Estimating and Plotting Logarithmic Error Bars

Eric M. Stuve

Department of Chemical Engineering
University of Washington

Box 351750, Seattle, WA 98195-1750, USA
stuve@uw.edu

http://faculty.washington.edu/stuve/uwess/log_error.pdf

©2004–12
Absolute Error Bars

- Suppose that one has a sufficient number of measurements to make an estimate of a measured quantity $y$ and report its absolute error, $\pm \delta y$.

- The absolute error $\pm \delta y$ is represented on a Cartesian plot by extending lines of the appropriate size above and below the point $y$. 

```
\begin{align*}
  y_i + \delta y \\
  y_i \\
  y_i - \delta y
\end{align*}
```
Absolute Error Bars on a log Plot

- If plotted on a logarithmic plot, however, absolute error bars that are symmetric on a $y$ vs. $x$ plot become asymmetric; the lower portion is longer than the upper portion.

$$\log(y_i + \delta y)$$  $$\log(y_i)$$  $$\log(y_i - \delta y)$$

- This gives a misleading view of measurement precision, especially when measured quantities vary by several orders of magnitude.
**Error in Logarithmic Quantities**

- To represent error bars correctly on a log plot, one must recognize that the quantity being plotted, which we call $z$, is different than the measured quantity $y$.

  $z = \log(y)$

- The error $\delta z$ is

  $\delta z = \delta \left[ \log(y) \right]$
log Error is Relative Error

• On the assumption of small errors, a differential analysis can be used

\[ \delta z \approx dz = d[\log(y)] = \frac{1}{2.303} \frac{dy}{y} \approx 0.434 \frac{\delta y}{y} \]

• The error \( \delta z \) is thus given by the relative error in \( y \)

\[ \delta z \approx 0.434 \frac{\delta y}{y} \]

• The error bars now display correctly on a logarithmic plot.
Example: log Error Bars

- Plot the following data with error bars on a log-log plot

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>δy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.011</td>
<td>0.003</td>
</tr>
<tr>
<td>0.1</td>
<td>0.042</td>
<td>0.006</td>
</tr>
<tr>
<td>0.2</td>
<td>0.093</td>
<td>0.018</td>
</tr>
<tr>
<td>0.5</td>
<td>0.21</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>0.28</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.53</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.77</td>
<td>0.12</td>
</tr>
<tr>
<td>20</td>
<td>1.88</td>
<td>0.3</td>
</tr>
<tr>
<td>50</td>
<td>3.56</td>
<td>0.4</td>
</tr>
<tr>
<td>100</td>
<td>8.10</td>
<td>1.58</td>
</tr>
</tbody>
</table>
**Example: log Error Bars**

- First we calculate the quantities we need to plot:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>δy</th>
<th>log(x)</th>
<th>log(y)</th>
<th>log(y - δy)</th>
<th>log(y + δy)</th>
<th>δy/y</th>
<th>0.434 δy/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.011</td>
<td>0.003</td>
<td>-1.523</td>
<td>-1.959</td>
<td>-2.097</td>
<td>-1.854</td>
<td>0.273</td>
<td>0.118</td>
</tr>
<tr>
<td>0.1</td>
<td>0.042</td>
<td>0.006</td>
<td>-1.000</td>
<td>-1.377</td>
<td>-1.444</td>
<td>-1.319</td>
<td>0.143</td>
<td>0.062</td>
</tr>
<tr>
<td>0.2</td>
<td>0.093</td>
<td>0.018</td>
<td>-0.699</td>
<td>-1.032</td>
<td>-1.125</td>
<td>-0.955</td>
<td>0.194</td>
<td>0.084</td>
</tr>
<tr>
<td>0.5</td>
<td>0.21</td>
<td>0.02</td>
<td>-0.301</td>
<td>-0.678</td>
<td>-0.721</td>
<td>-0.638</td>
<td>0.095</td>
<td>0.041</td>
</tr>
<tr>
<td>1</td>
<td>0.28</td>
<td>0.05</td>
<td>0.000</td>
<td>-0.553</td>
<td>-0.638</td>
<td>-0.481</td>
<td>0.179</td>
<td>0.078</td>
</tr>
<tr>
<td>2</td>
<td>0.53</td>
<td>0.12</td>
<td>0.301</td>
<td>-0.276</td>
<td>-0.387</td>
<td>-0.187</td>
<td>0.226</td>
<td>0.098</td>
</tr>
<tr>
<td>5</td>
<td>0.77</td>
<td>0.12</td>
<td>0.699</td>
<td>-0.114</td>
<td>-0.187</td>
<td>-0.051</td>
<td>0.156</td>
<td>0.068</td>
</tr>
<tr>
<td>20</td>
<td>1.88</td>
<td>0.30</td>
<td>1.300</td>
<td>0.274</td>
<td>0.199</td>
<td>0.338</td>
<td>0.160</td>
<td>0.069</td>
</tr>
<tr>
<td>50</td>
<td>3.56</td>
<td>0.40</td>
<td>1.699</td>
<td>0.551</td>
<td>0.500</td>
<td>0.598</td>
<td>0.112</td>
<td>0.049</td>
</tr>
<tr>
<td>100</td>
<td>8.10</td>
<td>1.58</td>
<td>2.000</td>
<td>0.908</td>
<td>0.814</td>
<td>0.986</td>
<td>0.195</td>
<td>0.085</td>
</tr>
</tbody>
</table>
Example (cont.)

- Compare the Cartesian (left) and log-log (right) plots.
- The log-log plot displays the data better.
- Many data points are lost in the lower left corner of the Cartesian plot.
Example (cont.)

- Plot on left shows absolute error bars
- Plot on right shows relative error bars

Incorrect
Asymmetric error bars

Correct
Symmetric error bars
Comments on Example

• The column $\delta y/y$ is the relative error. It varies from 10–27% in this example. The relative error is used for the error bars on a logarithmic plot.

• The asymmetric error bars on the Cartesian plot are best seen for the points with large errors, like the first point.

• The logarithmic error bars are plotted on the log(y) scale. That means on the scale that reads –3, –2, –1, 0, 1; not on the scale that reads 0.001, 0.01, 0.1, 1, 10.
Comments (cont.)

- The data in the example represent measurements of the amount ($y$) of methanol electrooxidation as a function of time ($x$) taken from:


- It was necessary to show that a straight line cannot be drawn through all of the points, which required correctly drawn error bars.

- The points can only be fit by a curved line, which meant that the reaction mechanism was more complex than thought: there were four rate determining steps instead of just one.
Reference

• The method for calculating \( \log \) error bars can be derived from discussions of measurement error as appear in texts on analytical chemistry.

• For a specific reference on this material, see: