Pricing in a Duopoly with Observational Learning

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Abstract

We look at the problem of pricing in a duopoly with observational learning. Prior literature shows that when the number of customers in a monopoly is sufficiently large and the monopolist is sufficiently patient, an informational cascade always happens; furthermore, the probability that the cascade is wrong, i.e., virtually all customers receive negative utility ex post, is always positive. We find similar results in a duopoly with static pricing. Interestingly, and in contrast to the previous literature, we show that in a duopoly with dynamic pricing, there are equilibria in which no cascade happens. More importantly, we show that a wrong cascade cannot happen in equilibrium. Our results could explain why, in practice, cascades, and specifically wrong cascades, are not as common as prior literature has predicted.

1 Introduction

Observational learning is learning the fundamental value of an object, e.g., product quality, by observing other decision-makers' actions, e.g., purchase decisions. It happens when customers can observe information regarding best selling, popular and trending products. Customers who engage in observational learning infer product qualities from other customers' choices; as such, popular products are perceived to be of high quality. For example, book buyers pursue bestsellers, restaurants with a long waiting list are often perceived to be of high quality, and Internet surfers want to watch trending videos or read trending articles.

Prior research shows that observational learning leads to *informational cascades* where virtually all customers make the same decision (Bikhchandani et al., 1992; Banerjee, 1992). Informational cascades occur when it is optimal for a customer, having observed the actions of those ahead of him, to follow the behavior of the preceding customers without regard to his own (incomplete and

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noisy) information. For example, a customer may choose to buy a best-seller book rather than a book with mediocre sales even if he has a higher prior expectation for the latter. Interestingly, informational cascades can sometimes be wrong (Bikhchandani et al., 1992; Banerjee, 1992; Welch 1992), i.e., customers' ex post utility from purchasing the product may be negative.

Previous literature on observational learning has primarily focused on customers' decision making process, and, with the exception of a few papers, has paid little attention to how observational learning affects firms' strategies such as pricing. Welch (1992), in the context of a sequential IPO, shows that observational learning can lead to a cascade among the investors; as such, in presence of observational learning, the issuer lowers the price to avoid a failure. The price in Welch (1992) is static. Bose et al. (2008) consider observational learning in a monopoly with dynamic pricing. In both papers, if the seller is patient enough and the number of customers is sufficiently large, a cascade always happens; furthermore, the cascade is wrong with a positive probability.

In this paper, we extend this literature to pricing decisions in a *duopoly*. In a duopoly with static pricing, our results are similar to those of a monopoly (Welch 1992); the presence of observational learning makes the demand more elastic and leads to lower equilibrium prices. Furthermore, a cascade always happens, and the probability that a wrong cascade emerges is positive. Interestingly, in a duopoly with dynamic pricing, we obtain very different results. First, we show that there are equilibria in which no cascade happens. More importantly, we find that in a duopoly with dynamic pricing, a wrong cascade cannot happen anymore. Our results could explain why, in practice, cascades, and specifically wrong cascades, are not as common as theory has predicted. Furthermore, they could explain why customer herding behavior is more commonly observed for product categories with static prices such as movies, music and books, compared to those with dynamic prices.

The rest of this paper is structured as follows. First, we review the related literature. In Section 2, we present the model. In Section 3, we present the results in a model with static pricing, and in Section 4, we discuss dynamic pricing. We conclude the paper and discuss the managerial implications in Section 5. All proofs are relegated to the appendix.

Related Literature

Much of the earlier work on observational learning focuses on showing the existence of informational cascades in certain domains (e.g. Bikhchandani et al. 1998), or measuring its effects by separating it from other confounding factors (e.g., Chen et al. 2010). Zhang (2010) studies how observational learning affects kidney transplant market. The author shows that observational learning and information sharing shape consumer choices in different ways. Chen et al. (2010) compare word-of-mouth to observational learning by studying their effects on sales and their interactions. In the context of microloan markets, Zhang and Liu (2012) show that observational learning leads to rational herding, and discuss how the herding behavior relates to observable characteristics of a product. Carare (2012) uses Apple's App Store data to show that consumers' willingness to pay for an app ranked as best seller is $4.5 higher than the same unranked app. Lee et al. (2015) show that informational cascades can affect a consumer's decision on how to rate a product post-purchase. Tucker et al. (2013) show that, because of observational learning, the number of days that a home has been on the market negatively affects buyers' willingness to pay. Ameri et al. (2016) study the effects of observational learning versus word-of-mouth on an anime platform with endogenous network formation. Cai et al. (2009) measure the effect of observational learning on consumers' choices using a natural field experiment conducted in a restaurant dining setting. They show that when customers are given ranking information of the five most popular dishes, the demand for those dishes increases by 13 to 20 percent, and that dining satisfaction also increases. Salganik et al. (2006) show that observational learning leads to informational cascades in an artificial music market. They use the results to explain why despite the fact that hit songs, books and movies are many times more successful than average, experts routinely fail to predict which products will succeed.

Another stream of research within this literature investigates how observational learning is affected by other parameters in the market. Tucker and Zhang (2011), using a field experiment, show that observational learning may benefit niche products with narrow appeal more than broad-appeal products. Zhang et al. (2015) study how network structures of friends versus strangers affect consumers' product choice and informational cascades. Hendricks et al. (2012) study observational learning when there is a search cost, and show that a wrong cascade, where a high quality product gets virtually zero sales, could happen with a positive probability. Our paper is different from the above papers as instead of focusing on customers' side, we explore how existence of observational learning affects the sellers' decisions. In particular, we look at how product prices are affected by existence of observational learning.

The question of how observational learning affects marketing decisions has been raised in the previous literature. Zhang (2010) makes an important distinction between observational learning, where customers only observe the actions and do not know the reason for other customers' decisions, and information sharing, where customers know the reasons for other customers' actions. Zhang (2010) suggests that "optimal marketing strategies should take into account how consumers learn from others." Godes (2016) and Jiang and Yang (2016) partially address this problem by studying how information sharing affects firms' product quality and pricing decisions. Our results complement their findings by analyzing how observational learning affects firms' pricing decisions.

Milkos-Thal and Zhang (2013) study how observational learning affects a firm's "marketing efforts." The authors show that a seller can benefit from de-marketing its product by toning down its marketing efforts. The managerial question addressed by Miklos-Thal and Zhang (2013) is similar to ours as both papers investigate the effects of observational learning on sellers' marketing decisions. However, marketing efforts in Miklos-Thal and Zhang (2013) correspond to promotional activities, timing of release, location, and ease of access. These factors influence the fraction of early customers who consider buying the product, and are different from price in their model. In contrast, the focus of our paper is to explore how observational learning affects pricing decisions. We should also note that the setting in Miklos-Thal and Zhang (2013) is a monopoly, whereas ours is a duopoly.

A few papers in the literature have studied the effects of observational learning on pricing decisions. Welch (1992) shows that when IPO shares are sold sequentially, observational learning can lead to informational cascade among the investors. As such, Welch (1992) shows that demand can be so elastic that even risk-neutral issuers underprice to avoid failure. In Welch (1992), price is static and the seller is a monopolist. While we extend the result of Welch (1992) to a duopoly setting, we also analyze a duopoly market with dynamic pricing.

Bose et al. (2008) consider observational learning in a monopoly with dynamic pricing. The main difference between their model and ours is that their setting is a monopoly whereas ours is a duopoly. In their model, an informational cascade always happens. However, we show that in a duopoly setting, under certain conditions, firms change their prices such that a cascade does not happen. Furthermore, contrary to the monopoly setting of Bose et al. (2008), we show that a wrong cascade, wherein customers regret their decisions, cannot emerge in equilibrium in a duopoly.

Garcia and Shelegia (2015) use a model of consumer search to show that observational learning increases price competition. In their model, consumers always start the search from the lowest priced firm, price cannot be used for signaling, and consumers only observe the decisions of a limited number of predecessors. Caminal and Vives (1996) examine a duopoly market where customers can infer product quality from market share, and where sellers secretly cut price to compete for market share. In contrast, we allow customers to fully observe prices, which distinguishes our model from the signal jamming mechanism that underlies Caminal and Vives (1996).

2 Model

The market consists of infinitely many homogenous customers and two firms, Firm 1 and Firm 2. Customers arrive sequentially with Customer t arriving at time t. Firms have the same discount factor δ which we assume is sufficiently large (close to 1). Firm i, $i \in \{L, H\}$, produces one product which we refer to as Product i. Firm i, decides the price $p_{i,t}$ of its product for Customer t before the arrival of the customer. One of the firms is high-type and the other is low-type. The quality of the product of the high-type firm, which we sometimes refer to as Firm H, is q_H, and the product quality of the low-type firm, which we refer to as Firm L, is q_L, where $q_H > q_L$. Customers know the values of q_L and q_H, but do not know the type of each firm.

To model observational learning, we use to the original model of Bikhchandani et al. (1992).[1] Customers arrive sequentially in an exogenous order, and cannot postpone their purchase decisions. Each Customer t has a unit demand, and buys the product that maximizes his expected utility. The expected utility of Customer t from purchasing the product of Firm i, is $\bar{q_{i,t}} - p_{i,t}$, where $\bar{q_{i,t}}$ is the inferred expected quality of Product i, and $p_{i,t}$ is the price at time t. If $\bar{q_{i,t}} - p_{i,t} < 0$ for both products, $i \in \{1, 2\}$, the customer does not purchase anything. Customers do not know the firms' types. However, each Customer t gets an independent noisy private signal $s_t \in \{1, 2\}$ before

[1] Our model is also similar to a special case of Banerjee (1992).

purchasing the product. This signal indicates which of the two products has higher quality. We assume that

$$Pr(s_t = H) = \rho. \tag{1}$$

In other words, the probability that a customer's signal indicates the high-type firm is $\rho > \frac{1}{2}$. Note that as ρ increases, the noise in the signal decreases, and if $\rho = 1$, each customer can identify the high-type firm with no uncertainty. While the realization of each signal s_t is private information to Customer t, the distribution in Equation (1) is common knowledge.

To study the effect of observational learning on the firms' strategies, we compare the situation in which customers cannot observe prior customers' decisions (e.g., because they cannot observe popularity information), as a benchmark, to the situation in which they know the decisions of the previous customers. Each customer gets his private signal, and observes the purchase decisions of all customers who arrived before him as well as all previous prices. The customer calculates the expected quality of each product and decides which product to buy, if any. If a customer is indifferent between the two products, he breaks the tie by purchasing the product that is indicated by his own private signal (Banerjee 1992; Bikhchandani et al. 1992).

The timing of the game is as follows. At time 0, the nature randomly assigns qualities q_H and q_L to q_1 and q_2. Firms observe each others' q_i's at time 0. At time t, firms simultaneously choose their prices $p_{1,t}$ and $p_{2,t}$. Then, Customer t arrives, and observes his own private noisy signal. When observational learning is possible, the customer also observes previous customers' decisions and all previous prices. The customer makes a rational inference about the quality of the products, and decides which product to purchase (if any). Firms observe Customer t's decision before moving to round $t + 1$.

Since we are interested in the long-term effect of observational learning on firms' profits, we assume that $\delta \to 1$ in our analysis. The total profit of Firm i is

$$\pi_i = \lim_{\delta \to 1} (1 - \delta) \sum_{t=1}^{\infty} \delta^t \pi_{i,t}$$

where $\pi_{i,t}$, the profit of Firm i from Customer t, is $\pi_{i,t} = p_{i,t}$ if the customer purchases the product from Firm i, and $\pi_{i,t} = 0$ otherwise.

We consider three different scenarios in our analysis: exogenous prices, endogenous static prices, and endogenous dynamic prices. To denote equilibrium prices, we use $p_{i,t}^{AB}$ where $i \in \{1, 2\}$ indicates the index of the firm, $A \in \{X, S, D\}$ denotes whether price is exogenous (X), endogenous static (S), or endogenous dynamic (D), and $B \in \{B, O\}$ denotes whether the analysis is for benchmark case (B) or for when observational learning is possible (O). For example, $p_{1,t}^{SB}$ is the equilibrium price set by Firm 1 for Customer t when price is static and observational learning is not possible (benchmark). We also use subscripts H and L to refer to high-type (high-quality) and low-type (low-quality) firms, respectively. For example, $p_{H,t}^{DO}$ is the equilibrium price that the high-quality firm sets for Customer t when price is dynamic and observational learning is possible. We solve for pure-strategy sub-game perfect equilibria of the game.

3 Static Pricing

In this section, we consider the situation in which the prices are static, i.e., the firms cannot change their prices after consumers start making purchase decisions. But before solving for firms' equilibrium pricing decisions, we discuss consumers' decision when prices are exogenously given.

3.1 Exogenous Price

We begin the analysis by assuming that both firms have the same exogenous price p, i.e., $p_{i,t} = p$ for all $i \in \{1, 2\}$ and $t \geq 0$. We assume that p is sufficiently low so that all customers purchase a product. Note that firms do not make any decisions when prices are exogenous; we simply provide the analysis to, first, build intuition for the effect of observational learning on customers' behavior. Second, this is a sub-game of the case with static endogenous prices that we will analyze in the next section; in other words, the results of this section will be used for calculating firms' equilibrium strategies in the next section.

Benchmark

In the benchmark model, each customer only observes his own private signal before deciding which product to purchase. It is easy to see that in this case, a customer's optimal decision is to purchase the product indicated by his signal. A fraction ρ of the customers purchase the product from Firm H and a fraction $1 - \rho$ purchase from Firm L. The expected revenue of Firm H is $\pi_H^{XB} = \rho p$

and, the expected revenue of Firm L is $\pi_L^{XB} = (1-\rho)p$.

Observational Learning

Following the previous literature on observational learning, we assume that customers arrive sequentially in an exogenous order. Each customer observes the decisions made by the previous customers, gets his private signal, and using all that information, decides which product to purchase.

Lemma 1 *If a customer has inferred/observed k signals, he purchases the product indicated by the majority of the signals. In case of a tie, he purchases the product indicated by his own private signal. (Bikhchandani et al., 1992)*

The First Customer: The first customer cannot observe the decisions of any other customers. Therefore, he can only use his private signal to decide which product to purchase. Using Lemma 1, we can see that the first customer purchases the product that his private signal indicates.

The Second Customer: The second customer can observe the first customer's purchase decision. Furthermore, he can infer the first customer's private signal from his decision. Therefore, the second customer can base his decision on two signals. If both of these signals are the same, using the same argument as in Lemma 1, he purchases the product indicated by those signals. However, if the two signals are different, the customer will be indifferent between the two products. In this case, the customer breaks the tie in favor of his own signal. Therefore, the second customer always purchases the product indicated by his own signal. In other words, the purchase decision of the first customer does not affect the decision of the second customer.

The Third Customer: The third customer observes the decisions of the first and the second customers. Furthermore, he can infer both the first and the second customers' signals based on their purchase decisions (each of the first and the second customers purchases the product indicated by their own private signals). Therefore, the third customer can base his decision on three signals. Using Lemma 1, it is easy to see that the third customer's optimal decision is to purchase the product that is indicated by the majority of the three signals.

If the first and the second customers got the same signal (i.e., made the same decision), the third customer follows their decision, regardless of his own private signal. If the signals of the first and the second customers are different, the third customer purchases the product indicated by his own signal.

The Fourth Customer and Beyond: Note that if the first and the second customers get the same signal, and make the same decision, the third customer also makes the same decision, regardless of his own private signal. The fourth customer cannot infer the private signal of the third customer, however, similar to the third customer, he also follows the decision of the previous customers, regardless of his own private signal. In other words, if the first and the second customers make the same decision, all other customers will follow that decision. As we show in the following lemma, if at any point in time, the difference between the number of customers who purchased each product becomes at least 2, a cascade happens.

Lemma 2 *A cascade happens if and only if at any point in time, the difference between the number of customers who purchased from each firm becomes* 2*. (Bikhchandani et al., 1992)*

Note that if the number of customers in the market is sufficiently large, a cascade happens with probability 1. Furthermore, it happens early enough such that the firm's profit only depends on the type of cascade.

Lemma 3 *A cascade happens with probability* 1*. With probability* $\frac{\rho^2}{\rho^2+(1-\rho)^2}$ *Firm* H *wins in the cascade, the profit of Firm* H *becomes* p *and the profit of Firm* L *becomes* 0*. With probability* $\frac{(1-\rho)^2}{\rho^2+(1-\rho)^2}$ *Firm* L *wins in the cascade, the profit of Firm* H *becomes* 0 *and the profit of Firm* L *becomes* p*.*

Lemma 3 shows that, in expectation, a cascade happens early enough such that the firms' profits only depend on who wins the cascade, and not on what happens before the cascade. The expected profits of the firms are as follows.

$$\pi_H^{XO} = \frac{\rho^2 p}{\rho^2 + (1-\rho)^2}$$

$$\pi_L^{XO} = \frac{(1-\rho)^2 p}{\rho^2 + (1-\rho)^2}.$$

Note that, since $\rho > \frac{1}{2}$, the expected profit of the low-type (high-type) firm is lower (higher) when observational learning exists than when it does not; in other words, $\pi_L^{XO} < \pi_L^{XB}$ and $\pi_H^{XO} > \pi_H^{XB}$.

3.2 Endogenous Pricing

In this section, we consider the situation in which the firms choose their prices endogenously, but prices are static. In other words, firms cannot change their prices after they set their prices in period 0, i.e., $p_{i,t} = p_{i,0}$ for all $i \in \{1, 2\}$ and $t \geq 0$. Customers observe the prices, but do not know the types. However, they may be able to infer the types from the prices. In other words, price can potentially be used to signal quality.

In a separating equilibrium, customers will infer the firms' types, and thus their qualities, from the observed prices. Since customers are homogenous, given the qualities and prices, they will all buy from the same producer—the producer that offers higher utility. This suggests that, if a separating equilibrium exists, one firm gets zero sales. The losing firm benefits from deviating by either lowering its price or mimicking the other firm's price. Therefore, as we show in Lemma 4, a separating equilibrium cannot exist.

Lemma 4 *When prices are static, a separating equilibrium (where the two firms set different prices) does not exist.*

Lemma 4 shows that a separating equilibrium where the two firms set two different prices cannot exist. Next, we show that a pooling equilibrium always exists. Among all pooling equilibria of the game, we select the one with the highest price since it is the "payoff dominant" (also known as "Pareto superior") equilibrium as it is preferred by both firms to any other equilibrium. Customers' belief is that any out-of-equilibrium deviation is made by a low-type firm. A necessary condition for a given price to be a pooling equilibrium is that the low-type firm cannot benefit from deviating to a sufficiently lower price such that all customers, despite knowing the type, buy from the low-type firm.

In a pooling equilibrium, customers cannot infer the firms' types from the price. Therefore, similar to the case with exogenous price, they use their private signals, and other customers' decisions when observational learning is possible, to choose between the two products.

Benchmark

When observational learning is not possible, using the benchmark case of Section 3.1, we know that the expected revenue of Firm i is ρp, where p is the equilibrium price in the pooling equilibrium. If the low-type firm wants to deviate to a lower price and win all the customers, for a possibly higher revenue, it has to set the price to at most $q_L - q_H + p$, otherwise, the customers will not buy from him. For this deviation to be unprofitable, we need the profit after deviation to be less than or equal to the profit before deviation, i.e., $q_L - q_H + p \leq (1-\rho)p$, which reduces to $p \leq \frac{q_H - q_L}{\rho}$. Furthermore, the price p should be such that a customer's expected utility from purchasing the product is non-negative. Therefore, $p \leq \rho q_H + (1-\rho)q_L$. In Lemma 5, we show that these conditions are also sufficient for a pooling equilibrium to exist.

Lemma 5 *When observational learning is not possible, in the unique payoff-dominant equilibrium, both firms set their prices to* $p_1^{SB} = p_2^{SB} = \min(\frac{q_H - q_L}{\rho}, \rho q_H + (1-\rho)q_L)$.

Lemma 5 presents the unique payoff-dominant equilibrium of the game when observational learning does not exist.

Observational Learning

When observational learning is possible, using Lemma 3, we know that the expected revenue of Firm i is $\frac{\rho^2 p}{\rho^2 + (1-\rho)^2}$ where p is the equilibrium price in the pooling equilibrium. If the low-type firm wants to deviate to a lower price and win all the customers, for a possibly higher revenue, it has to set the price to at most $q_L - q_H + p$, otherwise, the customers will not buy from him. For this deviation to be unprofitable, we need $q_L - q_H + p \leq \frac{(1-\rho)^2 p}{\rho^2 + (1-\rho)^2}$, which reduces to $p \leq \frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}$. Furthermore, we need $p \leq \rho q_H + (1-\rho)q_L$, otherwise, the first customer does not buy the product, and a cascade in which no one buys the product happens. In Lemma 6, we show that these conditions are also sufficient for a pooling equilibrium at price p.

Lemma 6 *When observational learning is possible, in the unique payoff-dominant equilibrium, both firms set their prices to* $p_H^{SO} = p_L^{SO} = \min(\frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}, \rho q_H + (1-\rho)q_L)$.

Lemma 6 characterizes the payoff-dominant equilibrium price of the game when observational learning exists. By comparing the price to the result of Lemma 5, we show in Proposition 1 that when price is static, observational learning weakly lowers the equilibrium price.

Proposition 1 *The price that the firms set when observational learning exists is less than or equal to the price that they set when observational learning does not exist; i.e.,* $p_i^{SO} \leq p_i^{SB}$ *for* $i \in \{1, 2\}$.

Proposition 1 shows that observational learning weakly lowers the equilibrium price. Intuitively, observational learning lowers the expected market share of the low-type firm. Therefore, in presence of observational learning, the low-type firm has more incentive to deviate to a lower its price to get the whole market. When the pooling price p decreases, the low-type firm's incentive to deviate to a lower price decreases. Therefore, the equilibrium pooling price when observational learning exists is lower than when it does not. This result extends the findings of Welch (1992) from a monopoly to a duopoly.

4 Dynamic Pricing

In this section, we assume that the firms can change their prices as customers make purchase decisions. Before any customer makes any purchase decision, firms set their initial prices. After each customer purchases the product, both firms observe the customer's decision and can change their prices. Each customer observes the new price before making a decision. When observational learning exists, each customer also observes all previous customers' decisions and previous prices.

In general, customers can have beliefs about any price strategy that a firm can use, without regard to their own signals or inferences of other customers' signals. For example, customers may believe that any firm that ever chooses a price different from a given price x is a low-type firm. This belief could induce an equilibrium wherein both firms set their prices to x, which in turn makes the belief consistent with the firms' equilibrium behavior. We exclude such equilibria by assuming that the customers only use their private signals and the decisions of the previous customers for inferring the firms' types; in other words, we assume that customers have the same ex ante beliefs for all price strategies.[2]

Let $p_{i,t}$ be the price of Firm i at time t. Suppose that the first $t-1$ customers have purchased a product, and consider the decision of customer t. First, note that the signal of a previous customer

[2]We do not need this assumption for Proposition 2.

cannot necessarily be inferred from his decision. Intuitively, if the price of Firm i is very low, and a customer buys from that firm, future customers cannot infer the signal of this customer from his decision. Let $n_{i,t}$ be the number of customers in $\{1, \ldots, t-1\}$ whose signals *can* be inferred, given their decisions and previous prices, and whose signals indicate Firm i. In Lemma 7, we characterize the decision of customer t as a function of $p_{i,t}$ and $n_{i,t}$.

Lemma 7 *Let indices $i, j \in \{1, 2\}$ be such that $n_{i,t} \geq n_{j,t}$. Also, assume that prices $p_{i,t}$ and $p_{j,t}$ are sufficiently small such that customer t buys a product. We have*

- *If*
$$p_{i,t} - p_{j,t} < \frac{\rho^{n_{i,t}-n_{j,t}-1}}{\rho^{n_{i,t}-n_{j,t}-1} + (1-\rho)^{n_{i,t}-n_{j,t}-1}}(q_H - q_L)$$
then customer t purchases the product of Firm i regardless of his own private signal.

- *If*
$$p_{i,t} - p_{j,t} > \frac{\rho^{n_{i,t}-n_{j,t}+1}}{\rho^{n_{i,t}-n_{j,t}+1} + (1-\rho)^{n_{i,t}-n_{j,t}+1}}(q_H - q_L)$$
then customer t purchases the product of Firm j regardless of his own signal.

- *If*
$$\frac{\rho^{n_{i,t}-n_{j,t}-1}}{\rho^{n_{i,t}-n_{j,t}-1} + (1-\rho)^{n_{i,t}-n_{j,t}-1}}(q_H - q_L) \leq p_{i,t} - p_{j,t} \leq \frac{\rho^{n_{i,t}-n_{j,t}+1}}{\rho^{n_{i,t}-n_{j,t}+1} + (1-\rho)^{n_{i,t}-n_{j,t}+1}}(q_H - q_L)$$
then customer t purchases the product that is indicated by his own private signal.

Lemma 7 shows how prices $p_{1,t}$ and $p_{2,t}$ affect the purchase decision of customer t. The lemma has two important implications. First, it shows that in our dynamic setting, the state of the game only depends on $n_{i,t} - n_{j,t}$. In other words, the continuation game for two different price histories that lead to the same $n_{i,t} - n_{j,t}$ is the same. We extensively use this property in the proofs of the following propositions.

Second, Lemma 7 shows that any firm can break a cascade at any point in time. A cascade happens when customer t, and all following customers, purchase from Firm i regardless of their own private signals. Since the firms can anticipate a cascade, using Lemma 7, a firm that is losing in a cascade can always break the cascade by lowering its price. In particular, if at any point in time, Firm j is about to lose in a cascade, i.e., $n_{i,t}$, $n_{j,t}$, $p_{i,t}$ and $p_{j,t}$ are such that

$$p_{i,t} - p_{j,t} < \frac{\rho^{n_{i,t}-n_{j,t}-1}}{\rho^{n_{i,t}-n_{j,t}-1} + (1-\rho)^{n_{i,t}-n_{j,t}-1}}(q_H - q_L)$$

the firm can lower its price to

$$p_{j,t} = p_{i,t} - \frac{\rho^{n_{i,t}-n_{j,t}-1}}{\rho^{n_{i,t}-n_{j,t}-1} + (1-\rho)^{n_{i,t}-n_{j,t}-1}}(q_H - q_L)$$

to break the cascade, so that customer t purchases the product indicated by his own private signal. It is worth noting that by doing so, if customer t's signal happens to indicate Firm j, not only Firm j could benefit from the purchase of customer t, it also indirectly benefits from the signal of customer t being revealed to the future customers (i.e., improving the state of the game).

4.1 Equilibria with No Cascades

In this section, we show that the duopoly game with dynamic pricing has an equilibrium with no cascade. Note that this is distinct from both a monopoly with dynamic pricing, and a duopoly with static pricing. In other words, we need both competition and dynamic pricing in order to get this result.

Using Bayes' rule, the expected quality of customer t for Firm i, before the customer observes his own signal, is

$$q_L + \frac{\rho^{n_{i,t}-n_{j,t}}(q_H - q_L)}{(1-\rho)^{n_{i,t}-n_{j,t}} + \rho^{n_{i,t}-n_{j,t}}},$$

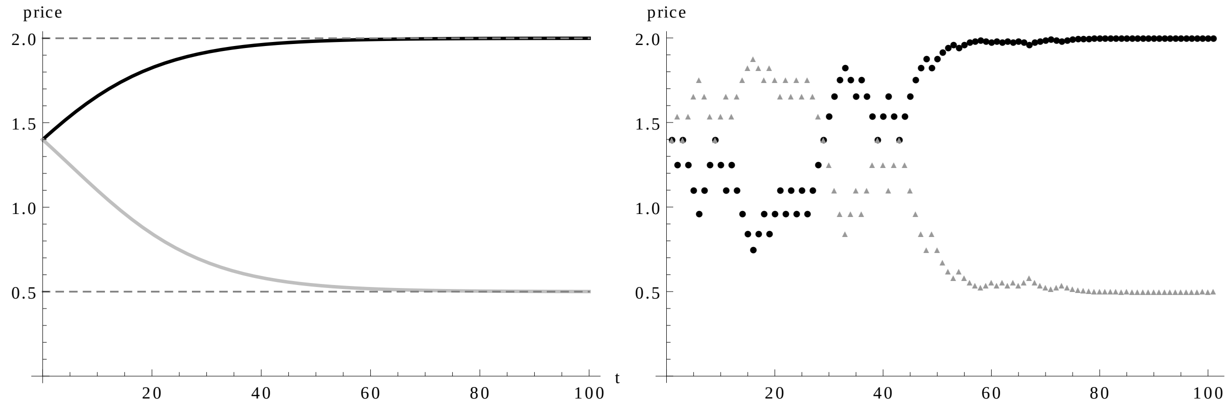
and if customer t's signal indicates Firm i, the expected quality increases to

$$q_L + \frac{\rho^{n_{i,t}-n_{j,t}+1}(q_H - q_L)}{(1-\rho)^{n_{i,t}-n_{j,t}+1} + \rho^{n_{i,t}-n_{j,t}+1}}.$$

Now, suppose that both firms set their prices to this value, i.e.,

$$p_{i,t} = q_L + \frac{\rho^{n_{i,t}-n_{j,t}+1}(q_H - q_L)}{(1-\rho)^{n_{i,t}-n_{j,t}+1} + \rho^{n_{i,t}-n_{j,t}+1}}.$$

At this price, customer t, before he observes his own signal, is indifferent between from Firm 1 and Firm 2, but has negative expected utility for both, i.e., he strictly prefers not to buy. However, after



(a) Expected trajectory of equilibrium prices

(b) Equilibrium prices using randomly generated customer signals

Figure 1: An equilibrium with no cascade: Figure (a) shows the expected trajectory of equilibrium prices, and Figure (b) shows an example of actual equilibrium prices when customers' private signals are randomly generated. In both figures, the high-type firm's prices are shown in black, and the low-type firm's prices are shown in gray. The parameters are set to $\rho = 0.6$, $q_L = \frac{1}{2}$, and $p_H = 2$.

the customer observes his private signal, depending on the signal, he prefers one firm to the other, and becomes indifferent between buying from his preferred firm and not buying.[3] In Proposition 2, we show that this pricing strategy is indeed an equilibrium.

Proposition 2 *There exists a sub-game perfect equilibrium of the game with no cascade.*

Proposition 2 shows that dynamic pricing allows the firms to coordinate on prices, and avoid the high demand elasticity that is otherwise created by informational cascades. In equilibrium, both firms set prices that are reflective of the customers' past purchase decisions. In the long-run, i.e., as t grows, $n_{H,t} - n_{L,t}$ grows unboundedly. As a result, the price of the high-type firm converges to q_H and the price of the low-type firm converges to q_L.

In this equilibrium, customers' uncertainty about the firms' types becomes smaller as t increases; however, since the firms adjust their prices, customers' expected utility also shrinks to zero. It is worth noting that in this equilibrium, while some customers have negative utility ex post, this negative amount converges to zero as t increases. In other words, even though the total customer surplus converges to zero, customer *regret* also converges to zero. This is an appealing property of this equilibrium that does not exists in other settings (e.g., monopoly or static pricing) of observational learning where wrong cascades can happen.

[3]We break the tie by assuming that when a customer is indifferent between buying and not buying he buys; otherwise, the firms lower their prices by a sufficiently small $\varepsilon > 0$.

Figure 1 shows an example of equilibrium prices for $\rho = 0.6$, $p_L = \frac{1}{2}$, and $p_H = 2$. As we see in Figrue 1(a), the expected price of each firm converges to its quality as t increases. Figure 1(b) shows that, while due to the noisy signals, the low-type firm may initially set a higher price than the high-type firm, as t increases, the prices converge to actual qualities. Customers' signals in Figure 1(b) could have led to a wrong cascade if the prices were not dynamic.

The equilibrium discussed in this section is not the only equilibrium with no cascades. However, it has two interesting properties. First, it is a symmetric equilibrium; in other words, firms' strategies only depend on past customers' decisions, and not on the firms' types. Second, this is a Pareto optimal equilibrium, meaning that there is no other equilibrium in which both firms have higher profits.

Before wrapping up this section, we should note that, while folk theorems intuitively suggest that firms should be able to cooperate to soften competition when they are sufficiently patient (i.e., $\delta \to 1$), the result of Proposition 2 cannot be directly derived from the variant of folk theorem for dynamic games (Dutta 1995). Basically, the requirements in Dutta (1995) for admissible punishments are too strong to allow us to use it in our setting. As such, we have provided a complete analysis of the equilibrium.

4.2 Wrong Cascades

So far we have shown the existence of equilibria with no cascades, but we have not discussed whether equilibria with (correct or wrong) cascades exist or not. In this section, we show that in a duopoly with dynamic pricing, equilibria with wrong cascades do not exist. As in Section 4.1, this result is unique to dynamic settings with competition; in other words, in a monopoly with dynamic pricing, or in a duopoly with static pricing, wrong cascades can happen in equilibrium.

From Lemma 7 we know that a firm can break a cascade at any point in time by sufficiently lowering its price. Therefore, a cascade could emerge in equilibrium only if the firm that loses, say Firm j, does not want to break the cascade. Note that breaking the cascade has two potential benefits for Firm j. First, the firm could benefit from selling its product to the current customer with a positive probability. Second, breaking the cascade could reveal the private signal of the current customer to all future customers, potentially improving the future expected revenue of Firm j. Therefore, for Firm j not to benefit from breaking the cascade, both of these effects have to be sufficiently small. In particular, Firm j must have to set a *negative* price[4] in order to break the cascade, otherwise, since it has 0 profit when losing in a cascade, it always benefits from breaking it.

If the expected revenue of future customers for Firm j is sufficiently large, the firm benefits from breaking the cascade even at a negative price. Therefore, for a cascade to exist, not only the price for Firm j to break it should be negative, the firm's expected profit (conditional on breaking the cascade) from future customers should also be sufficiently small. The latter condition is particularly important because it creates a distinction between a high-type firm and a low-type firm: since $\rho > \frac{1}{2}$, there could be a situation in which a low-type firm does not benefit from breaking a cascade because its expected revenue from future customers is not sufficiently large, whereas a high-type firm benefits from breaking the cascade. In Proposition 3, we show that the high-type firm always benefits from breaking a cascade in which it is losing; however, as we later show in Section 4.3, this is not always the case for the low-type firm.

Proposition 3 *No sub-game perfect equilibrium of the game has a wrong cascade.*

Proposition 3 shows that the high-type firm always benefits from breaking a cascade in which it is losing. Intuitively, the high-type firm breaks the cascade by setting a sufficiently low (possibly negative) price such that the next customer purchases the product indicated by his own signal. The immediate effect of this on the high-type firm's profit could be negative. However, since $\rho > \frac{1}{2}$, the value of $n_{H,t} - n_{L,t}$ increases in expectation; this has a positive long-term effect on the high-type firm's profit. In fact, if the high-type firm does this for a sufficiently large (but finite) number of customers, $n_{H,t} - n_{L,t}$ becomes sufficiently large. Since $\delta \to 1$, the negative short-term impact of breaking the cascade for a finite number of customers converges to zero, whereas the long-term impact of increasing $n_{H,t} - n_{L,t}$ remains. As such, the high-type firm always benefits from breaking a cascade in which it is losing.

4.3 Correct Cascades

So far, we have shown that a duopoly with dynamic pricing has equilibria with no cascades, and does not have any equilibria with wrong cascades. It remains to discuss the possibility of equilibria

[4] In a model with non-zero marginal cost of production, this translates into a price below the marginal cost of production.

with correct cascades, i.e., cascades in which almost all customers purchase from the high-type firm. In this section, we show that equilibria with correct cascades exist.

While there are many equilibria with correct cascades, in the equilibrium that we discuss, the low-type firm's price is always 0, i.e., $p_{L,t} = 0$ for all t. The price of the high-type firm has two phases. In the first phase, the high-type firm sets its price such that each customer purchases the product indicated by his private signal. This reveals the private signal of each customer to all future customers, and increases the expected value of $n_{H,t} - n_{L,t}$. The second phase begins when the value of $n_{H,t} - n_{L,t}$ becomes sufficiently large. In this phase, the high-type firm "settles" on a sufficiently low price such that all customers purchase from the high-type firm regardless of their private signals (the price in the second phase remains constant). Since $n_{H,t} - n_{L,t}$ could be made sufficiently large in the first phase, the price that the high-type firm settles on in the second phase could be made arbitrarily close to $q_H - q_L$. A correct cascade occurs in the second phase.

In the first phase, assuming that the price of the low-type firm is 0, the high-type firm chooses the lowest possible price at which the current customer purchases the product indicated by his own signal. Using Lemma 7, this price when $n_{H,t} < n_{L,t}$ is given by

$$p_{H,t} = -\frac{\rho^{n_{L,t}-n_{H,t}+1}(q_H - q_L)}{(1-\rho)^{n_{L,t}-n_{H,t}+1} + \rho^{n_{L,t}-n_{H,t}+1}}$$

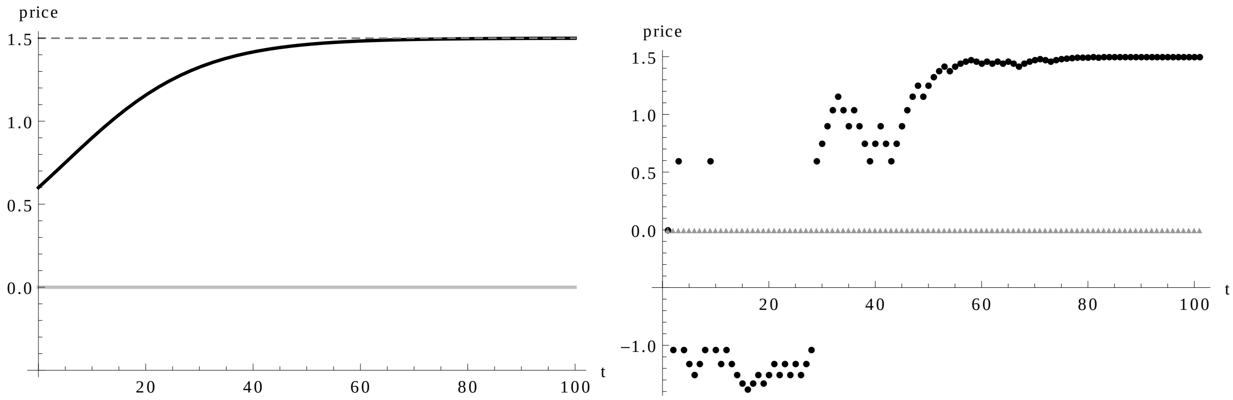
and when $n_{H,t} \geq n_{L,t}$ is

$$p_{H,t} = \frac{\rho^{n_{H,t}-n_{L,t}-1}(q_H - q_L)}{(1-\rho)^{n_{H,t}-n_{L,t}-1} + \rho^{n_{H,t}-n_{L,t}-1}}$$

Note that this strategy prevents the low-type firm from being able to profitably deviate to a higher price. As the first phase continues, since $\rho > \frac{1}{2}$, the expected value of $n_{H,t} - n_{L,t}$ increases. At any point in time, the high-type firm can start the second phase by settling on price

$$p_{H,t} = \frac{\rho^{n_{H,t}-n_{L,t}-1}(q_H - q_L)}{(1-\rho)^{n_{H,t}-n_{L,t}-1} + \rho^{n_{H,t}-n_{L,t}-1}} - \varepsilon$$

for a sufficiently small $\varepsilon > 0$. By doing so, the next customer purchases the product of the high-type firm regardless of his private signal. This increases the short-term profit of the high-type firm because it increases the probability of selling to customer t from ρ to 1. However, the customer's private signal does not become revealed to the future customers, i.e., $n_{H,t} - n_{L,t}$ does not increase;



(a) Expected trajectory of equilibrium prices

(b) Equilibrium prices using randomly generated customer signals

Figure 2: An equilibrium with a correct cascade: Figure (a) shows the expected trajectory of equilibrium prices, and Figure (b) shows an example of actual equilibrium prices when customers' private signals are randomly generated. In both figures, the high-type firm's prices are shown in black, and the low-type firm's prices are shown in gray. The parameters are set to $\rho = 0.6$, $q_L = \frac{1}{2}$, and $p_H = 2$.

this negatively affects the high-type firm's long-term profit.

In the first phase, as $n_{H,t} - n_{L,t}$ grows, $p_{H,t}$ becomes very close $q_H - q_L$, and therefore, the marginal benefit of increasing $n_{H,t} - n_{L,t}$ diminishes. However, the short-term loss of selling only with probability ρ, instead of probability 1, increases because the price increases. As such, when $n_{H,t} - n_{L,t}$ becomes sufficiently large, the firm benefits from transitioning to the second phase, and triggering a cascade. This result is formally presented in the following proposition.

Proposition 4 *There exists a sub-game perfect equilibrium of the game with a correct cascade.*

The equilibrium prices in Proposition 4 are depicted in Figure 2 for $\rho = 0.6$, $p_L = \frac{1}{2}$, and $p_H = 2$. As we see in Figure 2(a), the expected price of the high-type firm increases in equilibrium until it becomes very close $q_H - q_L = 1.5$, whereas the price of the low-type firm remains 0. Figure 2(b) shows an example of equilibrium prices when customers' signals are randomly generated.[5] The figure shows that, due to the noisy signals of the first few customers, the high-type firm may initially set a negative price; however, as t increases, the price of the high-type firm increases, and eventually becomes very close to $q_H - q_L$.

[5]Customers' private signals in Figure 2(b) are the same as those in Figure 1(b).

5 Conclusion

In this paper, we study the effects of observational learning among customers on firms' pricing decisions. Previous literature shows that observational learning leads to informational cascades wherein all customers purchase the same product. The effect of this herding behavior on firms' pricing strategies has been studied in the prior literature only when the seller is a monopolist. We extend this literature by analyzing how firms' pricing decisions are affected by observational learning, under dynamic and static pricing, in a duopoly.

When prices are static, i.e., firms cannot change their prices after customers start purchasing, our findings are inline with those of the previous literature. We extend the results of Welch (1992) from a monopoly to a duopoly by showing that observational learning makes the demand more elastic; as such, firms set lower prices when observational learning exists than when it does not. This result has implications for platforms managers as they could influence the degree of observational learning on their platforms. For example, releasing best-seller and trending products information increases the strength of observational learning, and the likelihood of informational cascades. In situations with static pricing (e.g., music and movies), our results indicate that observational learning lowers the equilibrium price of these products and increases the expected market share of the high-quality product.

Under dynamic pricing, i.e., when firms can adjust their prices in response to customers' purchase decisions, our findings depart from the previous literature in two respects. First, we show that observational does not necessarily lead to informational cascades. Second, we find that a wrong cascade, where customers regret their purchase decisions, does not happen in equilibrium. It is important to note that, for these findings to hold, both dynamic pricing and competition are required. These results are particularly relevant for managers of two-sided retail platforms (e.g., Amazon) as we show that, when observational learning exists, the equilibrium outcome could be drastically different from what the previous literature has suggested. For example, if Amazon allows observational learning, e.g., by providing best-seller and trending products information, since the sellers can change their prices dynamically, the outcome will not necessarily be a winner-takes-all competition. Furthermore, the platform does not have to be as concerned about a wrong cascade as prior literature has suggested. Finally, our results could explain why informational cascades, and customer herding behavior, have not been observed in two-sided platforms with dynamic pricing (e.g., eBay and Amazon) as commonly as in markets with static pricing (e.g., music and movies).

References

[1] Ameri, Mina, Elisabeth Honka, and Ying Xie. “Word-of-Mouth, Observational Learning, and Product Adoption: Evidence from an Anime Platform.” Available at SSRN (2016).

[2] Banerjee, Abhijit V. “A simple model of herd behavior.” The Quarterly Journal of Economics (1992): 797-817.

[3] Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. “A theory of fads, fashion, custom, and cultural change as informational cascades.” Journal of Political Economy (1992): 992-1026.

[4] Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. “Learning from the behavior of others: Conformity, fads, and informational cascades.” The Journal of Economic Perspectives 12.3 (1998): 151-170.

[5] Bose, Subir, Gerhard Orosel, Marco Ottaviani, and Lise Vesterlund. “Monopoly pricing in the binary herding model.” Economic Theory 37.2 (2008): 203-241.

[6] Cai, Hongbin, Yuyu Chen, and Hanming Fang. “Observational learning: Evidence from a randomized natural field experiment.” The American Economic Review 99.3 (2009): 864-882.

[7] Caminal, Ramon, and Xavier Vives. “Why market shares matter: An information-based theory.” The RAND Journal of Economics (1996): 221-239.

[8] Carare, Octavian. “The impact of bestseller rank on demand: Evidence from the app market.” International Economic Review 53.3 (2012): 717-742.

[9] Chen, Yubo, Qi Wang, and Jinhong Xie. “Online social interactions: A natural experiment on word of mouth versus observational learning.” Journal of marketing research 48.2 (2011): 238-254.

[10] Dutta, Prajit K. ”A folk theorem for stochastic games.” Journal of Economic Theory 66.1 (1995): 1-32.

[11] Garcia, Daniel, and Sandro Shelegia. “Consumer Search with Observational Learning.” Working Paper, 2015.

[12] Godes, David. “Product policy in markets with word-of-mouth communication.” Management Science (2016).

[13] Hendricks, Kenneth, Alan Sorensen, and Thomas Wiseman. “Observational learning and demand for search goods.” American Economic Journal: Microeconomics 4.1 (2012): 1-31.

[14] Jiang, Baojun, and Bicheng Yang. “Quality and pricing decisions in a market with consumer information sharing.” Available at SSRN 2675336 (2016).

[15] Lee, Young-Jin, Kartik Hosanagar, and Yong Tan. “Do I follow my friends or the crowd? Information cascades in online movie ratings.” Management Science 61.9 (2015): 2241-2258.

[16] Miklos-Thal, Jeanine, and Juanjuan Zhang. "(De) marketing to manage consumer quality inferences." Journal of Marketing Research 50.1 (2013): 55-69.

[17] Ross, Sheldon M. "Stochastic processes." Vol. 2. New York: John Wiley & Sons, 1996.

[18] Salganik, Matthew J., Peter Sheridan Dodds, and Duncan J. Watts. "Experimental study of inequality and unpredictability in an artificial cultural market." science 311.5762 (2006): 854-856.

[19] Tucker, Catherine, and Juanjuan Zhang. "How does popularity information affect choices? A field experiment." Management Science 57.5 (2011): 828-842.

[20] Tucker, Catherine, Juanjuan Zhang, and Ting Zhu. "Days on market and home sales." The RAND Journal of Economics 44.2 (2013): 337-360.

[21] Welch, Ivo. "Sequential sales, learning, and cascades." The Journal of finance 47.2 (1992): 695-732.

[22] Zhang, Juanjuan. "The sound of silence: Observational learning in the US kidney market." Marketing Science 29.2 (2010): 315-335.

[23] Zhang, Juanjuan, and Peng Liu. "Rational herding in microloan markets." Management science 58.5 (2012): 892-912.

[24] Zhang, Jurui, Yong Liu, and Yubo Chen. "Social learning in networks of friends versus strangers." Marketing Science 34.4 (2015): 573-589.

A Appendix: Proofs

Proof of Lemma 1

Suppose that k_1 signals indicate Product 1 and k_2 signals indicate Product 2, where $k_1 + k_2 = k$. Using the Bayes' rule, the probability that Product i is the better product, i.e., $H = i$, is

$$Pr(H = i) = \frac{\rho^{k_i}(1-\rho)^{k_j}}{\rho^{k_i}(1-\rho)^{k_j} + \rho^{k_j}(1-\rho)^{k_i}}$$

where $j = 2 - i$ is the index of the other firm. Since $\rho > \frac{1}{2}$, it is easy to see that $Pr(H = i) \geq Pr(H = j)$ if and only if $k_i \geq k_j$. When $k_i = k_j$, we have $Pr(H = i) = Pr(H = j)$. In this case, as we assumed in the model, the tie is broken in favor of the product indicated by the customer's own private signal.

Proof of Lemma 2

Let n_t^1 and n_t^2 denote the number of customers who have purchased Product 1 and Product 2, respectively, by time t (i.e., $n_t^1 + n_t^2 = t$, for any $t \geq 1$). We prove the lemma in two parts. First, we prove that as long as $|n_t^1 - n_t^2| \leq 1$ all customers purchase according to their own private signals. In the second part, we prove that if $|n_t^1 - n_t^2| = 2$ for some t, a cascade emerges, i.e., all of the following customers purchase the same product.

To prove the first part, we use induction on k, number of customers visited so far. Assume that for all customers $t \in \{1, \ldots, k\}$ we have $|n_t^1 - n_t^2| \leq 1$. Using the induction hypothesis, each of the customers $t \in \{1, \ldots, k\}$ acts according to his private signal, i.e., purchases from Firm s_t. Now, consider customer $k+1$. This customer can infer the private signals of all previous customers. He observers a total of $k+1$ signals, and since $|n_k^1 - n_k^2| \leq 1$, at least half of the $k+1$ signals are equal to his own private signal. Therefore, since he breaks the tie in favor of his own private signal, he acts according to his own private signal, i.e., purchases from Firm s_{k+1}.

To prove the second part, assume that k is the first customer (lowest index) for which $|n_k^1 - n_k^2| = 2$. Customer $k+1$ can infer the private signals of all customers $1, \ldots, k$. He also gets his own private signal. Since $|n_k^1 - n_k^2| = 2$, the majority of the total $k+1$ signals will be the same as the majority of the first k signals, i.e., s_k. Therefore, customer $k+1$ purchases the product of Firm s_k regardless of his own signal. Customer $k+2$ can infer the signals of customer $1, \ldots, k$, but cannot infer the signal of customer $k+1$. Therefore, similar to customer $k+1$, he also bases his decisions on a total of $k+1$ signals (his own, and those of the first k customers) majority of which is s_k. As such, customer $k+2$ also purchases the product of Firm s_k. Using the same argument, it is easy to see that all of the following customers will make the same decision and purchase from Firm s_k.

Proof of Lemma 3

Variable $n_t^H - n_t^L$ can be interpreted as a random walk that starts from 0 at $t = 0$, and increases or decreases by 1 for every increment of t. Using Lemma 2, if the variable ever becomes 2 (or -2), a cascade occurs where the variable keeps increasing (or decreasing). Before that, the variable increases by 1 with probability ρ, and decreases by 1 with probability $1 - \rho$.

Using random walks theory (e.g., see Ross 1996, page 188), we know that the expected number of steps before a cascade happens is $\frac{2}{\rho^2+(1-\rho)^2}$ which is less than 4. Furthermore, the probability that a cascade does not happen in the first $2k$ steps is $(2\rho(1-\rho))^k$ which is less than 2^{-k}. Therefore, the firms' average long-term profits converge to their profits in the cascade. The probability that the random walk hits +2 first (Firm H winning the cascade) is

$$\frac{\left(\frac{1-\rho}{\rho}\right)^2 - 1}{\left(\frac{1-\rho}{\rho}\right)^4 - 1} = \frac{\rho^2}{\rho^2 + (1-\rho)^2}.$$

Similarly, the probability that the random walk hits -2 first (Firm 2 winning the cascade) is $\frac{(1-\rho)^2}{\rho^2+(1-\rho)^2}$.

Proof of Lemma 4

Assume for sake of contradiction that the two firms set different prices p_1 and p_2. Given that this is a separating equilibrium, customers can infer the firms' qualities q_1 and q_2 in equilibrium.

First, consider the case where $q_1 - p_1 \neq q_2 - p_2$. In this case, all customers buy from the firm that offers higher utility, $\arg\max_i q_i - p_i$. If the firm with zero sales is the high type firm, he benefits from lowering its price to below the price of the low type firm, even if it leads to him being perceived as low type due to customers' beliefs. Since this is a profitable deviation, the high type firm cannot be the one who has zero sales in equilibrium. If the low type firm has zero sales, he could benefit from changing its price to the price of the other firm, pretending to be high type, where it would have positive expected sales. Therefore, a separating equilibrium in which the low type has zero sales is not possible either.

Finally, consider the case where $q_1 - p_1 = q_2 - p_2$, and assume that a non-zero fraction of customers purchase from each firm. In this case, the low type firm can lower its price by ε, for sufficiently small ε, so that all of the customers strictly prefer the low type firm. Note that since the low type firm is making the deviation, the deviation is profitable for any customers' belief on the new price $p_i - \varepsilon$. Therefore, a separating equilibrium where $q_1 - p_1 = q_2 - p_2$ and customers purchase from both firms cannot exist.

Proof of Lemma 5

Since observational learning is not possible, the revenue of Firm H is ρp and the revenue of Firm L is $(1-\rho)p$, where p is the equilibrium price in the pooling equilibrium. We want to find the highest price p that satisfies equilibrium conditions.

For price p to be a pooling equilibrium, customers' belief on any other (out-of-equilibrium) price should be that a firm making such deviation is low type. Since the low-type firm has lower expected revenue in this pooling equilibrium, any out-of-equilibrium deviation that is profitable for the high type would also be profitable for the low-type. Therefore, for p to be the price in a pooling equilibrium, the only necessary condition is that the low-type cannot benefit from deviating.

If the low-type firm deviates to a price p', customers would know that he is low type. Therefore, they would be able to infer his product quality q_L. Furthermore, customers also learn that the other firm is high type, and, therefore, has quality q_H. As such, Firm L's deviation to price p' could only be profitable if customers choose him because of low price p', even though they know that it has lower quality. In other words, unless $p' \leq q_L - q_H + p$, Firm L would have zero sale after deviation. Therefore, price p is a pooling equilibrium, if and only if Firm L's expected profit in the pooling equilibrium with price p is greater than or equal to its profit when it sells the product to all customers at price p'. In other words, the necessary and sufficient condition for p to be a pooling equilibrium is

$$q_L - q_H + p \leq (1-\rho)p$$

which simplifies to

$$p \leq \frac{q_H - q_L}{\rho}.$$

We also need the price p be such that customers expected utility from purchasing a product is non-negative. The expected quality of a product is $(1-\rho)q_L + \rho q_H$, therefore, price p has to be less than or equal to that. The equilibrium is payoff-dominant (Pareto-superior) when p is maximized; therefore, we have

$$p_1^{SB} = p_2^{SB} = \min\left(\frac{q_H - q_L}{\rho}, (1-\rho)q_L + \rho q_H\right).$$

Proof of Lemma 6

The proof is very similar to the proof of Lemma 5. Since observational learning is possible, using Lemma 3, the expected revenue of Firm H is $\frac{\rho^2}{\rho^2+(1-\rho)^2}p$ and the expected revenue of Firm L is $\frac{(1-\rho)^2}{\rho^2+(1-\rho)^2}p$, where p is the equilibrium price in the pooling equilibrium. We want to find the highest price p that satisfies equilibrium conditions.

For price p to be a pooling equilibrium, customers' belief on any other (out-of-equilibrium) price should be that a firm making such deviation is low type. Since the low type (low quality) firm has lower expected revenue in this pooling equilibrium, any out-of-equilibrium deviation that is profitable for the high type would also be profitable for the low type. Therefore, for p to be the price in a pooling equilibrium, the only necessary condition is that the low type cannot benefit from deviating.

If the low-type firm deviates to a price p', customers would know that he is low type. Therefore, they would be able to infer his product quality q_L. Furthermore, customers also learn that the other firm is high type, and, therefore, has quality q_H. As such, Firm L's deviation to price p' could only be profitable if customers choose him because of low price p', even though they know that it has lower quality. In other words, unless $p' \leq q_L - q_H + p$, Firm L would have zero sale after deviation. Therefore, price p is a pooling equilibrium, if and only if Firm L's expected profit in the pooling equilibrium with price p is greater than or equal to its profit when it sells the product to all customers at price p'. In other words, the necessary and sufficient condition for p to be a pooling equilibrium is

$$q_L - q_H + p \leq \frac{(1-\rho)^2}{\rho^2+(1-\rho)^2}p$$

which simplifies to

$$p \leq \frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}.$$

We also need the price p be such that customers expected utility from purchasing a product is non-negative. The expected quality of a product for the first customer ($t = 1$) is $(1-\rho)q_L + \rho q_H$; unless price p is less than that, the first customer, and consequently all following customers do not purchase any product. The equilibrium is payoff dominant (Pareto superior) when p is maximized;

therefore, we have

$$p_H^{SO} = p_L^{SO} = \min\left(\frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}, \rho q_H + (1-\rho) q_L\right).$$

Proof of Proposition 1

Using basic algebra, we can verify that

$$\frac{q_H - q_L}{\rho} \geq \frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}.$$

Therefore, we have

$$p_i^{SB} = \min\left(\frac{q_H - q_L}{\rho}, (1-\rho)q_L + \rho q_H\right) \geq \min\left(\frac{(q_H - q_L)(\rho^2 + (1-\rho)^2)}{\rho^2}, \rho q_H + (1-\rho) q_L\right) = p_i^{SO}.$$

Proof of Lemma 7

Let indices $i, j \in \{1, 2\}$ be such that $n_{i,t} \geq n_{j,t}$. Using Bayes' rule, the probability that Firm i is high-type, i.e., $H = i$, is

$$Pr(H = i) = \frac{\rho^{n_{i,t} - n_{j,t}}}{\rho^{n_{i,t} - n_{j,t}} + (1-\rho)^{n_{i,t} - n_{j,t}}}$$

Consequently,

$$Pr(H = i | s_t = j) = \frac{\rho^{n_{i,t} - n_{j,t} - 1}}{\rho^{n_{i,t} - n_{j,t} - 1} + (1-\rho)^{n_{i,t} - n_{j,t} - 1}}$$

and

$$Pr(H = i | s_t = i) = \frac{\rho^{n_{i,t} - n_{j,t} + 1}}{\rho^{n_{i,t} - n_{j,t} + 1} + (1-\rho)^{n_{i,t} - n_{j,t} + 1}}$$

The expected quality of product i for customer t is $Pr(H = i|s_t)q_H + (1 - Pr(H = i|s_t))q_L = q_L + Pr(H = i|s_t)(q_H - q_L)$. The expected utility of customer t, for product i, minus his expected utility for product j is

$$Pr(H = i|s_t)(q_H - q_L) - p_{i,t} + p_{j,t}$$

Customer t purchases product i if and only if the above expression is positive. This reduces to

- If

$$p_{i,t} - p_{j,t} < \frac{\rho^{n_{i,t} - n_{j,t} - 1}}{\rho^{n_{i,t} - n_{j,t} - 1} + (1-\rho)^{n_{i,t} - n_{j,t} - 1}}(q_H - q_L)$$

then customer t purchases the product of Firm i regardless of his own private signal.

- If

$$p_{i,t} - p_{j,t} > \frac{\rho^{n_{i,t}-n_{j,t}+1}}{\rho^{n_{i,t}-n_{j,t}+1} + (1-\rho)^{n_{i,t}-n_{j,t}+1}}(q_H - q_L)$$

then customer t purchases the product of Firm j regardless of his own signal.

- If

$$\frac{\rho^{n_{i,t}-n_{j,t}-1}}{\rho^{n_{i,t}-n_{j,t}-1} + (1-\rho)^{n_{i,t}-n_{j,t}-1}}(q_H - q_L) \leq p_{i,t} - p_{j,t} \leq \frac{\rho^{n_{i,t}-n_{j,t}+1}}{\rho^{n_{i,t}-n_{j,t}+1} + (1-\rho)^{n_{i,t}-n_{j,t}+1}}(q_H - q_L)$$

then customer t purchases the product that is indicated by his own private signal.

Proof of Proposition 2

Let indices $i, j \in \{1, 2\}$ be such that $n_{i,t} \geq n_{j,t}$. Firm i and j set prices

$$p_{i,t} = q_L + \frac{\rho^{n_{i,t}-n_{j,t}+1}(q_H - q_L)}{(1-\rho)^{n_{i,t}-n_{j,t}+1} + \rho^{n_{i,t}-n_{j,t}+1}}$$

and

$$p_{j,t} = q_L + \frac{(1-\rho)^{n_{i,t}-n_{j,t}-1}(q_H - q_L)}{(1-\rho)^{n_{i,t}-n_{j,t}-1} + \rho^{n_{i,t}-n_{j,t}-1}}.$$

As t grows, the prices of the low-type and high-type firms converge to q_L and q_H, respectively. The expected revenue of the low-type firm converges to $(1-\rho)q_L$ and the expected revenue of the high-type firm converges to ρq_H.

The price of each firm, for each customer, is set such that if the customer only uses the signals of the previous customers to calculate the probability of each firm being high-type (i.e., before the customer receives his own signal), he would be indifferent between buying from either firm, but would strictly prefer not to buy. However, when the customer observes his own private signal, he strictly prefers to purchase from the firm indicated by his signal, and becomes indifferent between buying from that firm and not buying (we break the tie by assuming that he buys). Therefore, all following customers who observe this customer's decision can infer his signal. The expected value of $n_{H,t} - n_{L,t}$ is $(\rho - (1-\rho))t = (2\rho - 1)t$. As t grows, since $\rho > \frac{1}{2}$, the value of $n_{H,t} - n_{L,t}$ grows unboundedly, and therefore, $p_{H,t}$ converges to q_H and $p_{L,t}$ converges to q_L. The revenue of the high-type firm converges to ρq_H, and the revenue of the low-type firm converges to $(1-\rho)q_L$.

In this equilibrium, the signal of each customer is revealed to all future customers. A firm may only benefit from a deviation in one stage if it sets a sufficiently low price in that stage so that the customer buys from him. Therefore, a deviation cannot improve the state of the game the deviating firm. Furthermore, note that the expected payoff of each firm in each round is strictly positive, whereas the minmax payoff of each player is zero (when the competitors sets a sufficiently low, possibly negative, price). Therefore, since $\delta \to 1$, each firm can punish a deviation by the other firm (by minmaxing the other firm for a finite number of periods) to make a deviation unprofitable.

Proof of Proposition 3

Sketch of the proof: We prove that for any $n_{H,t}$, $n_{L,t}$ and $p_{L,t}$, the high-type firm's expected profit from breaking a cascade in which it is losing is positive. Consider the strategy where the high-type firm sets its price such that every customer purchases the product indicated by his private signal. When $n_{H,t} < n_{L,t}$, the high-type firm loses at most $p_{L,t} - q_H + q_L$ each time a customer purchases its product. Random variable $n_{H,t} - n_{L,t}$ increases by 1 with probability ρ and decreases by 1 with probability $1-\rho$. Since $\rho > \frac{1}{2}$, Blackwell's theorem (see Ross 1996, pages 349-352) implies that for any $\hat{n}$, the expected number of customers for which $n_{H,t} - n_{L,t} < \hat{n}$ is finite. Therefore, in the limit ($\lim_{\delta \to 1}$), the revenue associated with customers for whom $n_{H,t} - n_{L,t} < \hat{n}$ converges to 0. This implies that the expected revenue of the high-type firm from future customers converges to $\rho(\bar{p_{L,t}} + q_H - q_L)$, where $\bar{p_{L,t}}$ is the average price set by the low-type firm. Since $\bar{p_{L,t}}$ cannot be negative (otherwise, the low-type firm would have negative total revenue which would violate the sub-game perfect assumption), the expected profit of the high-type firm when breaking the cascade is always positive, regardless of $n_{H,t}$, $n_{L,t}$ and $p_{L,t}$.

Proof of Proposition 4

Sketch of the proof: Consider the following strategies:

If $n_{H,t} < n_{L,t}$, the high-type firm sets its price to

$$p_{H,t} = -\frac{\rho^{n_{L,t}-n_{H,t}+1}(q_H - q_L)}{(1-\rho)^{n_{L,t}-n_{H,t}+1} + \rho^{n_{L,t}-n_{H,t}+1}}$$

if $n_{H,t} \geq n_{L,t}$ the high-type firm sets its price to

$$p_{H,t} = \min\left(\frac{\rho^{n_{H,t}-n_{L,t}-1}(q_H - q_L)}{(1-\rho)^{n_{H,t}-n_{L,t}-1} + \rho^{n_{H,t}-n_{L,t}-1}}, p_\delta\right)$$

and

$$p_{L,t} = 0.$$

for any $t \geq 1$, where p_δ is defined below. At price $p_{H,t} = p_\delta$, a cascade in which all customers purchase the product from the high-type firm happens. Note that as long as $p_{H,t} < p_\delta$, each customer buys the product indicated by his private signal. Therefore, as t increases, $n_{H,t} - n_{L,t}$ grows. When a new customer arrives, the high-type firm could set the price low enough such that the customer purchases the product with probability 1 or could keep it high enough so that the customer purchases the product indicated by his private signal (i.e., purchases from the high-type firm with probability ρ). The upside of the lower price this is that the sale happens with probability 1 (instead of probability ρ); the downside is that with the lower price $n_{H,t} - n_{L,t}$ does not increase (it increases with probability ρ with the higher price, which leads to higher long-term revenue). It is easy to see that as $n_{H,t} - n_{L,t}$ increases, the long-term benefit of increasing $n_{H,t} - n_{L,t}$ diminishes (as $\frac{\rho^{n_{H,t}-n_{L,t}-1}(q_H-q_L)}{(1-\rho)^{n_{H,t}-n_{L,t}-1}+\rho^{n_{H,t}-n_{L,t}-1}}$ becomes sufficiently close to $q_H - q_L$) whereas the short-term loss (of the purchase happening with probability ρ instead of 1) remains constant. Therefore, for any value of δ, there exists a p_δ (an increasing function of δ that converges to $q_H - q_L$ as δ approaches 1) such that at p_δ the high-type firm prefers to keep the price constant and get all customers (i.e., triggering a cascade) than to increase the price for higher long-term revenue. This shows that the high-type firm does not want to deviate from this equilibrium. The low-type firm's probability of selling drops to zero if it increases its price to any $p_{L,t} > 0$. Lowering the price to $p_{L,t} < 0$ could increase the probability of selling, however, it leads to negative profit and does not increase $n_{L,t} - n_{H,t}$. As such, the low-type firm cannot benefit from deviating either.