All consumers visit the publisher's website, and the publisher shows each consumer one advertisement. This creates a unit supply of impressions (also known as ad views) for the publisher to sell to the advertisers. Consumers are uniformly located on a Hotelling line represented by the $[0, 1]$ interval. The advertisers are located at the two endpoints of the interval: Advertiser 1 at 0, and Advertiser 2 at 1. The Hotelling distance between a consumer and an advertiser corresponds to the mismatch between the consumer's need and the advertiser's offering. For example, the proximity on the Hotelling line could represent the likelihood of purchase if exposed to an ad from that advertiser. As such, advertisers have higher valuation for impressions (i.e., showing their ads) to consumers that are closer to them. The valuation of Advertiser 1 for an impression to a consumer located at x is $1-tx$, and the valuation of Advertiser 2 for showing his ad to the same consumer is $1-t(1-x)$, where $t \leq 1$ is the travel cost in the Hotelling model, and corresponds to consumers' heterogeneity in taste.

Advertisers typically know more than publishers about consumers, particularly, regarding consumers' interest in the products that they advertise. Advertisers track consumers on several websites, and regularly purchase tracking information from data brokers, activities that publishers rarely engage in (Yuan et al. 2013). For example, consumers that have previously been searching for vacations in Barbados are very valuable to some travel companies, but a publisher might not even know that those consumers have been visiting his website. As such, we assume that the advertisers know the location of a consumer on the Hotelling line, but the publisher does not. In practice, a publisher may have some information about consumers, e.g., age range, gender, geographic location, etc., and this information could be allowed for targeting in reservation contracts. For example, advertisers may specify gender or geographic location in their reservation contracts. However, the amount of information that the publisher has, and the targeting capability of reservation contracts is much more limited than what advertisers can achieve in RTB. To facilitate exposition, we consider an extreme case[11] where no targeting is possible in reservation contracts, and the publisher has no information about the consumers. To reconcile this with reality, we can think of the consumers in our model as a segment of all consumers that is already narrowed down by the information (e.g., age range, gender and location) available to the publisher.

The advertisers could buy impressions in reservation contracts before consumers arrive, and in RTB after consumers arrive. When an advertiser buys some impressions in reservation contracts, he is guaranteed to receive that many impressions at the pre-committed price of the contract; however, the advertiser does not know where on the Hotelling line those impressions will be located. In RTB, on the other hand, an advertiser can observe the location of an impression on the Hotelling line before bidding for that impression, but the cost of the impressions could vary, and the advertiser is not guaranteed to receive (i.e., to be able to win) any certain number of impressions.

We assume that the advertisers are profit maximizers. The profit of Advertiser i is the sum of his valuation for all impressions that he purchases, minus the total cost (what he pays the publisher). The publisher maximizes his revenue, i.e., what he collects from the advertisers. The timing of the

[11] We should note that we get similar results in a model where the publisher has some information about consumers. For example, consider the model where the publisher knows which subinterval $[0, \frac{1}{3}), [\frac{1}{3}, \frac{2}{3}]$ or $(\frac{2}{3}, 1]$ a consumer is located in, but does not know the exact location within that subinterval. In this model, the publisher sells all impressions in the first subinterval to Advertiser 1 and all impressions in the third subinterval to Advertiser 2, all at per-impression price $\frac{5}{6}$ in reservation contracts. The remaining impressions (subinterval $[\frac{1}{3}, \frac{2}{3}]$) are sold in an equilibrium very similar to what we find in Proposition 3: the impressions in $[\frac{5}{9}, \frac{2}{3}]$ are sold in RTB to Advertiser 2, and the impressions in $[\frac{1}{3}, \frac{5}{9}]$ are sold in reservation contracts to Advertiser 1.

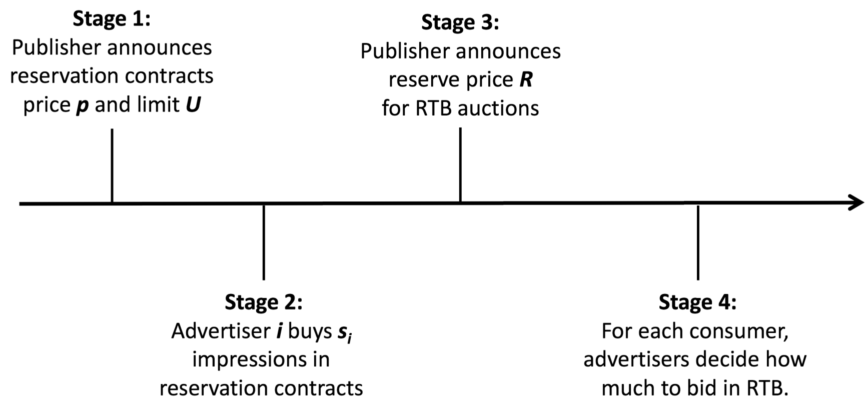

Figure 1: Timeline of The Game

game is as follows, and is summarized in Figure 1.

Stage 1: In the first stage, the publisher announces a per-impression price p and a maximum amount U available for sale in reservation contracts.[12]

Stage 2: In the second stage, advertisers announce their demand, d_i (where $d_i \leq U$), for impressions in reservation contracts; in other words, d_i denotes how many impressions Advertiser i *wants* to buy in reservation contracts. Then, the publisher decides how many impressions to *sell* to Advertiser i, denoted by s_i (where $s_i \leq d_i$), in reservation contracts. If the total demand is less than or equal to U, i.e., $d_1 + d_2 \leq U$, each advertiser is sold d_i impressions ($s_1 = d_1$ and $s_2 = d_2$). If the total demand is more than U, i.e., $d_1 + d_2 > U$, we assume that the publisher prioritizes Advertiser 1 by setting $s_1 = d_1$ and $s_2 = U - s_1$.[13]

The impressions sold in reservation contracts are guaranteed to be delivered by the publisher, i.e., Advertiser i is guaranteed to get s_i impressions when consumers arrive; however, the advertiser does not know where on the Hotelling line those impressions will be located.

Stage 3: In the third stage, consumers arrive. The publisher chooses R, and makes each impression

[12]While the fixed price selling is the common practice, in Section 4.4, we propose an alternative selling mechanism for reservation contracts that could potentially improve the publisher's revenue.

[13]We show that other rationing rules, such as one where the advertiser who is prioritized is selected randomly, or one where the inventory is divided between the two advertisers proportionally (i.e., $s_i = \frac{Ud_i}{d_1+d_2}$), both lead to the same equilibrium outcome.

available on RTB in a second-price auction with reserve price R.[14] Note that since the publisher does not know the consumers' locations, the same reserve price will be used for all impressions.[15]

Stage 4: In the fourth stage, advertisers decide how much they want to bid for each impression. The publisher then allocates the impressions as follows:

- If the highest bid is greater than or equal to R, the highest bidder wins the impression and pays the maximum of the second highest bid and R.[16]

- If the highest bid is less than R, and the impressions sold in reservation contracts are not delivered yet, the impression will be allocated randomly as one of the impressions sold in reservation contracts (i.e., the impression is allocated to Advertiser i with probability $\frac{s_i}{s_1+s_2}$).

- If the highest bid is less than R, and the impressions sold in reservation contracts are already delivered, the impression will be left unallocated.

- Finally, if at any point in time, the amount of inventory sold in RTB becomes $1 - s_1 - s_2$, the publisher stops holding auctions, and allocates the rest of the impressions to reservation contracts; in other words, the publisher ensures that the s_1+s_2 impressions sold in reservation contracts are delivered.

Note that we are not incorporating two features of RTB that we discussed in the introduction into the model yet. Specifically, at this point, we do not consider the publisher's supply uncertainty and the effectiveness of ad customization in improving consumers' probability of conversion. After establishing the key insights from the main model, in Section 4, we add these features to the model and discuss their roles.

[14] In practice, RTB auction is a slight variation of the second-price auction. Publishers set two reserve prices: a hard reserve price R, and a soft reserve price R', where $R' > R$. If the highest bid is below R, no one wins. If the highest bid is above R', then the highest bidder pays the maximum of R' and the second highest bid. Finally, if the highest bid is between R and R', then the highest bidder pays his own bid (i.e., similar to a first-price auction). In this paper, since advertisers know each others' valuations, the soft reserve price R' does not affect equilibrium payoffs. We should also mention that even in a more general setting where advertisers do not know each others' valuations, Zeithammer (2017) shows that soft reserve prices do not affect equilibrium payoffs.

[15] In practice, publishers set their reserve prices in real-time, after consumers arrive. This is why we assume that R is set in Stage 3, and not in Stage 1. When we look at the publisher's supply uncertainty in Section 4.1, we show that the realized number of visitors, which is unknown in Stages 1 and 2, affects the equilibrium value of R.

[16] In practice, the publisher is not committed to assigning the impression to the winner of the RTB auction. In other words, after receiving RTB bids, the publisher may still decide to assign the impression to reservation contracts, even if the winning bid in RTB is above the reserve price. However, we can show that when the reserve price in RTB is optimized, the publisher never assigns an impression for which the RTB bid is above the reserve price to a reservation contract (even when there is no commitment). In other words, while this assumption simplifies the model, it does not affect the equilibrium outcome of the game. A formal proof is available upon request.

3 Analysis

Before solving the game, we analyze two benchmarks for the publisher's revenue: when all impressions are sold in RTB, and when all impressions are sold in reservation contracts.

Bechmarks

First, we assume that RTB is not available. In this case, the publisher sells all of the impressions in reservation contracts. Since the publisher does not observe the location of each customer on the Hotelling line, the impressions will be allocated to the advertisers randomly. The expected value of each advertiser for each impression is $1 - \frac{t}{2}$. Using a posted price $p = 1 - \frac{t}{2}$, the publisher could extract all of the advertisers' surplus, and make a total revenue of $1 - \frac{t}{2}$. Note that there are multiple equilibria in this case: the publisher could sell any fraction of the impressions to one advertiser, and the rest to the other, all for the per-impression price of $1 - \frac{t}{2}$. However, all of these equilibria lead to the same total revenue for the publisher, and zero surplus for the advertisers.

Next, as another benchmark, we assume that all impressions have to be sold in RTB. In this case, the optimal reserve price is $R^* = 1 - \frac{t}{2}$.[17] Since R^* is greater than or equal to the second-highest valuation, thus the second-highest bid, for all impressions, all impressions are sold at the reserve price; therefore, the publisher's total revenue is, again, $1 - \frac{t}{2}$. Advertisers, however, have non-zero surplus in this case. Each advertiser wins the half of the impressions that are closest to him on the Hotelling line. The expected valuation of each advertiser for each impression that he wins is $1 - \frac{t}{4}$, and the payment is $1 - \frac{t}{2}$. Therefore, each advertiser's expected surplus per impression is $(1 - \frac{t}{4}) - (1 - \frac{t}{2}) = \frac{t}{4}$, and each advertiser's total surplus is $\frac{1}{2} \times \frac{t}{4} = \frac{t}{8}$.

It is interesting to note that both benchmarks lead to the same revenue for the publisher. In the first benchmark, the publisher could extract all of the advertisers' surplus. In this case, the publisher's revenue is limited because of the inefficient allocation. In the second benchmark, allocation is efficient, but the publisher cannot extract all of the advertisers' surplus. This is due to the competition dispersion effect of RTB and the fact that the publisher does not know the advertisers' valuations for each impression. In Proposition 3, we show that when the publisher uses reservation contracts and RTB together, it could achieve a higher revenue than each of the two

[17] For a formal proof, please refer to Proposition 2 for the case of $s_1 = s_2 = 0$

benchmarks.

In the following, we use backward induction to find subgame perfect equilibria of the game.

Fourth Stage

In the fourth stage, advertisers observe the location of each consumer and decide how much they want to bid. Note that since the unsold impressions in RTB are assigned to reservation contracts, advertisers may still win an impression that they did not bid for. In particular, even if an advertiser does not bid for an impression in RTB, he could still win that impression if the competitor does not win the impression in RTB either, *and* he is "lucky enough" so that the impression is assigned to his reservation contract (rather than his competitor's). The price of an impression sold through RTB is at least R, and the price of an impression in reservation contracts is p. Therefore, when $R > p$, if the valuation of an advertiser for an impression is not sufficiently high (even if the valuation is higher than R), the advertiser may prefer to wait and win the impression at price p with some probability, than to bid and win the impression at price R.

In equilibrium, each advertiser bids for impressions that are most valuable to him, i.e., impressions that he does not want to risk losing. Let m_1 and m_2 be such that Advertiser 1 bids for and wins the impressions in $[0, m_1]$, and Advertiser 2 bids for and wins the impressions in $[m_2, 1]$. In equilibrium, we must have $m_2 - m_1 \geq s_1 + s_2$, i.e., the number of unsold impressions in RTB should be greater than or equal to the number of impressions sold in reservation contracts. Assuming that the reserve price R is sufficiently high,[18] the expected profit of Advertiser 1 is

$$\pi_1 = \int_0^{m_1} (1 - tx - R)\,\mathrm{d}x + \int_{m_1}^{m_2} \left(\frac{s_1}{m_2 - m_1} (1 - tx - p) \right) \mathrm{d}x. \tag{1}$$

The first term on the right-hand side is the expected profit from RTB, and the second term is the expected profit from reservation contracts. Fraction $\frac{s_1}{m_2 - m_1}$ is the probability that an unsold impression in RTB is assigned to reservation contract of Advertiser 1. Similarly, the expected profit of Advertiser 2 is

$$\pi_2 = \int_{m_2}^{1} (1 - t(1-x) - R)\,\mathrm{d}x + \int_{m_1}^{m_2} \left(\frac{s_2}{m_2 - m_1} (1 - t(1-x) - p) \right) \mathrm{d}x. \tag{2}$$

[18]In the Appendix, we prove that this condition holds.

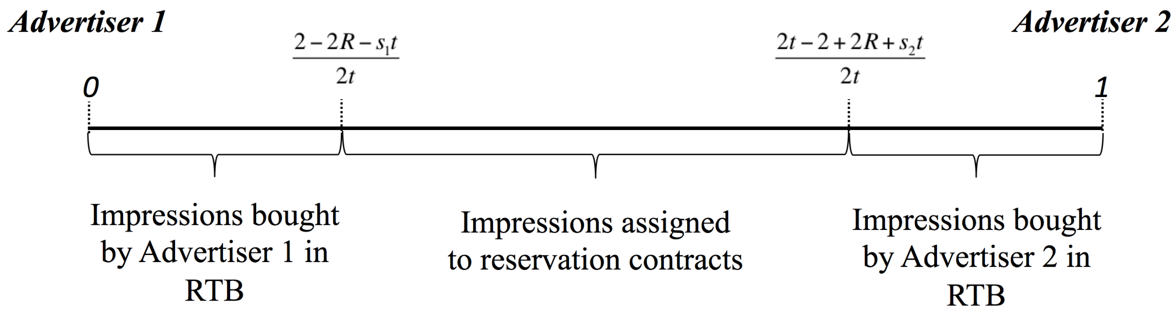

Figure 2: Allocation of impressions to RTB and reservation contracts as presented in Proposition 1.

Maximizing π_1 with respect to m_1, and π_2 with respect to m_2 give us advertisers' bidding strategies in RTB auctions, summarized in Proposition 1.

Proposition 1 *Advertiser* 1 *bids truthfully for impressions to consumers located in* $[0, \frac{2-2R-s_1t}{2t}]$*, and does not bid for impressions to other consumers. Similarly, Advertiser* 2 *bids truthfully for impressions to consumers located in* $[\frac{2t-2+2R+s_2t}{2t}, 1]$*, and does not bid for impressions to other consumers.*

Proposition 1 shows that advertisers cherry-pick when bidding in RTB auctions. In other words, each advertiser only bids for his most valuable segment of consumers. As the reserve price increases, the segment of consumers for which each advertiser bids shrinks. This is an intuitive result and is driven by the fact that as the reserve price increases, the expected payment of each advertiser in an RTB auction increases. Interestingly, Proposition 1 also shows that as the size of Advertiser i's reservation contract, s_i, increases, Advertiser i participates in fewer RTB auctions. There are two forces that drive this result. First, as s_i increases, the probability that Advertiser i is assigned an unsold impression in RTB increases. As such, the advertiser is more willing to take the risk of not bidding in the RTB auction. Second, buying in RTB negatively affects the expected value of impressions that are allocated to reservation contracts. In other words, when Advertiser i cherry-picks using RTB, he is negatively affecting the average value of his own impressions in reservation contracts. Therefore, as s_i increases, Advertiser i participates in fewer RTB auctions. Proposition 1 implies that Advertiser i bids for an impression in RTB if and only if his valuation is at least $R+\frac{s_it}{2}$. The result of Proposition 1 is depicted in Figure 2

Third Stage

Given the advertisers' bidding strategies, the publisher sets a reserve price that maximizes his revenue. The publisher sells $s_1 + s_2$ impressions at price p in reservation contracts and $\min(1 - s_1 - s_2, m_1 + 1 - m_2)$ impressions in RTB at price R (assuming that R is sufficiently large). The publisher's revenue can be written as

$$\Pi = (s_1 + s_2)p + R \min(1 - s_1 - s_2, m_1 + 1 - m_2).$$

By using the values of m_1 and m_2 from Proposition 1, we can write the publisher's revenue as

$$\Pi = (s_1 + s_2)p + R \min\left(1 - s_1 - s_2, \frac{2 - 2R - s_1 t}{2t} + 1 - \frac{2t - 2 + 2R + s_2 t}{2t}\right).$$

Maximizing Π with respect to R gives us the optimal reserve price of RTB auctions.

Proposition 2 *For given s_1, s_2 and t, the optimal reserve price for the publisher is given by*

$$R^* = 1 - \frac{(2 - s_1 - s_2)t}{4}.$$

Proposition 2 shows that as the amount of inventory sold in reservation contracts, $s_1 + s_2$, increases, the publisher increases the reserve price of RTB auctions. This is despite the result of Proposition 1 which shows that as s_i increases, Advertiser i's willingness to participate in RTB auctions decreases. In other words, given the result of Proposition 1, one would expect the publisher to lower his reserve price as $s_1 + s_2$ increases to compensate for the advertisers' lower interest in RTB auctions. However, Proposition 2 shows the opposite.

Intuitively, as s_1+s_2 increases, the publisher can allocate more unsold inventory of RTB auctions to reservation contracts. This allows the publisher to set a higher reserve price. If the unsold impressions in RTB auctions could not be "repurposed," as is the case in (non-RTB) second-price auctions in many other contexts, the optimal reserve price would have been $1 - \frac{t}{2}$. This is the same as R^* for $s_1 = s_2 = 0$. Since reservation contracts enable the publisher to repurpose the unsold impressions of RTB auctions, their existence allows the publisher to set a higher reserve price, and ultimately extract more revenue in RTB.

Second Stage

In the second stage, the advertisers have to decide how many impressions they want to buy in reservation contracts. In Equations (1) and (2), if we replace the values of R and m_i with those from Propositions 1 and 2, the profit of Advertiser i can be written as

$$\pi_i = \frac{\left(5s_i^2 + s_j^2 - 6s_is_j - 20s_i - 4s_j + 4\right)t + 32(1-p)s_i}{32}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5t}{16} > 0$ which implies that π_i a convex function of s_i.

There are two effects that make Advertiser i's profit a convex function of s_i. First, as an advertiser buys more impressions in reservation contracts (i.e., s_i increases), according to Proposition 2, the optimal reserve price in RTB increases. Therefore, as s_i increases, impressions in reservation contracts become more appealing than before compared to the impressions in RTB. Second, since impressions are cherry-picked in RTB, as s_i increases, the marginal expected value of Advertiser i for an impression in reservation contracts increases. In other words, every impression that moves from RTB to reservation contracts (as s_i increases) is a "cherry" that was being picked in RTB before, and is more valuable than all previous impressions assigned to reservation contracts. Therefore, as s_i increases, Advertiser i's incentive to buy impressions in reservation contracts also increases.

Since π_i is a convex function of s_i, an advertiser either buys no impressions in reservation contracts (when the price p is too high), or buys the maximum available quantity. The condition on price p is presented in Lemma 1.

Lemma 1 *Suppose that quantity $U - s_j$ is available for Advertiser i to buy in reservation contracts. If $p > \frac{5Ut+32-20t-11s_jt}{32}$, Advertiser i does not buy any impressions in reservation contracts; otherwise, he buys all of the available impressions, $U - s_j$.*

Lemma 1 shows that as the amount of inventory available for reservation contracts, $U - s_j$, increases, Advertiser i's willingness to pay per impression in reservation contracts increases. As discussed before, this is driven by the fact that the advertiser's profit is a convex function of the amount of inventory that he buys in reservation contracts.

Lemma 1 also shows that as the size of the competitor's reservation contract, s_j, increases, Advertiser i's willingness to pay for impressions in reservation contracts decreases. There are two reasons for this effect. The first reason is that, as s_j increases, there are fewer impressions available for Advertiser i, and due to the convexity Advertiser i's profit in s_i, Advertiser i's willingness to pay decreases. However, this is not the only reason. In fact, if we replace U with $U + s_j$ so that the number of impressions available for Advertiser i is kept constant, as s_j increases, Advertiser i's willingness to pay for impressions in reservation contracts still decreases.

This is a counter-intuitive effect because an increase in s_j leads to a higher reserve price in RTB, which, intuitively, should motivate Advertiser i to buy more impressions in reservation contracts; but Lemma 1 shows the opposite. This is because as s_j increases, according to Proposition 1, Advertiser j bids in fewer RTB auctions. Therefore, many of the impressions that are valuable to Advertiser j, thus not valuable to Advertiser i, will be left unsold in RTB. This reduces the expected valuation of Advertiser i for the unsold impressions in RTB, i.e., impressions that are assigned to reservation contracts. As such, Advertiser i's willingness to pay per impression in reservation contracts decreases as the size of the reservation contract of Advertiser j, s_j, increases.

First Stage

Each advertiser's willingness to pay per impression in reservation contract increases as the number of impressions in the contract increases. Therefore, for a given total quantity U, the publisher's revenue is maximized if all of the impressions are sold to one advertiser. The publisher sets the price of reservation contracts, p, such that the advertisers are indifferent between buying nothing and buying all of the available inventory U. In equilibrium, one advertiser, which we assume without loss of generality is Advertiser 1, buys ad inventory U in reservation contracts, and Advertiser 2 buys nothing.

Proposition 3 *The publisher makes $U = \frac{2}{3}$ impressions available for reservation contracts at price $p = 1 - \frac{25t}{48}$, and sets the reserve price of RTB auctions to $R^* = 1 - \frac{t}{3}$. Advertiser 1 buys all of the U impressions in reservation contracts, and does not bid in RTB. Advertiser 2 only bids for consumers who are located in $[\frac{2}{3}, 1]$. The publisher's total revenue is $1 - \frac{11t}{24}$, Advertiser 1's profit is $\frac{t}{8}$, and Advertiser 2's profit is $\frac{t}{18}$.*

Proposition 3 summarizes how RTB affects advertisers' and publishers' selling and buying strategies in equilibrium. In Corollary 1, we use the result of Proposition 3 to show that the publisher's revenue under the current selling mechanism is higher than two alternatives.

Corollary 1 *Given the publisher's equilibrium revenue in Proposition 3, we have:*

- *The publisher's revenue when selling $U = \frac{2}{3}$ of the impressions in reservation contracts and the rest in RTB is higher than when selling all of the impressions in RTB.*
- *The publisher's revenue when he makes all of the impressions available on RTB, and allocates the impressions that do not receive sufficiently high RTB bids to reservation contracts, is higher than when he allocates all of the impressions to reservation contracts until reservation contracts are fulfilled, and makes the leftover available on RTB.*

The results of Proposition 3 and Corollary 1 have several interesting implications for advertisers and publishers:

1. *Future of Display Advertising.* Since RTB is a new technology that allocates impressions to advertisers more efficiently, many analysts believe that RTB will replace reservation contracts in the future. In contrast, Corollary 1 shows that, since the publisher uses reservation contracts to increase the reserve price of RTB auctions, RTB will not replace reservation contracts in equilibrium. In fact, existence of reservation contracts allows the publisher to achieve a higher total revenue.

   Proposition 3 shows that two third of the impressions are sold in reservation contracts in equilibrium. While this number is close to industry reports as of 2017, we should note that in the extensions that we consider in Section 4, depending on the model parameters, this number might change. In particular, we show that as the publisher's uncertainty about the number of his future visitors increases (Section 4.1), as the number of premium consumers increases (Section 4.2), or as the effectiveness of ad customization increases (Section 4.3), the fraction of impressions that are sold in reservation contracts decreases.

2. *Price of Impressions in Reservation Contracts.* Since advertisers can cherry-pick impressions in RTB, impression assigned to reservation contracts become less valuable when RTB exists. In other words, RTB negatively affects advertisers' willingness to pay for impressions in reservation contracts. A few papers in computer science and operations management literature

(e.g., see Ghosh et al. 2009 and Balseiro et al. 2014) tackle this problem by proposing methods to maintain the quality of impressions assigned to reservation contracts, e.g., by not making all of the impressions available in RTB.[19]

In contrast, we show that the publisher's optimal strategy is to allow cherry-picking in RTB, and to set a lower price for impressions in reservation contracts. In particular, as we show in Proposition 3, the price of impressions in reservation contracts when RTB is available, $1 - \frac{25t}{48}$, is lower than when it is not, $1 - \frac{t}{2}$. Interestingly, despite the reduction in the price of impressions in reservation contracts, Proposition 3 shows that the publisher's total revenue increases with RTB. In other words, we show that as long as the reserve price in RTB is set optimally, the high price of impressions in RTB compensates for the low price of impressions in reservation contracts such that the publisher's total revenue when RTB exists is higher than when it does not.

3. *Advertisers' Asymmetric Strategies.* Since advertisers know that buying in reservation contracts increases the reserve price of RTB, they may want to avoid reservation contracts even if the price is lower than their expected valuation. However, for a sufficiently low price of impressions in reservation contracts, advertisers use asymmetric strategies where Advertiser 1 buys all of his ad inventory in reservation contracts, and, therefore, is not affected by the higher reserve price in RTB. Advertiser 2 buys the inventory at a high reserve price in RTB, but, since RTB allows for cherry-picking, still benefits from existence of RTB.

   We should note that the result that Advertiser 1 does not bid in RTB is moderated under two of the extensions that we consider in Section 4. In particular, when there is a segment of premium consumers for which both advertisers have high valuation (Section 4.2), we show that both advertisers bid in RTB for premium consumers. Furthermore, if we consider publisher's supply uncertainty (Section 4.1), again, both advertisers participate in RTB auctions. Nonetheless, in both extensions, the advertisers still use asymmetric strategies, and the key insights of Proposition 3 continue to hold.

4. *Advertisers' and Publisher's Profits.* Since RTB increases the overall efficiency of allocation,

[19] For example, Ghosh et al. (2009) address this issue by directly bidding in RTB auctions on behalf of contract-buying advertisers, randomly varying the bid in order to get a representative/fair allocation in reservation contracts.

all parties benefit from its existence. Proposition 3 shows that RTB improves the expected revenue of the publisher from $1 - \frac{t}{2}$ to $1 - \frac{11t}{24}$, and the expected profits of Advertisers 1 and 2 from 0 to $\frac{t}{8}$ and $\frac{t}{18}$, respectively. Interestingly, Advertiser 1 who does not use RTB benefits more from existence of RTB than Advertiser 2 who only uses RTB.

5. *Header Bidding.* As discussed in the introduction, header bidding changes the order in which the publisher allocates impressions to reservation contracts and RTB. Before header bidding, the publisher allocated all of his impressions to reservation contracts until those contracts were fulfilled; then, made the leftover available on RTB. With header bidding, the publisher receives bids on all of his impressions, and then decides whether to allocate each impression to RTB or reservation contracts. Corollary 1 shows that the publisher's revenue under the new mechanism, i.e., with header bidding, is higher than the old mechanism. This finding is consistent with the trend in RTB market that shows a rapid adoption of header bidding by publishers in 2015 and 2016.

4 Extensions

In the previous section, we used a simple model to discuss the effects of RTB on advertisers' and publishers' strategies. In this section, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings.

In developing our model, we made a few simplifying assumptions to facilitate the exposition of our key results. For example, thus far we have assumed that the publisher knows the exact number of future visitors at the time of selling impressions in reservation contracts. In Section 4.1, we relax this assumption and show the robustness of our results. Furthermore, we show that under supply uncertainty, RTB increases social surplus, and that both advertisers and the publisher benefit from the increase in social surplus.

In the preceding analysis, we examined a market where consumers are located on a Hotelling line. In Section 4.2, we consider two additional segments of consumers: "premium consumers," which both advertisers highly value, and "unsusceptible consumers," for which both advertisers have low valuation. We find that both advertisers bid for impressions to premium consumers in RTB, and that all such impressions are sold in RTB. However, advertisers do not bid for impressions to unsusceptible consumers, and all those impressions are assigned to reservation contracts. We show that the advertisers still use the asymmetric strategies discussed in Proposition 3 for buying the impressions to consumers on the Hotelling line.

In Section 4.3, we analyze the ad customization (also referred to as "personalization" in the literature) effect of RTB. Since advertisers can customize their ads for each individual consumer in RTB, they can increase the probability of conversion given impression, and therefore, would have higher willingness to pay for the same impression in RTB than in reservation contracts. In Section 4.3, we assess the robustness of our results in presence of ad customization. Interestingly, we also find that the advertisers' profits may decrease as their ad customization becomes more effective (i.e., as the probability of conversion due to ad customization increases). Finally, in Section 4.4, we discuss other selling mechanisms that could potentially improve the publisher's revenue.

4.1 Publishers' Supply Uncertainty

In practice, publishers do not know how many visitors their website will have in the future. As such, publishers are uncertain about the size of their supply when selling impressions in reservation contracts. Before RTB, this uncertainty, along with the fact that reservation contracts include penalties for underdelivery, made the decision of how many impressions to sell in reservation contracts a major challenge for many publishers, and publishers were often left with unsold impressions. RTB was introduced as a mechanism to sell the leftover inventory in real-time. In this section, we study how RTB affects advertisers' and publishers' strategies under supply uncertainty.

We assume that the supply of impressions (e.g., number of visitors) is $1 + D$ where D is either 0 or $d \geq 0$, each with probability $\frac{1}{2}$. The size of the supply (i.e., the value of D) is unknown in Stages 1 and 2, during the sale of impressions in reservation contracts. We assume that the penalty of underdelivery in reservation contracts is sufficiently high so that the publisher does not sell more than 1 unit in reservation contracts. Since the publisher can change the reserve price in real-time in response to changes in the number of visitors, we assume that the supply is realized at the beginning of Stage 3, before the publisher sets the reserve price. Proposition 4 summarizes the equilibrium strategies under supply uncertainty.

Proposition 4 *The publisher makes* $U = \frac{4(d+1)}{3(d+2)}$ *impressions available for reservation contract* *at price* $p = 1 - \frac{25t}{48}$. *Advertiser* 1 *buys all of the* U *impressions in reservation contracts. The publisher's expected revenue is* $\frac{1}{64}\left(\frac{16(d+1)t}{3(d+2)} + 16(d+2)(2-t)\right)$, *the expected profit of Advertiser* 1 *is* $\pi_1 = \frac{1}{16}(d+2)t$, *and the expected profit of Advertiser* 2 *is* $\pi_2 = \frac{(d(9d+16)+16)t}{144(d+2)}$. *When the supply is* $1+d$, *the publisher sets the reserve price of RTB to* $R = 1 - \frac{(3d+4)t}{6(d+2)}$, *and when the supply is* 1, *the reserve price is set to* $R = 1 - \frac{(d+4)t}{6(d+2)}$. *Advertiser* 1 *bids for consumers located in* $[0, \frac{1-R}{t} - \frac{U}{2(1+D)}]$, *and Advertiser* 2 *bids for consumers located in* $[\frac{R+t-1}{t}, 1]$.

Proposition 4 shows the robustness of our findings in Proposition 3. In particular, it shows that advertisers use asymmetric strategies where one advertisers buys most of his impressions in reservation contracts and the other advertiser buys all of his impressions in RTB. Proposition 4 also shows that under supply uncertainty, RTB increases the social surplus, and that both advertisers and the publisher benefit from this increase.

4.2 Premium Consumers and Unsusceptible Consumers

In this section, in addition to the main segment of consumers with mass 1 that we considered before, we consider two new segments. First, we consider a segment of *premium consumers* with mass P for which both advertisers have high valuation. This segment represents consumers that are identified by both advertisers as having high probability of conversion. For half of these consumers (i.e., $\frac{P}{2}$ impressions), we assume that Advertiser 1 has valuation 1 and Advertiser 2 has valuation $\lambda > 1$; for the other half, we assume that Advertiser 1 has valuation λ and Advertiser 2 has valuation 1. Parameter λ captures the degree of advertisers' differentiation in regard to premium consumers, and is common knowledge. Second, we consider a segment of *unsusceptible consumers* with mass Q for which both advertisers have low valuation. This segments corresponds to consumers who are identified by both advertisers as having low probability of conversion (e.g., because they use ad blockers, or have already purchased the product). We assume that the value of each advertiser for an impression to a consumer in this segment is 0. As before, the advertisers know their valuation for each consumer. However, the publisher does not know the advertisers' valuations, and, specifically, does not know if a consumer is in premium segment or unsusceptible segment. Proposition 5 shows that the existence of these segments does not affect our main findings from Proposition 3.

Proposition 5 • *If* $\lambda \leq 1 + \frac{t}{24P}$*, the publisher makes quantity* $U = \frac{2}{3} + Q$ *available for reservation contracts at price* $p = \frac{48-25t}{72Q+48}$*, and sets the reserve price of RTB to* $R^* = 1 - \frac{t}{3}$*. Advertiser 1 buys all of the* U *impressions in reservation contracts. Both advertisers bid truthfully for RTB impressions to premium consumers, and all of those impressions are sold in RTB at price 1. Advertiser 1 does not bid for non-premium impression in RTB, and Advertiser 2 only bids for non-premium impressions to consumers who are located in* $[\frac{2}{3}, 1]$*. The publisher's total revenue is* $1 + P - \frac{11t}{24}$*, Advertiser 1's profit is* $\frac{t}{8} + \frac{P(\lambda-1)}{2}$ *and Advertiser 2's profit is* $\frac{t}{18} + \frac{P(\lambda-1)}{2}$*.*

- *If* $\lambda > 1 + \frac{t}{24P}$*, the publisher sets the reserve price of RTB to* $R^* = \lambda$*, sells all of the premium impressions in RTB, and sells the rest of the impressions (i.e.,* $U = 1 + Q$*) for a total price* $1 - \frac{t}{2}$ *in reservation contracts. Advertiser 1 buys all of the* U *impressions in reservation contracts. Both advertisers bid truthfully for RTB impressions to premium consumers. The publisher's total expected revenue is* $\lambda P + 1 - \frac{t}{2}$*, and each advertiser's expected surplus is 0.*

Proposition 5 shows that when the advertisers have similar valuations for premium consumers (i.e., when λ is small), competition is maximized, and advertisers' surplus is small. As the difference in advertisers' valuations for premium consumers increases (i.e., as λ increases), competition decreases, and advertisers' surplus increases. When λ becomes sufficiently large (i.e., competition between the advertisers for premium consumers becomes sufficiently weak), the publisher cannot rely on advertisers' competition for surplus extraction anymore; as a result, the publisher increases the reserve price to $R^* = \lambda$, and all of the RTB impressions are sold at the reserve price. In this case, the publisher sells all of the non-premium impressions in reservation contracts.

When λ is not too large (i.e., $\lambda \leq 1 + \frac{t}{24P}$), since the advertisers' valuations for premium consumers are sufficiently high, both advertisers bid for those impressions in RTB. These impressions are sold in auctions, and each advertiser's (per-impression) surplus is given by how much he values the impression more than the competitor, i.e., $\lambda - 1$. As such, the existence of premium consumers increases each advertiser's expected profit by $\frac{P(\lambda-1)}{2}$.

Since the advertisers' valuations for unsusceptible consumers are sufficiently low, advertisers do not bid for those impressions in RTB. All those impressions are assigned to reservation contracts, and, therefore, the per-impression price of reservation contracts in presence of unsusceptible consumers is lower than in the main model. The advertiser that buys the impressions in bulk in reservation contracts gets the Q impressions in unsusceptible segment, but since he pays a lower price for impressions in reservation contracts, his profit remains unaffected.

We should also note that by using a Hotelling line in the main model, we implicitly assumed that the advertisers' valuations for impressions are negatively correlated. As in most papers that use the Hotelling model, this assumption is motivated by the facts that firms are differentiated in their offerings and consumers are heterogenous in their tastes. This assumption also allowed us to capture the competition dispersion effect of RTB, which is known to be an issue in online advertising markets (Levin and Milgrom 2010; Rafieian and Yoganarasimhan 2017). In practice, however, advertisers' valuation may be positively correlated for some consumers. Proposition 5 shows the robustness of our results in this regard.[20]

4.3 Ad Customization

In the main model, we only considered the fine-grained targeting effect of RTB. In practice, advertiser can also customize (also referred to as personalize) their ads when using RTB. This allows the advertisers to increase the probability of conversion given an impression. We assume that the value of an advertiser for a consumer with distance x, when customizing the ad using RTB, is $1 - \alpha t x$ (instead of $1 - tx$ in the main model) where $\alpha < 1$. As α decreases, the advertisers' ability to influence consumers by customizing their ads increases. When α is 0, the customization is so strong such that each consumer perceives the advertiser as being located at the place as himself on the Hotelling line (i.e., perceived perfect match). Note that α only affects advertisers' valuations for impressions that are sold in RTB; the advertisers' valuation for impressions in reservation contracts remain the same as before.

Proposition 6 *The publisher makes $U = \frac{2\alpha}{2\alpha+1}$ impressions available for reservation contracts at price $p = \frac{32\alpha+16-t(4\alpha(\alpha+5)+1)}{32\alpha+16}$. Advertiser 1 buys all of the U impressions in reservation contracts.*

[20] We can also show that when advertisers' valuations are independent draws from uniform distribution $U[0,1]$, the equilibrium outcome is similar to that of Proposition 3. The publisher sets the price of impressions in reservation contracts to $p = \frac{1}{24}\left(11 - \sqrt{5}\right)$ ($\simeq 0.365$, less than $\frac{1}{2}$, the price that it would set if RTB did not exist), sets $U = \frac{1}{2}\left(\sqrt{5}-1\right) \simeq 0.618$, and sets the reserve price of RTB to $R = \frac{1}{2}\left(\sqrt{5}-1\right)$ ($\simeq 0.618$, which is higher than $\frac{1}{2}$, the optimal reserve price if all impressions were sold in RTB). Advertiser 1 buys U of the inventory in reservation contracts and does not bid in RTB. Advertiser 2 bids for impressions for which his valuation is at least R, and wins all of them at price R.

The publisher's expected revenue is $\frac{\alpha(16-t(2\alpha+9))+8}{16\alpha+8}$*, the expected profit of Advertiser* 1 *is* $\pi_1 = \frac{t\alpha}{8}$*, and the expected profit of Advertiser* 2 *is* $\pi_2 = \frac{t\alpha}{2(2\alpha+1)^2}$*. The publisher sets the reserve price of RTB to* $R = \frac{2\alpha+1-t\alpha}{2\alpha+1}$*. Advertiser* 1 *does not bid in RTB, and Advertiser* 2 *bids for consumers located in interval* $[\frac{2\alpha}{2\alpha+1}, 1]$.

Proposition 6 shows that as customization becomes stronger, the publisher sells fewer impressions in reservation contracts. Interestingly, although customization is not possible in reservation contracts, the publisher increases the price of reservation contracts as customization becomes stronger. This is because customization increases the advertisers' willingness to pay, and hence the reserve price, in RTB auctions. As the reserve price in RTB increases, the publisher could also set a higher price for impressions in reservation contracts.

Another interesting finding in Propostion 6 is the effect of customization on the advertisers' profits. Intuitively, one would expect the advertisers' profits to increase as they become better in ad customization. However, Proposition 6 shows that as customization becomes stronger (i.e., α decreases), Advertiser 1's equilibrium profit decreases and Advertiser 2's equilibrium profit first increases and then decreases.

As α decreases, the price of reservation contracts, p, increases, and the number of impressions sold in reservation contracts, U, decreases. Both of these effects lower the expected profit of Advertiser 1. Furthermore, since Advertiser 1 does not bid in RTB auctions, he does not benefit from stronger customization. Therefore, as customization becomes stronger (i.e., α decreases), the expected profit of Advertiser 1 decreases.

As α decreases, Advertiser 2's valuation for each RTB impression increases. Furthermore, since lower α leads to lower U, the number of impressions that Advertiser 2 buys in RTB, $1 - U$, also increases. Both of these forces positively affect the expected profit of Advertiser 2. However, as α decreases, the publisher increases the reserve price of RTB auctions. This negatively affects the expected profit of Advertiser 2. In the most extreme case, where $\alpha = 0$, Advertiser 2's valuation for all impressions is 1. In this case, the publisher sets the reserve price to 1 which leaves Advertiser 2 with 0 profit. Overall, as customization becomes stronger (i.e., α decreases), the expected profit of Advertiser 2 first increases, and then decreases.

4.4 Other Selling Mechanisms

In this section, we study whether the publisher could increase its revenue by using a different selling mechanism. In other words, since the publisher can choose the rules of the game, we investigate what rules maximize the publisher's revenue. We first show that if the publisher has full control over the rules of the game, it can achieve the optimal allocation and extract all of the advertisers' surplus (optimal mechanism). Then, we discuss some practical constraints that the publisher has when choosing the rules of the game, and discuss how the publisher can improve its revenue under those constraints.

Optimal Selling Mechanism

Before discussing the optimal mechanism, note that if the impressions are allocated efficiently, each advertiser gets the half of the impressions that are closest to it, for a total (per-advertiser) valuation $\int_0^{\frac{1}{2}}(1-tx)\,\mathrm{d}x = \frac{1}{2} - \frac{t}{8}$. Therefore, total advertisers' valuation can be at most $2 \times (\frac{1}{2} - \frac{t}{8}) = 1 - \frac{t}{4}$. This is an upper-bound on the publisher's revenue.

We show that the publisher can achieve this upper-bound by using the following two-part tariff mechanism. The publisher sets a fixed fee of $F = \frac{t}{4} - \varepsilon$ for each advertiser in order to let them participate in RTB auctions (where $\varepsilon > 0$ is a sufficiently small number). The publisher sets the reserve price of RTB auctions to $R = 0$, and does not sell any impressions in reservation contracts.

In equilibrium, by bidding truthfully in RTB auctions, each advertiser wins the half of the impressions that is closet to him. The expected valuation of each advertiser for each impression that he wins $1-\frac{t}{4}$, and the expected payment is $1-\frac{3t}{4}$. Therefore, each advertiser's expected surplus from each impression that he wins is $\frac{t}{2}$, and each advertiser's total surplus is $\frac{t}{4}$. The publisher is extracting all of this surplus using the fixed fee F. The publisher's total revenue is the sum of the expected payments of the advertisers in RTB, $2 \times \frac{1}{2} \times 1 - \frac{3t}{4} = 1 - \frac{3t}{4}$, and the total fixed fee, $2F = \frac{t}{2} - 2\varepsilon$. The publisher's revenue simplifies to $1 - \frac{t}{4} - 2\varepsilon$ which is almost the same as the upper-bound, $1 - \frac{t}{4}$, when ε is sufficiently small.

The above two-part tariff mechanism is simple and optimal, yet it is not common in practice for various reasons (e.g., see Balseiro et al. 2017). One of the problems is that this mechanism requires the publisher to limit the advertisers' free access to RTB exchanges (and to credibly commit to that). This requirement is hard to implement because advertisers often work with several agencies who buy impressions on their behalf. Furthermore, open exchanges are typically run by third-party intermediaries, and the publisher gets to know who won the impression only after the auction is ended.[21] Next, we explore whether the publisher could increase its revenue without limiting the advertisers' ability to buy impressions in RTB (i.e., keeping Stages 3 and 4 of the game in the main model intact). Specifically, we study whether the publisher could increase its revenue by selling the impressions in reservation contracts differently.

Selling Impressions in Reservation Contracts Differently

In this section, we show that by selling the impressions in reservation contracts using an auction, instead of a posted price, the publisher can increase its revenue. Using Proposition 2, we know that as the number of impressions sold in reservation contracts increases, the reserve price of RTB increases. In other words, an advertiser's expected profit in RTB decreases when the competitor buys impressions in reservation contracts. Due to this negative externality, advertisers are willing to pay a higher price for impressions in reservation contracts *if* they know that by not doing so, their competitor would get those impressions. As such, the publisher could use the *threat* of giving the impressions to the competitor to increase the price of impressions in reservation contracts.

When the price of impressions in reservation contracts is set to $p = 1 - \frac{25t}{48}$, Advertiser 2 is indifferent between two outcomes: one in which he buys all U impressions in reservation contracts, and one in which neither advertiser buys any impressions in reservation contracts. Since the tie is broken in favor of Advertiser 1, Advertiser 2 knows that he would only get the impressions in reservation contracts if Advertiser 1 is not interested in buying them. Therefore, if the price is set to $p > 1 - \frac{25t}{48}$, since Advertiser 2 can assume that Advertiser 1 is not buying the impressions in reservation contracts, Advertiser 2 would not buy the impressions in reservation contracts. Knowing that Advertiser 2 will not buy the impressions in reservation contracts when $p > 1-\frac{25t}{48}$, Advertiser 1 also refuses to buy impressions in reservation contracts for $p > 1 - \frac{25t}{48}$. Therefore, the publisher's revenue would be maximized at $p = 1-\frac{25t}{48}$ when impressions in reservation contracts are sold using a posted price.

[21] With the growth of (publisher-owned) private exchanges, some of these challenges might become easier to address in the future. For example, publishers may become able to charge fixed fees (e.g., in the form of monthly subscriptions) for giving advertisers access to their private exchanges.

However, if impressions in reservation contracts are sold in an auction, so that each advertiser knows that if he does not win the reservation contract his competitor does, advertisers would pay a higher price for impressions in reservation contracts. In other words, each advertiser has to decide between these two outcomes: one in which he buys all U impressions in reservation contracts, and one in which his competitor buys all U impressions in reservation contracts. Therefore, as we show in Proposition 7, advertisers pay a higher price for impressions in reservation contracts in equilibrium.

Proposition 7 *If the publisher sets $U = \frac{3}{4}$ and sells the impressions in reservation contracts in a second price auction, both advertisers bid $1 - \frac{5t}{12}$ per impression in equilibrium. Advertiser 1 wins the reservation contract at price $1 - \frac{5t}{12}$ per impression, and does not bid in RTB. The publisher sets the reserve price of RTB impressions to $R = 1 - \frac{t}{4}$. Advertiser 2 only bids in RTB for impressions for which his valuation is above the reserve price (the same strategy as in Proposition 3). The expected total profit of each advertiser is $\pi_1 = \pi_2 = \frac{t}{32}$, and the expected revenue of the publisher is $1 - \frac{3t}{8}$.*

It is interesting to note that Advertiser 1's advantage over Advertiser 2, in the main model, disappears when the impressions in reservation contracts are sold in auctions. In the main model, since the publisher breaks the tie in favor of Advertiser 1 when both advertisers want the U impressions in reservation contracts, Advertiser 1 earns a higher profit, $\pi_1 = \frac{t}{8}$, than Advertiser 2, $\pi_2 = \frac{t}{18}$. However, when the impressions in reservation contracts are sold in an auction, both advertisers have the same profit, $\pi_1 = \pi_2 = \frac{t}{32}$.

Finally, we should mention that changing the selling mechanism in practice might be challenging for publishers for two reasons. First, advertisers need to change their equilibrium strategies for the publisher to gain a higher revenue. For example, if advertisers (temporarily) keep using the same strategies as in Proposition 3 after the mechanism changes, the publisher's revenue from reservation contracts would (temporarily) drop to 0. In general, this is a common obstacle when a seller wants to change his selling mechanism.[22] Second, since reservation contracts are traditionally negotiated in person, implementing a new mechanism, in particular a simultaneous auction, might be challenging from an operational point of view. However, given that a growing number

[22]For example, Varian and Harris (2014) mention this as one of the reasons that Google did not change its search advertising auctions from GSP to VCG.

of impressions in reservation contracts are being bought and sold by computer programs (Fisher and Liu 2016), we expect the implementation of the proposed mechanism to become easier in the future.

5 Conclusion

Real-time bidding in display advertising is a new and growing model for buying and selling advertising space online. In this paper, we study the implications of RTB on advertisers' and publishers' strategies and their profits. Our theoretical analysis offers useful insights on several issues of managerial significance.

Implications for Publishers. RTB has several advantages for publishers. First, since it allows for fine-grained targeting and ad customization, RTB increases advertisers' willingness to pay for impressions. Second, since RTB unbundles the impressions in display advertising, it increases the overall efficiency of ad allocation, and improves social surplus in the market. Third, since impressions are sold in real-time, RTB resolves publishers' supply uncertainty by allowing them to sell the impressions that they could not anticipate.

RTB also has some negative effects on publishers' revenue. First, RTB could lower publishers' revenue by making ad auctions thinner, i.e., dispersing competition. In fact, RTB auctions are known to have relatively few bids per impression. Second, since RTB allows for cherry-picking of impressions in real-time, it negatively affects the quality of impressions assigned to reservation contracts. As such, advertisers' willingness to pay for impressions in reservation contracts declines, which in turn lowers publishers' revenue from reservation contracts. Our results have the following implications for publishers.

- Despite RTB's higher efficiency in ad allocation, and advertisers' higher willingness to pay for RTB impressions, publishers should not phase out reservation contracts. Reservation contracts allow publishers to set higher reserve prices in RTB, and achieve higher total revenue.
- Since advertisers can cherry-pick impressions in RTB, their willingness to pay for impressions assigned to reservation contracts decreases. Publishers should not fight this cherry-picking. Instead, they should lower the price of impressions in reservation contracts so that advertisers

still buy impressions in reservation contracts, and increase the reserve price of RTB auctions to achieve a higher total revenue.

- As advertisers become more effective in ad customization, i.e., the probability of conversion due to ad customization increases, publishers should lower the amount of inventory allocated to reservation contracts and increase the reserve prices of RTB.

Implications for Advertisers. RTB has several advantages for advertisers as well. First, RTB allows advertisers to only bid, and pay, for impressions that they want. Second, advertisers can customize their ads, and thus increase the probability of conversion, for each consumer in RTB. Third, since RTB allows advertisers to bid only on impressions to consumers in their target market, advertisers can differentiate on the impressions they bid on, and thus, can avoid head-on competition for all impressions.

RTB could also negatively affect advertisers' profits. Advertisers typically have to pay a higher price for impressions in RTB. Furthermore, since impressions are cherry-picked in RTB, advertisers who rely on reservation contracts receive lower-quality impressions on average. Our results have the following implications for advertisers.

- Advertisers use asymmetric strategies. An advertiser could focus on reservation contracts by purchasing large quantities of impressions at low prices (lower than before RTB), or focus on RTB by cherry-picking impressions at high prices. While both advertisers benefit from existence of RTB, the advertiser that focuses on reservation contracts benefits more.

- An advertiser who focuses on reservation contracts should still participate in RTB. In particular, when there is a segment of premium consumers that both advertisers highly value, even an advertiser who focuses on reservation contracts should bid for impressions to premium consumers in RTB. Furthermore, in case of publisher's supply uncertainty, an advertiser that focuses on reservation contracts should also participate in RTB, but should only bid on impressions that he *highly* values, i.e., impressions for which the valuation is sufficiently higher than the reserve price.

- An advertiser who focuses on RTB should bid on all impressions in RTB for which his valuation is above the reserve price, and should not buy impressions in reservation contracts.

Future Research. Our work is a first step towards studying the impact of real-time bidding in display advertising on publishers' and advertisers' strategies and their profits. The are several opportunities for further research. For example, we do not model budget constraints in this paper. Modeling budget constraints and understanding their effects on advertisers' adoption of RTB could lead to valuable insights. Next, advertisers and publishers often use agencies for buying and selling RTB impressions. It would be interesting to investigate how the insertion of an intermediary, which may introduce agency considerations, affects our insights. Exchange platforms such as OpenX and Google's DoubleClick are trying to consolidate the market by also acting as advertisers' and publishers' agents. It would be interesting to understand whether exchange platforms have the same incentives as third-party agents or not. Future work can also study the competition between exchange platforms, and how it affects advertisers' and publishers' strategies.

References

[1] Adams, William James, and Janet L. Yellen. "Commodity bundling and the burden of monopoly." The Quarterly Journal of Economics (1976): 475-498.

[2] Amaldoss, Wilfred, Preyas S. Desai, and Woochoel Shin. "Keyword search advertising and first-page bid estimates: A strategic analysis." Management Science 61.3 (2015): 507-519.

[3] Amaldoss, Wilfred, Kinshuk Jerath, and Amin Sayedi. "Keyword management costs and 'broad match' in sponsored search advertising." Marketing Science 35.2 (2015): 259-274.

[4] Anand, Bharat N., and Ron Shachar. "Advertising, the matchmaker." The RAND Journal of Economics 42.2 (2011): 205-245.

[5] Balseiro, Santiago R., Jon Feldman, Vahab Mirrokni, and S. Muthukrishnan. "Yield optimization of display advertising with ad exchange." Management Science 60, no. 12 (2014): 2886-2907.

[6] Balseiro, Santiago, Vahab Mirrokni, and Renato Paes Leme. "Dynamic mechanisms with martingale utilities." Working Paper (2017).

[7] Bart, Yakov, Andrew T. Stephen, and Miklos Sarvary. "Which products are best suited to mobile advertising? A field study of mobile display advertising effects on consumer attitudes and intentions." Journal of Marketing Research 51.3 (2014): 270-285.

[8] Bergemann, Dirk, and Alessandro Bonatti. "Targeting in advertising markets: implications for offline versus online media." The RAND Journal of Economics 42.3 (2011): 417-443.

[9] Berman, Ron. "Beyond the last touch: Attribution in online advertising." Working Paper (2016).

[10] Berman, Ron, and Zsolt Katona. "The role of search engine optimization in search marketing." Marketing Science 32.4 (2013): 644-651.

[11] Bharadwaj, Vijay, Wenjing Ma, Michael Schwarz, Jayavel Shanmugasundaram, Erik Vee, Jack Xie, and Jian Yang. "Pricing guaranteed contracts in online display advertising." In Proceedings of the 19th ACM international conference on Information and knowledge management, pp. 399-408. ACM, 2010.

[12] Bleier, Alexander, and Maik Eisenbeiss. "Personalized online advertising effectiveness: The interplay of what, when, and where." Marketing Science 34.5 (2015): 669-688.

[13] Cao, Xinyu, and T. Tony Ke. "Cooperative Search Advertising." Working Paper (2016).

[14] Chen, Ye, Pavel Berkhin, Bo Anderson, and Nikhil R. Devanur. "Real-time bidding algorithms for performance-based display ad allocation." In Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining, pp. 1307-1315. ACM, 2011.

[15] Chen, Yuxin, Chakravarthi Narasimhan, and Z. John Zhang. "Individual marketing with imperfect targetability." Marketing Science 20, no. 1 (2001): 23-41.

[16] Chen, Bowei, Shuai Yuan, and Jun Wang. "A dynamic pricing model for unifying programmatic guarantee and real-time bidding in display advertising." Proceedings of the Eighth International Workshop on Data Mining for Online Advertising. ACM, 2014.

[17] Desai, Preyas S., Woochoel Shin, and Richard Staelin. "The company that you keep: when to buy a competitor's keyword." Marketing Science 33, no. 4 (2014): 485-508.

[18] Fisher, Lauren, and Cindy Liu. "US Programmatic Ad Spending Forecast." eMarketer Report, September (2016).

[19] Ghose, Anindya, and Sha Yang. "An empirical analysis of search engine advertising: Sponsored search in electronic markets." Management Science 55, no. 10 (2009): 1605-1622.

[20] Ghosh, Arpita, Preston McAfee, Kishore Papineni, and Sergei Vassilvitskii. "Bidding for representative allocations for display advertising." In International Workshop on Internet and Network Economics, pp. 208-219. Springer Berlin Heidelberg, 2009.

[21] Goldfarb, Avi, and Catherine Tucker. "Online display advertising: Targeting and obtrusiveness." Marketing Science 30.3 (2011): 389-404.

[22] Hoban, Paul R., and Randolph E. Bucklin. "Effects of internet display advertising in the purchase funnel: Model-based insights from a randomized field experiment." Journal of Marketing Research 52.3 (2015): 375-393.

[23] Hu, Yu, Jiwoong Shin, and Zhulei Tang. "Incentive Problems in Performance-Based Online Advertising Pricing: Cost per Click vs. Cost per Action." Management Science 62, no. 7 (2015): 2022-2038.

[24] Hummel, Patrick, and R. Preston McAfee. "When Does Improved Targeting Increase Revenue?." Proceedings of the 24th International Conference on World Wide Web. ACM, 2015.

[25] Iyer, Ganesh, David Soberman, and J. Miguel Villas-Boas. "The targeting of advertising." Marketing Science 24, no. 3 (2005): 461-476.

[26] Jerath, Kinshuk, Liye Ma, Young-Hoon Park, and Kannan Srinivasan. "A 'position paradox' in sponsored search auctions." Marketing Science 30, no. 4 (2011): 612-627.

[27] Johnson, Garrett A. "The impact of privacy policy on the auction market for online display advertising." Working Paper (2013).

[28] Katona, Zsolt, and Miklos Sarvary. "The race for sponsored links: Bidding patterns for search advertising." Marketing Science 29, no. 2 (2010): 199-215.

[29] Korula, Nitish, Vahab Mirrokni, and Hamid Nazerzadeh. "Optimizing display advertising markets: Challenges and directions." Available at SSRN 2623163 (2015).

[30] Krishna, Vijay. Auction theory. Academic press, 2009.

[31] Lambrecht, Anja, and Catherine Tucker. "When does retargeting work? Information specificity in online advertising." Journal of Marketing Research 50.5 (2013): 561-576.

[32] Lee, Kuang-Chih, Ali Jalali, and Ali Dasdan. "Real time bid optimization with smooth budget delivery in online advertising." Proceedings of the Seventh International Workshop on Data Mining for Online Advertising. ACM, 2013.

[33] Levin, Jonathan, and Paul Milgrom. "Online advertising: Heterogeneity and conflation in market design." The American Economic Review 100, no. 2 (2010): 603-607.

[34] Liu, Cindy. "US Ad Spending: eMarketer's Updated Estimates and Forecast for 2015-2020." Industry Report, November (2016).

[35] Lu, Shijie, Yi Zhu, and Anthony Dukes. "Position Auctions with Budget Constraints: Implications for Advertisers and Publishers." Marketing Science 34.6 (2015): 897-905.

[36] Lu, Shijie, and Sha Yang, "A Two-Sided Market Analysis of Behaviorally Targeted Display Advertising." Working Paper (2016).

[37] Manchanda, Puneet, Jean-Pierre Dube, Khim Yong Goh, and Pradeep K. Chintagunta. "The effect of banner advertising on internet purchasing." Journal of Marketing Research 43.1 (2006): 98-108.

[38] McAfee, R. Preston, John McMillan, and Michael D. Whinston. "Multiproduct monopoly, commodity bundling, and correlation of values." The Quarterly Journal of Economics 104.2 (1989): 371-383.

[39] Muthukrishnan, S. "Ad exchanges: Research issues." In International Workshop on Internet and Network Economics, pp. 1-12. Springer Berlin Heidelberg, 2009.

[40] Radovanovic, Ana, and William D. Heavlin. "Risk-aware revenue maximization in display advertising." Proceedings of the 21st international conference on World Wide Web. ACM, 2012.

[41] Rafieian, Omid, and Hema Yoganarasimhan. "The Value of Information in Mobile Ad Targeting." Working Paper (2017).

[42] Rutz, Oliver J., and Randolph E. Bucklin. "From generic to branded: A model of spillover in paid search advertising." Journal of Marketing Research 48, no. 1 (2011): 87-102.

[43] Sayedi, Amin, Kinshuk Jerath, and Kannan Srinivasan. "Competitive poaching in sponsored search advertising and its strategic impact on traditional advertising." Marketing Science 33, no. 4 (2014): 586-608.

[44] Shin, Woochoel. "Keyword search advertising and limited budgets." Marketing Science 34, no. 6 (2015): 882-896.

[45] Tucker, Catherine E. "Social networks, personalized advertising, and privacy controls." Journal of Marketing Research 51.5 (2014): 546-562.

[46] Urban, Glen L., Guilherme Liberali, Erin MacDonald, Robert Bordley, and John R. Hauser. "Morphing banner advertising." Marketing Science 33, no. 1 (2013): 27-46.

[47] Varian, Hal R., and Christopher Harris. "The VCG auction in theory and practice." The American Economic Review 104.5 (2014): 442-445.

[48] Wilbur, Kenneth C., and Yi Zhu. "Click fraud." Marketing Science 28, no. 2 (2009): 293-308.

[49] Yang, Sha, and Anindya Ghose. "Analyzing the relationship between organic and sponsored search advertising: Positive, negative, or zero interdependence?." Marketing Science 29, no. 4 (2010): 602-623.

[50] Yao, Song, and Carl F. Mela. "A dynamic model of sponsored search advertising." Marketing Science 30, no. 3 (2011): 447-468.

[51] Yuan, Shuai, Jun Wang, and Xiaoxue Zhao. "Real-time bidding for online advertising: measurement and analysis." Proceedings of the Seventh International Workshop on Data Mining for Online Advertising. ACM, 2013.

[52] Zeithammer, Robert. "The futility of soft floors in auctions" Working Paper (2017).

[53] Zia, Mohammad, and Ram Rao. "Search Advertising: Budget Allocation Across Search Engines" Working Paper (2016).

[54] Zhang, Weinan, Shuai Yuan, and Jun Wang. "Optimal real-time bidding for display advertising." Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining. ACM, 2014.

[55] Zhu, Yi, and Kenneth C. Wilbur. "Hybrid advertising auctions." Marketing Science 30, no. 2 (2011): 249-273.

Proofs

Proof of Proposition 1

Assume that Advertiser 1 buys the impressions in interval $[0, m_1]$ and Advertiser 2 buys the impressions in $[m_2, 1]$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = m_1 \left((1 - t\frac{m_1}{2}) - R \right) + s_1 \left((1 - t\frac{m_1+m_2}{2}) - p \right)$

- $\pi_2 = (1-m_2)\left(1 - t\frac{1-m_2}{2} - R\right) + s_2\left(1 - t(1 - \frac{m_1+m_2}{2}) - p\right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to $m_1 = \frac{2-2R+s_1t}{2t}$ and $m_2 = \frac{2t-2+2R+s_2t}{2t}$. If $R \geq 1 - \frac{t}{4}(2 - s_1 - s_2)$, total sales in RTB, i.e., $m_1 + 1 - m_2$, will be less than or equal to $1 - s_1 - s_2$, so the above equilibrium exists.

Next, we show that the publisher's revenue when setting the reserve price to less than $\underline{R} = 1 - \frac{t}{4}(2 - s_1 - s_2)$ is less than when setting the reserve price to $\underline{R}$. Suppose that the reserve price is set to $R' < \underline{R}$. The price of each impression is the maximum of the second-highest bid and R'. For a consumer located at x, the second-highest bid cannot be more than the second highest valuation, i.e., $\min(1 - tx, 1 - t(1-x))$, which cannot be more than $1 - \frac{t}{2}$ (which is less than or equal to $\underline{R}$, and strictly less than $\underline{R}$ if $s_1 + s_2 > 0$). Since we also have $R' < \underline{R}$, when the reserve price is R', each impression sold in RTB is sold at a lower price than when the reserve price is $\underline{R}$. The total number of impressions sold in RTB when the reserve price is $\underline{R}$ is $m_1 + 1 - m_2 = 1 - s_1 - s_2$. Since the publisher cannot sell more than this many impressions under any conditions, the publisher's revenue when the reserve price is R' is strictly less than the publisher's revenue when the reserve price is $\underline{R}$ for any s_1 and s_2 such that $0 < s_1 + s_2 < 1$.

Proof of Proposition 2

Given Proposition 1, the revenue of the publisher from the auction is given by $R\min(\frac{4-4R-(s_1+s_2)t}{2t}, 1 - s_1 - s_2)$ which is maximized at $R = 1 + \frac{(s_1+s_2-2)t}{4}$.

Proof of Lemma 1

Substituting R and m_i values from Propositions 1 and 2, the profit of Advertiser i is:

$$\pi_i = \frac{1}{32}\left(t\left(-6s_is_j + 5(s_i - 4)s_i + s_j^2 - 4s_j + 4\right) - 32(p-1)s_i\right)$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5t}{16} > 0$ which implies that π_i a convex function of s_i. Therefore, the optimal value of s_i is either 0 or $U - s_j$. Using basic algebra, it is easy to see that if $5Ut + 32 - 20t - 11s_jt < 32p$ then Advertiser i prefers 0 to $U - s_j$.

Proof of Proposition 3

Using the results of Proposition 1, 2 and Lemma 1, the revenue of the publisher is

$$Up + (1-U)R = \frac{1}{32}(t((4-3U)U-16)+32).$$

This function is maximized at $U = \frac{2}{3}$. The optimum price of reservation contracts is, $p = 1 - \frac{25t}{48}$, and the optimum reserve price of RTB auctions is $R = 1 - \frac{t}{3}$. The publisher's total revenue is $1 - \frac{11t}{24}$, Advertiser 1's profit is $\frac{t}{8}$ and Advertiser 2's profit is $\frac{t}{18}$.

Proof of Corollary 1

If the publisher does not sell any impressions in reservation contracts, his revenue would be the same as when $U = 0$. From proof of Proposition 3, we know that the publisher's revenue when $U = 0$ is $\Pi = 1 - \frac{t}{2}$, which is lower than the publisher's revenue when U is optimally set to $\frac{2}{3}$.

If the publisher allocates the first U impressions to reservation contracts, assuming that consumers along the Hotelling line arrive uniformly, the expected value of each advertiser for each impression in reservation contracts would be $1 - \frac{t}{2}$. The remaining $1-U$ impressions will be made available on RTB. Using Proposition 2, it is easy to see that the optimal reserve price in RTB is $1 - \frac{t}{2}$, and all of those impressions will be sold at the reserve price. Therefore, the publisher's total revenue is

$$\Pi = U(1-\frac{t}{2}) + (1-U)(1-\frac{t}{2}) = 1 - \frac{t}{2}$$

which is again smaller than the revenue of the publisher in Proposition 3.

Proof of Proposition 4

Let m_1 and m_2 be such that the first advertiser bids for consumers in $[0, m_1]$ and Advertiser 2 bids for consumers located in $[m_2, 1]$. Suppose that the supply is $1 + D$ where $D = 0$ or $D = d$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 + D - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = (D+1)m_1\left(-\frac{m_1 t}{2} - R + 1\right) + s_1\left(-\frac{1}{2}t(m_1+m_2) - p + 1\right)$
- $\pi_2 = (D+1)(1-m_2)\left(-\frac{1}{2}(1-m_2)t - R + 1\right) + s_2\left(-t\left(\frac{1}{2}(-m_1-m_2)+1\right) - p + 1\right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to

$$m_1 = \frac{1-R}{t} - \frac{s_1}{2(D+1)}$$

and

$$m_2 = \frac{s_2}{2D+2} + \frac{R+t-1}{t}.$$

If $R \geq \frac{-2D(t-2)+t(s_1+s_2-2)+4}{4(D+1)}$, total sales in RTB, i.e., $(m_1 + 1 - m_2)(1 + D)$, will be less than or equal to $1 + D - s_1 - s_2$, so the above equilibrium exists. Setting the reserve price below $\frac{-2D(t-2)+t(s_1+s_2-2)+4}{4(D+1)}$ leads to other equilibria; however, it is dominated because the publisher cannot sell more than $1+D-s_1-s_2$ in RTB, and therefore, cannot make more than $R(1+D-s_1-s_2)$ in RTB.

The revenue of the publisher from the auction is given by $R\min((1+D)(m_1+1-m_2), 1+D-s_1-s_2)$, and is maximized at

$$R = \frac{-2d(t-2) + t(s_1 + s_2 - 2) + 4}{4(d+1)}.$$

Substituting R and m_i from above, the profit of Advertiser i can be written as:

$$\pi_i = \frac{-64(d+1)(p-1)s_i + 5(d+2)s_i^2 t - 2s_i t(3(d+2)s_j + 20(d+1)) + t\left((d+2)s_j^2 - 8(d+1)s_j + 4(d+1)(d+2)\right)}{64(d+1)}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5(d+2)t}{32(d+1)} > 0$ which implies that π_i a convex function of s_i. Therefore, the optimal value of s_i is either 0 or $U - s_j$. Using basic algebra, it is easy to see that if

$$5U(d+2)t < 64(d+1)(p-1) + t(11(d+2)s_j + 40(d+1))$$

then Advertiser i prefers 0 to $U - s_j$. The publisher sets the price such that Advertiser 1 is indifferent between buying U impressions in reservation contracts, and buying 0. Advertiser 1 buys U impressions in reservation contracts, and Advertiser 2 buys 0. The expected revenue of the publisher, hence, can be written as

$$Up + (1 + D - U)R_D = \frac{1}{64}\left(-\frac{3(d+2)tU^2}{d+1} - 16(d+2)(t-2) + 8tU\right)$$

where R_D is the reserve price conditional on supply $1+D$. This function is maximized at $U = \frac{4(d+1)}{3(d+2)}$. The optimum price of reservation contracts is, $p = 1 - \frac{25t}{48}$. Using the equilibrium values of U and

p, we can calculate other parameters as functions of t and d. The optimum reserve price of RTB auctions when supply is $1+d$ is $R = 1 - \frac{(3d+4)t}{6(d+2)}$, and when the supply is 1 is $R = 1 - \frac{(d+4)t}{6(d+2)}$. The publisher's expected total revenue is

$$\frac{1}{64}\left(\frac{16(d+1)t}{3(d+2)} - 16(d+2)(t-2)\right).$$

Advertiser 1's profit is

$$\pi_1 = \frac{1}{16}(d+2)t,$$

and Advertiser 2's profit is

$$\pi_2 = \frac{(d(9d+16)+16)t}{144(d+2)}.$$

Proof of Proposition 5

First, note that advertisers do not bid on impressions to unsusceptible consumers. As such, all those impressions are assigned to reservation contracts.

To find the optimum reserve price, we consider two cases. First, we assume that the reserve price is greater than 1 and calculate the publisher's revenue; then, we assume that it is less than or equal to 1 and calculate the publisher's revenue. Comparing the two revenues allows us to find the optimal reserve price and equilibrium strategies.

Assume that $R > 1$. In this case, none of non-premium impressions will be sold in RTB. Therefore, conditioned on $R > 1$, the optimal reserve price is $R^* = \lambda$. The publisher has to sell all of non-premium impressions in reservation contracts. Because those impressions cannot be allocated efficiently, the maximum revenue that the publisher could achieve is $1 - \frac{t}{2}$, and the publisher could achieve that by setting $U = 1 + Q$ (for a total price of $1 - \frac{t}{2}$). The publisher's revenue in this case is $P\lambda + 1 - \frac{t}{2}$.

Next, we assume that $R \leq 1$. In this case both advertisers will bid on all premium impressions. Since the reserve price does not affect the price of premium impressions (because the second-highest bid for those impressions is always 1), the publisher only takes non-premium impressions into account when calculating the optimal reserve price. Therefore, the optimal reserve price, and the advertisers' strategies for impressions to consumers on the Hotelling line remain the same as those in Proposition 3. We have $R^* = 1 - \frac{t}{3}$, $U = \frac{2}{3} + Q$, and

$$p = (1 - \frac{25t}{48})\frac{\frac{2}{3}}{Q + \frac{2}{3}} = \frac{48 - 25t}{72Q + 48}.$$

The publisher's revenue when setting $R < 1$ is $1 - \frac{11t}{24} + P$. Therefore, the publisher sets $R = \lambda$ if and only if

$$P\lambda + 1 - \frac{t}{2} > 1 - \frac{11t}{24} + P$$

which simplifies to $\lambda > 1 + \frac{t}{24P}$.

Proof of Proposition 6

Assume that the first advertiser buys the interval $[0, m_1]$ and the other buys $[m_2, 1]$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = m_1 \left((1 - \alpha t\frac{m_1}{2}) - R\right) + s_1 \left((1 - t\frac{m_1+m_2}{2}) - p\right)$
- $\pi_2 = (1 - m_2)\left(1 - \alpha t\frac{1-m_2}{2} - R\right) + s_2 \left(1 - t(1 - \frac{m_1+m_2}{2}) - p\right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to $m_1 = -\frac{2R+s_1t-2}{2t\alpha}$ and $m_2 = \frac{2R+s_2t-2}{2t\alpha} + 1$. If $R \geq \frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4)$, total sales in RTB, i.e., $m_1 + 1 - m_2$, will be less than or equal to $1 - s_1 - s_2$, so the above equilibrium exists. Setting the reserve price below $\frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4)$ leads to other equilibria; however, it is dominated because the publisher cannot sell more than $1 - s_1 - s_2$ in RTB, and therefore, cannot make more than $R(1 - s_1 - s_2)$ in RTB.

Given m_1 and m_2, the revenue of the publisher from the auction is given by $R \min(m_1 + 1 - m_2, 1 - s_1 - s_2)$ which is maximized at

$$R = \frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4).$$

Substituting R and m_i values from above, the profit of Advertiser i is:

$$\pi_i = \frac{-4\alpha\left(8(p-1)s_i + t\left(s_i^2 + s_i(2s_j + 3) + (s_j - 1)s_j\right)\right) + t\left(5s_i^2 - 6s_is_j + s_j^2\right) + 4t\alpha^2(s_i + s_j - 1)^2}{32\alpha}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{t(4(\alpha-1)\alpha+5)}{16\alpha} > 0$ which implies that π_i is a convex function of s_i. Therefore, the optimal value of s_i is either 0 or $U - s_j$. Using basic algebra, it is easy to see that if

$$Ut(4(\alpha-1)\alpha+5) < 4\alpha(8p+2t\alpha+3t-8)+s_2t(11-4(\alpha-1)\alpha)$$

then Advertiser i prefers 0 to $U - s_j$.

Using the above expressions, the revenue of the publisher could be written as

$$Up + (1-U)R = \frac{5tU^2 - 4t(3(U-2)U+4)\alpha^2 + 4\alpha(t(U-5)U+8)}{32\alpha}.$$

This function is maximized at $U = \frac{2\alpha}{2\alpha+1}$. Using the optimum value U, we can calculate other parameters of the models as functions of t and α. The optimum price of reservation contracts is,

$$p = \frac{32\alpha+16-t(4\alpha(\alpha+5)+1)}{32\alpha+16},$$

and the optimum reserve price of RTB auctions is $R = \frac{-t\alpha+2\alpha+1}{2\alpha+1}$. The publisher's total revenue is $\frac{\alpha(16-t(2\alpha+9))+8}{16\alpha+8}$, Advertiser 1's profit is $\pi_1 = \frac{t\alpha}{8}$ and Advertiser 2's profit is $\pi_2 = \frac{t\alpha}{2(2\alpha+1)^2}$. Finally, note that the derivative of π_2 with respect to α is $\frac{\partial \pi_2}{\partial \alpha} = \frac{t-2t\alpha}{2(2\alpha+1)^3}$ which is positive if and only if $\alpha < \frac{1}{2}$. In other words, the profit of Advertiser 2 increases as α decreases (i.e., customization becomes stronger) if and only if $\alpha > \frac{1}{2}$.

Proof of Proposition 7

First, note that, using Proposition 3, the expected profit of the firm that does not win the impressions in reservation contract is $(1-U)\frac{1-U}{2}t$. Therefore, when bidding for impressions in reservation contracts, Advertiser i's willingness to pay per impression, say w, is such that his total profit is at least $(1-U)\frac{1-U}{2}t$. In other words, we have

$$U(1 - \frac{Ut}{2} - w) = (1-U)\frac{1-U}{2}t$$

which, by solving for w, gives us

$$w = 1 + t - tU - \frac{t}{2U}.$$

The publisher's total revenue is given by its revenue from RTB, $(1-U)(1-(1-U)t)$, plus the revenue from reservation contracts, Uw. In other words, the publisher's revenue is given by

$$(1-U)(1-(1-U)t)+U(1+t-tU-\frac{t}{2U})$$

which is maximized at $U=\frac{3}{4}$. At $U=\frac{3}{4}$, the per-impression price of reservation contracts simplifies to $w=1-\frac{5t}{12}$, the expected profit of each advertiser becomes $\pi_1=\pi_2=\frac{t}{32}$, and the publisher's total revenue becomes $1-\frac{3t}{8}$.