The Logic of the Third Man

S. Marc Cohen

THE LOGIC OF THE THIRD MAN

THE MAIN PROBLEMS facing the interpreter of the Third Man Argument (*TMA*) in Plato's *Parmenides* (132a1-b2) arise not so much from what Plato says as from what he does not say. Gregory Vlastos, in his famous paper of 1954,\(^1\) points out that the argument is formally a *non sequitur* and sets out to discover the suppressed premises of the argument. The literature dealing with the *TMA*, already large in 1954, has become enormous since then, and all of the authors I have read have followed Vlastos at least this far. But beyond a shared belief that the *TMA* as written is formally invalid and that in order to understand the argument we must identify its suppressed premises, there has been little agreement among the commentators. What are the suppressed premises? Is Plato committed to holding them? Is the argument, with the addition of such premises, valid? Did Plato think it was? What does it prove? What did Plato think it proves? Radically different answers have been offered to these questions, and I do not expect to offer definitive answers to any of them in this paper. What I hope to do is to show in what way the main lines of interpretation offered to date are inadequate, and to advance a formalization of the *TMA* which avoids these inadequacies and seems to me better to reveal the logical structure of the argument. On the basis of my examination of the logic of the *TMA* I conclude that the philosophical point of the argument is different from what it has been generally supposed to be.

I

The text, in Cornford’s translation, reads as follows:

I imagine your ground for believing in a single form in each case is this. When it seems to you that a number of things are large, there seems,\(^1\)

---

\(^1\) "The Third Man Argument in the *Parmenides*," *Philosophical Review*, LXIII (1954), 319-349; reprinted with an addendum in *Studies in Plato's Metaphysics*, ed. by R. E. Allen (London, 1965), pp. 231-263. Subsequent references will be to the reprinted version, which will be cited hereafter as "*TMA* I."
I suppose, to be a certain single character which is the same when you look at them all; hence you think that largeness is a single thing. . . . But now take largeness itself and the other things which are large. Suppose you look at all these in the same way in your mind’s eye, will not yet another unity make its appearance—a largeness by virtue of which they all appear large? . . . If so, a second form of largeness will present itself, over and above largeness itself and the things that share in it, and again, covering all these, yet another, which will make all of them large. So each of your forms will no longer be one, but an indefinite number.

Vlastos, in his original account of the *TMA*, transcribes what he identifies as the first two steps of the argument in the following way:

(A1) If a number of things, \(a, b, c\), are all F, there must be a single Form F-ness, in virtue of which we apprehend \(a, b, c\), as all F.

(A2) If \(a, b, c\), and F-ness are all F, there must be another Form, F-ness\(_1\), in virtue of which we apprehend \(a, b, c\), and F-ness as all F.\(^2\)

It is obvious that (A2) does not follow from (A1), and so Vlastos concludes that “there must have been something more in Plato’s mind than the information supplied at (A1)”\(^3\) to make the inference to (A2) seem plausible. Now the question of what was in Plato’s mind at this point is admittedly a difficult one; but Vlastos is content to raise “a more modest question: What are the simplest premises, not given in the present Argument, which would have to be added to its first step, to make (A2) a legitimate conclusion?”\(^4\) In answer to this question he produces two premises, one to justify the antecedent of (A2) and one to justify its consequent. The two premises are the well-known self-predication (*SP*) and non-identity (*NI*) Assumptions:

(SP) Any Form can be predicated of itself. Largeness is itself large. F-ness is itself F.\(^5\)

---


\(^3\) *TMA* I, p. 236.

\(^4\) Ibid.

\(^5\) Ibid.
(NI) If anything has a certain character, it cannot be identical with the Form in virtue of which we apprehend that character. If x is F, x cannot be identical with F-ness.⁶

Given (A1), these two premises are supposed to yield (A2) in the following way. It is a commonplace that there are F things—say, a, b, c. (A1) tells us that there is a Form, F-ness, in virtue of which we apprehend these as F. (SP) tells us that this Form, F-ness, is another F thing. But (NI) tells us that the Form in virtue of which we apprehend all of a, b, c, and F-ness as F things cannot be F-ness. Hence, it must be a second Form, F-ness₁. And this amounts to an assertion of (A2).

But, as Vlastos himself noticed, there is something strange about the way in which these new premises operate. For it is obvious that (SP) and (NI) are inconsistent; together they entail that F-ness is not identical with F-ness, which is self-contradictory. Indeed, they are formal contradistinctions, as Peter Geach pointed out;⁷ (SP) is the assumption that F-ness is an F thing, and (NI) amounts to the assumption that F-ness is not an F thing. But this does not discourage Vlastos from insisting upon (SP) and (NI) as being the TMA's implicit premises. For they are surely sufficient to generate (A2) (and hence the entire regress) just because they are inconsistent and can generate any conclusion we like. But since they are inconsistent, Vlastos feels he must conclude that Plato did not realize that these were the argument's implicit premises: "If Plato had identified all the premises which are necessary (and sufficient) to warrant the second step of the Third Man Argument, he would not have produced the Third Man Argument at all."⁸

That the premises necessary to generate the TMA are inconsistent is thus a cornerstone of Vlastos' interpretation—for it is on this basis that he concludes that Plato did not know what premises he was using, on the charitable assumption a philosopher—or Plato, anyway—will not produce an argument whose premises are inconsistent unless he is unaware of the inconsistency of the

---

⁶ TMA I, p. 237.
⁸ TMA I, p. 241.
THE LOGIC OF THE THIRD MAN

premises. And in the case of a premise set consisting of the formally contradictory pair \((SP)\) and \((NI)\), the only way to be unaware of the inconsistency of the premises is to be unaware of one or both of the premises.

It seems to me to be a matter of some importance for our understanding of the \(TMA\) to determine whether these premises are necessary to generate the regress. Let us be clear that we understand what is involved in the claim that \((SP)\) and \((NI)\) are indispensable premises. If \((SP)\) and \((NI)\), as formulated, were required as \(TMA\) premises, the conclusion of the argument would itself have to be logically inconsistent. A proposition is itself inconsistent if the only premise set from which it will follow is an inconsistent one. But, worse still, if the conclusion were inconsistent, then it would make no sense to say that \((SP)\) and \((NI)\) in particular are required as premises—for any other inconsistent set of premises would do just as well. But is the conclusion of Plato’s argument inconsistent? The conclusion, we recall, is this: “Each of your forms will no longer be one, but an indefinite number.” I suppose there may be some inclination to regard this conclusion as logically inconsistent, for it seems to say of the Forms that each one is not one, but many. But this inclination finds no real support in Plato’s text and is fostered only by a peculiarity in Cornford’s translation.\(^9\) The conclusion of the \(TMA\) (οὐκέτα δὴ ἐν ἑκαστὸν σοι τῶν ἑιδῶν ἑσται . . .) is explicitly the contradictory of that thesis (ἐν ἑκαστὸν ἑιδὸς) of the Theory of Forms which Parmenides cites at \(132a1\) and then sets out to refute in the regress argument. And the thesis that Parmenides sets out to refute is not the triviality that each Form is one (Form), but rather, as Cornford correctly puts it, that there is “a single Form in each case.” (I will have more to say later about how the phrase ἐν ἑκαστὸν ἑιδὸς should be taken.) So the conclusion of the argument should read: “And so there will no longer be one Form for you in each case, but infinitely many.”\(^10\) So formulated, the conclusion no longer has even the look of a logical inconsistency.

\(^9\) Cornford’s reading is, of course, grammatically possible. My point is that it is not the only possible reading, and that the only reasonable way to understand the conclusion is as the denial of the ἐν ἑκαστὸν ἑιδὸς thesis.

\(^10\) Vlastos’ translation in his most recent \(TMA\) paper (“Plato’s ‘Third Man’

451
S. MARC COHEN

Thus it simply cannot be true that an inconsistent premise set — \{ (SP), (NI) \} or any other, for that matter—is necessary for generating the infinite regress of Forms that the TMA purports to generate; the proposition that there are an infinite number of Forms of Largeness, for example, may be a most peculiar proposition, but it is not an inconsistent one. And if these assumptions are not necessary for generating the regress, there can be no good reason for trying to foist them on Plato. For (SP) and (NI) were introduced in the first place on the basis of their logical, not textual, credentials. And even if texts can be found which show that Plato was, after all, committed to each of these inconsistent assumptions, this will still not justify their introduction as premises of the TMA. The TMA intrigued Plato as it has countless of his readers; and Vlastos’ reconstruction of it has the defect of robbing the regress of its interest.

II

None of what I have said so far is really new. Vlastos’ critics from the first have been dissatisfied with his reconstruction of the TMA for just this reason. The first and one of the most powerful of these critics, Wilfrid Sellars, proposed a formalization of the argument with a consistent premise set. Since I have argued that such a formalization is a desideratum, a look at the Sellars version will be in order.

Sellars’ main point is that the self-predication and non-identity assumptions do not have to be understood as contradictories. His argument turns on the question of how we are to treat the expression “F-ness” in the formalization of the argument. More precisely, his question is this: to what syntactic category do

Argument [Parm. 132a1-b2]: Text and Logic,” Philosophical Quarterly, 19 (1969), 289-301, henceforth cited as “TMA II”), p. 293. Interestingly, nowhere in TMA I does Vlastos actually produce a translation of the TMA’s conclusion, although there are numerous allusions to it.

11 “Vlastos and ‘The Third Man,’ ” Philosophical Review, LXIV (1955), 405-437; reprinted in Philosophical Perspectives (Springfield, Ill., 1967), pp. 23-54. Subsequent references will be to the reprinted version.

452
THE LOGIC OF THE THIRD MAN

we assign the substituends for “F-ness”? One possibility is to regard substituends for “F-ness” as proper names of Forms: “Largeness,” for example, or “Redness.” In this case “F-ness” would be what Sellars calls a representative symbol or representative name. Another possibility is to regard substituends for “F-ness” as variables proper. To do so entitles us to quantify with respect to the substituends for F-ness and say, “There is a Redness such that . . .” or “For all Largenesses . . .,” and so forth, which would be syntactically inappropriate if “Redness” and “Largeness” were names. Now the expression “F-ness” combines what Sellars calls these “modes of variability,” and is a representative variable. That is, “F-ness” stands in place of, or represents, not a class of names but a class of variables.

Looked at in this way, (SP) and (NI) are defective in that they contain free occurrences of the representative variable “F-ness.” The defect can be remedied with the aid of quantifiers; the result is the Sellars version of the two assumptions:

(SP’) All F-nesses are F.

(NI’) If x is F, then x is not identical with the F-ness by virtue of which it is F. And, as Sellars correctly points out, “the inconsistency vanishes.”

Sellars is now able to generate the regress from a consistent premise set containing, in addition to (SP’) and (NI’), the following two premises:

13 I use “substituends for ‘F-ness’ ” here as short for the more appropriate expression: “expressions which result from ‘F-ness’ when ‘F’ is replaced by one of its substituends.”

14 Strictly speaking, the defect is this: every substitution instance of each of (SP) and (NI) contains free occurrences of the variables represented by “F-ness.”

15 Sellars’ formulation, (NI’), is not quite right as it stands. For (NI’), together with the other assumptions, will not generate the regress as Plato envisages it. Plato thinks of the particulars a, b, c as being F in virtue of the first Form, F-ness I, and all of these, in turn, as being F in virtue of a second Form, F-ness II. But (NI’) disallows this, since it requires that there be, for each F thing, such a thing as the F-ness by virtue of which it is F. Hence F-ness II cannot cover any of the particulars that F-ness I covers, and the regress will not develop. The formulation of the non-identity assumption that Sellars requires would be, rather, this: If x is F, then x is not identical with any of the F-nesses by virtue of which it is F.

453
S. MARC COHEN

\((G)\) If a number of entities are all \(F\), there must be an \(F\)-ness by virtue of which they are all \(F\).

\((P)\) \(a, b, c\), and so forth, particulars, are \(F\).

The proof is a non-terminating sequence which proceeds in this way: \((P)\) provides us with a stock of \(F\)'s, \((G)\) generates a Form by virtue of which they are all \(F\), \((NI')\) establishes that none of the \(F\)'s in the stock is identical with the Form \((G)\) has generated, and \((SP')\) establishes that the Form just generated is an \(F\). Thus our stock of \(F\)'s is increased by one, and we are ready for new applications of \((G)\), \((NI')\) and \((SP')\) which will generate fresh Forms, ad infinitum.

This argument Vlastos himself regards as "incomparably better"\(^{15}\) than an argument whose premise set is inconsistent, as all versions Vlastos has produced have been. The only thing wrong with it, according to Vlastos, is that it is not supported by the text and so cannot be regarded as a version of the argument Plato presented. The reason it does not fit the text, according to Vlastos,\(^{16}\) is that \((G)\) represents Plato as saying that there is at least one

\(^{15}\) TMA II, p. 293.

\(^{16}\) Vlastos has located the difficulty in Sellars' account in different places at different times. In his 1955 reply to Sellars ("Addenda to the Third Man Argument: A Reply to Professor Sellars," Philosophical Review, LXIV [1955], 438-448) he claimed that substituends for "\(F\)-ness" are not variables but proper names of Forms. This takes us back to \((SP)\) and \((NI)\) as Vlastos originally formulated them: an inconsistent pair. He now maintains that the self-predication and non-identity assumptions were defectively formulated in TMA I, that they are not, when properly formulated, an inconsistent pair, but that the TMA premise set is still inconsistent since it must contain a version of \((G)\) according to which "the Form corresponding to \(F\) is unique" (TMA II, p. 300, n. 39; cf. p. 292).

That the TMA premise set is an inconsistent triad (rather than an inconsistent pair) was first put forward, to my knowledge, by Anders Wedberg (Plato's Philosophy of Mathematics [Stockholm, 1955] Ch. III, esp. pp. 36-37). Wedberg's premise set is this:

\((i)\) A thing is \(\tau\) if and only if it participates in the Idea of \(\tau\)-ness.

\((ii)\) An Idea is never one among the objects participating therein.

\((iii)\) The Idea of \(\tau\)-ness is \((a)\) \(\tau\).

This premise set has \((ii)\), a non-self-participation assumption, in place of Vlastos' \((NI)\). And while \((ii)\) and \((iii)\) are consistent (i.e., self-predication is compatible with non-self-participation) the addition of \((i)\) produces an inconsistent set. On the inconsistency of \{(\(SP'\), \((NI')\), \((G1)\)\}, see n. 20 below.

454
Form corresponding to a given character, whereas Plato’s own words, both throughout the TMA and elsewhere in the dialogues, make clear that he means to be saying that there is just one Form corresponding to a given character. The word “one” (ἐν or μία) occurs five times in the TMA, and at each occurrence, Vlastos argues, it means “just one” and not “at least one.” And, as Vlastos further argues, in numerous other places where Plato uses the phrases ἐν ἔδος or μία ῖδεα (or their equivalent) he means “one Form,” “a single Form,” not “at least one Form.” Here Vlastos is surely correct: when Parmenides concludes that there is not one Form in each case, but rather an infinite number, he means to be denying the ἐν ἐκαστὸν ἔδος thesis. So that thesis must surely be that there is exactly one Form in each case; if ἐν meant “at least one,” the conclusion of the TMA would not contradict that thesis. It seems to me that the best reason for trying to read ἐν here as “at least one” would have been this: if ἐν means “exactly one,” then (G) cannot be correct as a formulation of the TMA’s first premise. Rather, that premise would apparently have to be, as Vlastos suggests,

\[ (G1) \text{ If a number of entities are all } F, \text{ there must be exactly one Form corresponding to the character, } F; \text{ and each of those entities is } F \text{ by virtue of participating in that Form.} \]

and it is easy to see that \( (G1), (SP') \) and \( (NT') \) form an inconsistent set.\(^\text{20}\)

\(^{17}\) Rep. 476a, 507b, 596a; Parm. 131a8-9, 132b5, 132c3-4, 133b1-2.

\(^{18}\) Another possible interpretation has been offered by Colin Strang (“Plato and the Third Man,” Proceedings of the Aristotelian Society, supp. vol. XXXVII [1963], 147-163). Strang argues that although the occurrences of ἐν in α1 and β2 must be taken to mean “exactly one,” the occurrences of μία and ἐν in α2, α3, and α7 need only be taken to mean “at least one.” But why should we assume that Plato is using ἐν equivocally in the TMA, shifting senses from one line to the next? Strang’s only reason seems to be that the assumption of such a shift in senses enables him to reconstruct the TMA as a valid argument with consistent premises and a conclusion which is the denial of the uniqueness thesis. But, as I hope to show below, it is possible to produce such a reconstruction without assuming any equivocation on ἐν. If I am right in this contention, Strang’s interpretation should lose much of its appeal.

\(^{19}\) Adapted from Vlastos, TMA II, p. 290.

\(^{20}\) Given the assumption that there are } F \text{ things. For suppose there are;
III

We seem to be faced with the following dilemma: when Plato introduces a Form for the “many large things” with the words μία ἰδέα, we must interpret him as meaning either “at least one” or “exactly one.” If we take the former reading we can generate the regress from a consistent premise set but only at the cost of misreading the text; if we take the latter reading, we will be fair to the text but only at the cost of leaving the argument’s premise set inconsistent. Neither of these alternatives is very attractive.

Fortunately, there is a way out of the dilemma. It is to show that the second horn contains a mistake, and that we can read μία and ἐπιθετικά throughout as “exactly one” and still have a consistent premise set. We can make a beginning in this direction by noticing that, even if we agree that μία means “exactly one,” Vlastos’ (G1) is not the only alternative to Sellars’ (G). Another alternative would be

(G2) If a number of entities are all F, there must be exactly one Form by virtue of which they are all F.

Two points should be noted about (G2). First, it is a more reasonable alternative to (G) than is (G1), since it differs from (G) only in that it replaces “an F-ness” with “exactly one Form,” which is really all one is entitled to if one’s only objection to (G) is that (G) is based on a misreading of μία ἰδέα. Second, (G2) does not assert that there is a unique Form corresponding to the character F, as (G1) does, but only that, given a number of F’s, there is a unique Form corresponding to them, in virtue of which they are all F. Thus (G2) leaves open the possibility, as (G1) does not, that there is more than one Form corresponding to the character F. It does not assert this—for, after all, that is the conclusion of the argument, and we should hardly expect the conclusion itself to be baldly asserted in a single premise—but it does not rule it out, either. And

then by (G1) there is exactly one Form—call it “F-ness”—corresponding to the character F. By (SP) F-ness is itself an F thing and by (NT) F-ness is not identical with the Form by virtue of which it is F. But according to (G1) F-ness is the Form by virtue of which each F thing is F, so F-ness is, after all, identical with the Form by virtue of which it is F. So F-ness both is, and is not, identical with the Form by virtue of which it is F.

456
it leaves this possibility open in spite of the fact that it reads μία as "exactly one" and not "at least one." ①

But are there not difficulties with \((G2)\)? \((G2)\) seems to tell us this: (1) if \(a, b,\) and \(c\) are all \(F\), then there is exactly one Form by virtue of which they are all \(F\), and (2) if \(h, i,\) and \(j\) are all \(F\), then there is exactly one Form by virtue of which they are all \(F\). The question whether the Form introduced in (2) is the same Form introduced in (1) is left open. But not for long; for \((G2)\) also tells us that (3) if \(a, b, c, h, i,\) and \(j\) are all \(F\), then there is exactly one Form by virtue of which they are all \(F\). And now our option to treat the Forms introduced at (1) and (2) as distinct seems to be canceled. For the Form introduced at (3) must be identical with the Form introduced at (1), for it is by virtue of just one Form that \(a, b,\) and \(c\) are \(F\). But, by parity of reasoning, the Form introduced at (3) must be identical with the Form introduced at (2). So we have one Form after all, and not two or three. And now it seems that \((G2)\) has been reduced to \((G1)\), with the result that we are still faced with the dilemma that \((G2)\) was supposed to get us out of.

This is one way of reading \((G2)\), but it is not the only way. As we have been reading \((G2)\) it comes to this. If, say, \(F\)-ness \(i\) is the one Form corresponding to a given set of \(F\)'s, then \(F\)-ness \(i\) is the one Form corresponding to any subset of that set; members of that set participate in \(F\)-ness \(i\) and are \(F\) by virtue of that participation and they participate in no other \(F\)-ness. But it is possible to read \((G2)\) differently; we can suppose it comes to this: if \(F\)-ness \(i\) is a Form corresponding to a given set of \(F\)'s, then \(F\)-ness \(i\) is the only \(F\)-ness corresponding to precisely that set. Other Forms

① There are good logical reasons for insisting that \((G1)\) simply cannot be an indispensable premise. For \((G1)\) embodies (in part) the uniqueness claim:

\((U)\) There is exactly one Form corresponding to each character or property

which is precisely what the conclusion of the \(TMA\) denies. Since not-\((U)\) is the conclusion, \((U)\) cannot be required as a premise. Vlastos' reply to Sellars ("Addenda to the Third Man Argument," p. 446) suggests that he would justify the inclusion of \((U)\) in the premise set on the ground that the \(TMA\) is a \textit{reductio}. But this would be to confuse the argument with its proof. If not-\((U)\) is a consequence of a set of premises which includes \((U)\), then it is a consequence of that set with \((U)\) deleted. Indeed, this is the leading principle of \textit{reductio} proofs. Hence \((G1)\), which entails \((U)\), cannot be required as a \(TMA\) premise.
might correspond to subsets of that set, but no other Form will correspond to that set itself. If we read \(G_2\) this way, the argument of the preceding paragraph designed to show that \(G_2\) reduces to \(G_1\) will fail; for that argument depended on the assumption that the Form corresponding to a given set of \(F\)'s is the Form corresponding to each of its subsets.

Put another way, our difficulty so far has been this: \((G)\) and \((G_2)\), the two versions of Plato's one-over-many principle that we have been considering, make reference to \(F\)'s but not to sets of \(F\)'s. Since they make no reference to sets of \(F\)'s, the force of \((G)\) and \((G_2)\), respectively, can be given in these two quantificational versions:

\[(G_3)\] For any \(x\), if \(x\) is \(F\) then there is at least one \(F\)-ness in which \(x\) participates.

\[(G_4)\] For any \(x\), if \(x\) is \(F\) then there is exactly one \(F\)-ness in which \(x\) participates.

But neither of these is acceptable. \((G_3)\) is unacceptable for Vlastos' reasons: Plato's one-over-many principle is meant to introduce exactly one, not (merely) at least one, Form. \((G_4)\) is unacceptable because it is inconsistent with the introduction of a second Form.

---

22 Sellars seems to be making substantially the same point when he writes ("Vlastos and 'The Third Man,' " pp. 29-30):

[A]s being large by virtue of participating in a given Largeness, an item is a member of a certain class of large items. Thus, \(a\), \(b\), \(c\), etc., would be members of the class of large particulars by virtue of the fact that each participates in the first largeness. On the other hand, \(a\), \(b\), \(c\), etc., together _with this first Largeness_ are members of a more inclusive class by virtue of their common participation in the second Largeness, and so on. Thus it does not follow from Plato's premises that the members of _one and the same class_ of large items, e.g., the class of large particulars, are members of that class by virtue of two different Largenesses. The latter would indeed be a gross inconsistency. . . . [T]he regress as Plato sets it up requires that it be incorrect to speak of the Form by virtue of which an item, \(x\), is large, without going on to specify the class of large things with respect to which it is being considered.

Even though his reasoning here commits him to saying that there will be exactly one \(F\)-ness for a given set of \(F\)'s, however, Sellars goes on to formulate his one-over-many premise as \((G)\), thus leaving himself open—unnecessarily—to Vlastos' objection.

23 \((G_1)\) has already been dismissed. Cf. n. 21.
into the TMA: the second Form introduced has all of the participants of the first, plus one. If we are going to come up with an adequate formulation of (G2), then, we will have to shift to a version which quantifies over sets of F's as well as over F's.

IV

We might try to formulate our set-theoretic version of (G2) in this way:

(G5) For any set of F's, there is exactly one Form over that set.

But there is something intuitively unsatisfactory about (G5); for it introduces a new relation, the "over" relation, which holds between Forms and sets of F's, and we have, thus far, no idea of what that relation might be. It is natural to suppose that the relation of a Form to the set it is "over" can be analyzed in terms of the participation relation between that Form and members of that set. Unfortunately, there is no easy way of doing this. Suppose we try:

(G6) For any set of F's, there is exactly one Form in which all members of that set participate.24

Clearly this will not do. (G6), like (G4), conflicts with the second step of the TMA. All members of the set {Mt. Everest, Mt. McKinley} participate in Largeness I; but they both, together with Largeness I, participate in Largeness II, . . . and so on. There may be more than one Form in which all members of a given set of F's participate. We might alter (G6) to read:

(G7) For any set of F's, there is exactly one Form in which only members of that set participate.

24 (G6) is essentially identical to Colin Strang's (loc. cit.) "strong OM," the strong version of the one-over-many thesis. Strang agrees that strong OM is inconsistent with the TMA premises (giving roughly the same argument I give), but he is content to rest the TMA on "weak OM" (essentially [G6] with "at least" in place of "exactly").
This will not do either. \((G7)\) tells us that, given a set of \(F\)'s, there will be one Form all of whose participants are members of that set. But this seems a most unlikely assumption. Consider the set \{Everest, McKinley\}. There is no Form of Largeness whose participants are limited to the pair \{Everest, McKinley\}. Hence there may be no Form in which only members of a given set of \(F\)'s participate.

Perhaps we should combine \((G6)\) and \((G7)\), yielding:

\[(G8)\] For any set of \(F\)'s, there is exactly one Form in which all and only members of that set participate.

But this is no better. For while the objection to \((G6)\) will not work against \((G8)\), the objection to \((G7)\) will; nothing in Plato's theory tells us that there should be one Form of Largeness over the set \{Everest, McKinley\} and another Form over the set \{Everest, Kilimanjaro\}. Plato's one-over-many principle will have to allow for more than one Form corresponding to the predicate "\(F\)"; but it should not require as many Forms corresponding to "\(F\)" as there are sets of \(F\)'s. Some sets of \(F\)'s, such as the ones mentioned above, are just not interesting enough to require their own special Forms.

But some sets are—the set of \(F\) particulars, for example. So perhaps something like \((G8)\) would do as a formulation of the one-over-many principle if there were some way of specifying which set of \(F\)'s is involved. As a start, we might try:

\[(G9)\] For any set which is the set of \(F\) particulars, there is exactly one Form in which all and only members of that set participate.

But \((G9)\), while unobjectionable as a Platonic truth, is too weak to be of much help in generating a regress. For \((G9)\) is equivalent to:

\[(G10)\] There is exactly one Form in which all and only members of the set of \(F\) particulars participate.

And \((G10)\) is silent about sets containing things other than \(F\) particulars, whereas it is just such a set that pops up in the second step of the \textit{TMA}.
THE LOGIC OF THE THIRD MAN

Clearly what is wanted is a more restricted version of \((G8)\) that is not so restricted, as \((G9)\) is, that it defuses the infinite regress. It will be my aim in the next section to produce such a version of the one-over-many principle.

V

Let me begin with a series of definitions. These definitions will be given in terms of a single undefined relational predicate, “participates in,” and the schematic letter “\(F\),” which will serve as a dummy predicate and will play the role that “large” does as a sample predicate in the \(TMA\).

\((D1)\) By an \(F\)-object (hereafter “object,” for short) I will mean any \(F\) thing (anything, that is, whether a particular or a Form, of which “\(F\)” can be truly predicated).

\((D2)\) An \(F\)-particular (hereafter “particular,” for short) is an object in which nothing participates.\(^{25}\)

\((D3)\) A Form is an object that is not a particular.

\((D4)\) I will also speak of a particular as an object of level \(O\).

\((D5)\) An object is an object of level one if

\((a)\) All of its participants are particulars, and

\((b)\) all particulars participate in it.

\((D6)\) In general, an object is an object of level \(n\) \((n \geq 1)\) if

\((a)\) All of its participants are of level \(n - 1\) or lower, and

\((b)\) all objects of level \(n - 1\) or lower participate in it.

I will define the level of a set of objects as the level of its highest-level member. Thus,

\((D7)\) A set of objects is a set of level \(n\) if it contains an object of level \(n\) and no higher-level object. Finally,

\(^{25}\) Strictly, this should be modal: a particular is an object in which nothing can participate. For the subsequent definition of a Form as a non-particular should have it that a Form is an object in which something can participate, in order to leave open the possibility of there being a Form which lacks participants. But no harm is done here by simplifying the definitions, since the \(TMA\) assumes the existence of particulars, which, in turn, guarantees that no Form (in this discussion) will go unpaticipated in.
(D8) A set of level \( n \) will be said to be a maximal set if it contains every object of level \( m \) for every \( m \leq n \). In other words, a maximal set contains every object on every level equal to or less that the level of its highest-level member.

The one-over-many principle that seems to be operative in the TMA can now be stated. I will label it "OM-axiom" to try to emphasize its deductive power, since it turns out to be the only assumption needed to generate not only the TMA but a number of important theorems as well.\(^\text{26}\)

(OM-axiom) For any maximal set there is exactly one Form in which all and only members of that set participate.

(Thus [OM-axiom] is simply [G8] restricted to maximal sets.) That the infinite regress of the TMA is a consequence of (OM-axiom) can be proved formally; the proof will proceed in roughly the following way. Assume the existence of the set of particulars—that is, the set of non-Forms of which "\( F \)" can be truly predicated; since this is a maximal set, (OM-axiom) gives us one Form over that set;\(^\text{27}\) the addition of this Form to the set of particulars gives us a new maximal set; (OM-axiom) then gives us a new Form; and so on. Now two questions arise about the proof as just sketched. (1) How do we know that each application of (OM-axiom) gives us a "new" Form—that is, one not identical with any of the objects introduced up to that point in the proof? (2) How do we know that the addition of a Form to the set it is over produces a maximal set? It is clear that we must have answers to these questions; if we cannot answer (1) we cannot guarantee that there will be a regress, and if we cannot answer (2) we cannot guarantee that we will keep producing sets to which (OM-axiom) will be applicable. Before setting out the TMA formally, then, it will be useful to mention two consequences of the axiom and definitions which will enable us to answer these questions. They are the following two theorems:

\(^{26}\) Except, of course, the assumption that there are particulars; we must be given a non-empty set of particulars to which to apply the OM-axiom.

\(^{27}\) "Over" will be used (until further notice) to abbreviate "participated in by all and only the members of."
THE LOGIC OF THE THIRD MAN

(T₁) No object is on more than one level.

(T₂) There is exactly one object on each level (greater than O).

(T₁) is derivable from (G₂), (D₄) and (D₆); (T₂) is derivable from (D₆), (D₈), and (OM-axiom). The proofs will be omitted. Our formalization of the TMA can now be sketched more fully.

THE TMA (FIRST VERSION)

1. Let α be the set of all particulars.
2. α is a maximal set (level O).
3. There is exactly one Form over α, call it "F-ness 1."
   (1), (D₄), (D₇), (D₈)
4. F-ness 1 is of level one.
   (1), (3), (D₅)
5. F-ness 1 is not a member of α.
   (2), (4), (T₁), (D₇)
6. α ∪ {F-ness 1} is maximal (level one).
   (2), (4), (T₂), (D₇), (D₈)
7. There is exactly one Form over α ∪ {F-ness 1}, call it "F-ness 11."
   (6), (OM-axiom)
8. F-ness 11 is of level 2.
   (6), (7), (D₆)
9. F-ness 11 is not a member of α ∪ {F-ness 1}.
   (6), (8), (T₁), (D₇)
10. F-ness 11 ≠ F-ness 1.
   (9)

---

28 Roughly, the proofs would run as follows.

For (T₁): Suppose an object, y, to be on more than one level, say levels i and i + j, for some i ≥ O and j ≥ 1. Then, by (D₆), y participates in itself, since an object of level i + j is participated in by every lower-level object and hence by any object of level i. But then y must also be on a level lower than i, since, by (D₆), all participants of an object of level i are on a level lower than i. Iteration of this reasoning will show that y must also be on level O. But then, by (D₄), y is a particular; and by (D₂) nothing participates in y. Hence, y does not participate in itself. But this contradicts the assumption that y is on both of levels i and i + j.

For (T₂): To show that, for any n ≥ 1, there is exactly one object of level n, let α be a maximal set of level n — 1. (This assumption is justified by the fact that it is provable that, for every n, there is a maximal set of level n.) Then by (OM-axiom) there is exactly one Form over α. But, by (D₈), the members of α are all and only those objects of level n — 1 or lower. Hence there is exactly one Form participated in by all and only objects of level n — 1 or lower, which means, by (D₆), that there is exactly one object of level n.
11. $\alpha \cup \{F\text{-ness 1, } F\text{-ness 11}\}$ is maximal (level 2).

12. There is exactly one Form over $\alpha \cup \{F\text{-ness 1, } F\text{-ness 11}\}$, call it "$F\text{-ness 111.}"

Etc.

The sequence, of course, is non-terminating; but since this is where Parmenides left off we, too, can stop at this point and examine the results.

The most important point about the argument whose proof is sketched above is the absence of explicit self-predication and non-indentity assumptions. This is not to say that self-predication and non-identity are not involved in the TMA as I have presented it; they are, but not as explicit premises in the argument. This seems to me to mark the point of greatest similarity between Plato's statement of the argument and my formalization of its proof. Self-predication is presupposed in the definitions of "Form" and "object"; non-identity comes in not as a premise but (at step 10) as a consequence of the line which is an instance of the theorem that a Form is not a member of the set it is over. It may be felt that it is perverse deliberately to conceal just those "assumptions" that some have argued are really the ones responsible for the TMA. On the contrary, I feel that it is a virtue of this way of looking at the TMA that it directs our attention to the one-over-many principle, which has been the least discussed of the TMA's assumptions, even though it was the only one Plato explicitly formulated.

But how well does (OM-axiom) represent the one-over-many principle Plato employs in the TMA? The most glaring difference is this: Plato does not say anything that suggests that the "many" to which the one-over-many principle will be applied must be (what I have called) a maximal set. Quite to the contrary, the text suggests that Plato is prepared to apply the principle to non-maximal sets; it is applied to $\pi\delta\lambda\prime\ \alpha\tau\alpha\ \mu\varepsilon\gamma\alpha\lambda\alpha$, "some plurality of large things" (Vlastos)—that is, some set of many large things. If Plato is prepared, as he seems to be, to start the TMA with any set of large things, then (OM-axiom) cannot be adequate as a formulation of the relevant one-over-many principle.
THE LOGIC OF THE THIRD MAN

So our problem is this: if we think of the TMA as starting with some non-maximal set, we do not yet have a principle which will provide us with exactly one Form over that set, in some suitable sense of "over." Given (OM-axiom), the best we can do for a general one-over-many principle would be this: no matter what set we start with, there will be exactly one Form over the lowest-level maximal set which includes that set—that is,

\((G_{11})\) For any set \(\alpha\), there is exactly one Form participated in by all and only members of the lowest-level maximal set which contains every member of \(\alpha\).

The Form \((G_{11})\) generates will not, however, be said to be over, in the sense given to that term above, the set to which \((G_{11})\) is applied. For "\(x\) is over \(\alpha\)" has been abbreviating "\(x\) is participated in by all and only the members of \(\alpha\)." And so unless \(\alpha\) is a maximal set, the Form \((G_{11})\) introduces will not be over \(\alpha\).

It should now be apparent that "participated in by all and only the members of" does not, after all, capture the intuitive sense of "over" \(\text{\(\varepsilon_{\pi}\)}}\) in "the one over the many." For one thing, the over relation ought to be understood to be a one-many relation; for another, the one which is over a set of many things ought to be understood to be over each of them. Yet the over relation, as defined thus far, has neither of these features; it is a relation that obtains between a Form and a maximal set, and hence is a one-one, not a one-many, relation; consequently a Form cannot be said to be over each of the members of the set it is over.

VI

The formalization of the TMA proposed in the last section suffered from the defect of requiring a maximal set at step one. Since it is at precisely that point that it seems to diverge from Plato's argument, I shall try to remedy the defect in the present section.

I shall begin by providing a definition of the over relation which will be closer, I think, to Plato's notion of that relation:

\((D_9)\) \(x\) is over \(y = df y\), or, if \(y\) is a set, every member of \(y\), participates in \(x\).
The \textit{over} relation will clearly not be a one-one relation. But it will not be a one-many relation, either. For to suppose it is would be to assume \((G6)\) once again, and \((G6)\) has already been rejected as inconsistent with the second step of the \textit{TMA}. The \textit{over} relation must—unhappily, it seems—be a many-many relation.

The \textit{over} relation is many-many because not only is \(F\)-ness \(I\) over each of the particulars \(a, b, c\), but so is \(F\)-ness \(II\), and so forth. But still, it is \(F\)-ness \(I\) and not \(F\)-ness \(II\) (or any of the others in the hierarchy) that makes the first appearance in the \textit{TMA}. That is, it is not just any Form over the initial set that appears at the first step of the \textit{TMA}; it is, one might say, the Form \textit{immediately} over the initial set that appears first. The Form immediately over the particulars \(a, b, c\) will be the Form whose participants are particulars only; it may be over other particulars, but it will not be over any Forms. We can make the sense of "immediately over" more precise:

\[(D10)\] \(x\) is immediately over \(y = df x\) is over \(y\) and \(x\) is over all and only those sets whose level is equal to or less than that of \(y\).

Thus, while \(F\)-ness \(I\) and \(F\)-ness \(II\) are both \textit{over} particulars \(a, b, c\), only the former is \textit{immediately} over them, for the latter is not over sets of level \(O\) only, being over the level one object \(F\)-ness \(I\). So while the \textit{over} relation may be many-many, the \textit{immediately over} relation is one-many. And since it is, the one-over-many principle required for the \textit{TMA} can be stated in terms of it:

\((IOM\text{-axiom})\) For any set of \(F\)'s, there is exactly one Form immediately over that set.

This axiom, it turns out, is equivalent to \((G11)\) and entails \((OM\text{-axiom})\);\(^{29}\) hence by using it in place of \((OM\text{-axiom})\) we can pro-

---

\(^{29}\) That \((IOM\text{-axiom})\) entails \((OM\text{-axiom})\) can be seen as follows. Let \(\alpha\) be a maximal set of level \(n\) (cf. n. 28); by \((IOM\text{-axiom})\) there is exactly one Form, say \(x\), immediately over \(\alpha\); by \((D10)\) \(x\) is over all and only sets of level \(n\) or lower; hence \(x\) is over \(\alpha\) and over no higher-level set; by \((D9)\) \(x\) is participated in by all members of \(\alpha\), and, by the previous step, participated in by nothing else; hence \(x\) is participated in by all and only members of \(\alpha\). Therefore, if \(\alpha\) is a maximal set, there is exactly one Form participated in by all and only members of \(\alpha\)—which is \((OM\text{-axiom})\).
duce a formalization of the TMA which is not open to the objections raised against that of the previous section. Once again it will be helpful if we can make use of an additional theorem in our proof:

\[(T_3)\] If \(x\) is immediately over \(y\), then the level of \(x\) is one greater than the level of \(y\).

\((T_3)\) is derivable from \((D6)-(D10)\).\(^{30}\) The formalization of the TMA follows.

**The TMA (final version)**

1. Let \(\alpha\) be any set of \(F\)'s (of level \(n\)).
2. There is exactly one Form immediately over \(\alpha\), call it "\(F\)-ness 1."
3. \(F\)-ness 1 is of level \(n + 1\).
4. \(F\)-ness 1 is not a member of \(\alpha\).
5. \(\alpha \cup \{F\text{-ness }1\}\) is of level \(n + 1\).
6. There is exactly one Form immediately over \(\alpha \cup \{F\text{-ness }1\}\), call it "\(F\)-ness 11."
7. \(F\)-ness 11 is of level \(n + 2\).
8. \(F\)-ness 11 is not a member of \(\alpha \cup \{F\text{-ness }1\}\).
9. \(F\)-ness 11 \(\neq F\)-ness 1.

Etc.

Once again, self-predication and non-identity assumptions are built in but not made explicit. The difference between this version of the TMA and the first lies in the different ways in which the one-over-many principle is formulated. The main advantage of \((IOM\text{-axiom})\) over its predecessor is that it makes clearer Plato's

\(^{30}\) Proof of \((T_3)\): let \(y\) be of level \(n\), and let \(x\) be immediately over \(y\); then by \((D10)\) \(x\) is over all and only sets of level \(n\) or lower; hence \(x\) is over the maximal set of level \(n\), and over no higher-level set; by \((D9)\) every member of the maximal set of level \(n\) participates in \(x\), and nothing else does; hence all of \(x\)'s participants are of level \(n\) or lower and all objects of level \(n\) or lower participate in \(x\); thus by \((D6)\) \(x\) is of level \(n + 1\).
S. MARC COHEN

inclination to think that while the one-over-many principle yields exactly one Form for the set under consideration at each step, that principle is consistent with there being more than one Form over the set with which we start. This inclination comes out, I think, in Plato’s use of verbs like δικεῖ and φαίνεται to introduce the Forms at each step. Over the first set of large things just one Form “appears” or “comes into view,” even though, as it turns out, there will be others. The one which appears will be the one immediately over that set. There may be more than one Form over a given set, but there would not appear to be to someone asked to pick out the one over the many. Clearly, Plato thinks of the Form introduced at each step as just overtopping, as it were, the set of things over which it is introduced. Over the set of particulars with which, presumably, we begin there will be just one object of the next level. But the uniqueness of the Form on each level is insufficient to prove the uniqueness thesis in which Plato is interested—namely, that there is exactly one Form corresponding to each predicate.

All of this fits perfectly the over-all structure of the TMA. Plato offers the one-over-many principle (at 132a2-3) as a reason for holding that the Forms are unique (ἐν ἑκάστων εἴδως, 132a1). The reasoning, presumably, would go like this: when you consider a set of large things, exactly one Form of Largeness will come into view, immediately over that set; so there is exactly one Form of Largeness. What Parmenides sets out to show is that this reasoning is inconclusive; indeed, it is the point of the TMA to show that the

---

31 The only part of my reconstruction for which there is no direct textual support is the division of objects into levels. Plato does not, of course, have a word for “levels,” nor does he explicitly divide objects in the way I have in my reconstruction. But I would defend this division on the grounds that it gives a fairly precise formulation of the logical structure implicit in Plato’s argument. Any account of the TMA must, it seems to me, take very seriously the one-over-many principle, and part of doing this is to say what is involved in the claim that a Form can be “over” its participants. It is clear that Plato thought of Forms as being on a higher “ontological level” than particulars (cf., e.g., Rep. 515d; 477a ff.; Tim. 28a, 49e; Phed. 74a, 78d ff., et passim). The TMA seems to extend this notion by assuming, in general, that a Form is on a higher level than its participants.

32 The importance of this line has not, I think, been sufficiently appreciated. It seems to me to show conclusively that the TMA is not, as has been generally supposed, a reductio argument directed against the uniqueness thesis.
one-over-many principle, far from supporting the uniqueness thesis, leads to its denial.\textsuperscript{33}

VII

If my account of the \textit{TMA} is, at least in its essentials, correct, then the difficulty in the Theory of Forms that is being shown up lies in the one-over-many principle. The argument of one over many, thought to be a safe route to the uniqueness thesis, has been shown to be defective. This diagnosis of the \textit{TMA}, however, will be unacceptable to those who think that in \textit{Republic} X Plato has shown us that he knows very well how to disable objections to the uniqueness thesis.\textsuperscript{34} There (597c-d) Plato argues in the following way. There is just one Form of Bed (literally, “bed in nature,” \(\epsilon\nu \tau\eta \varphi\epsilon\sigma\epsilon\iota\kappa\lambda\iota\nu\eta\)); for suppose there were two; immediately, another would crop up whose \(\epsilon\iota\delta\omicron\sigma\iota\) they would both have, and it, not they, would be the Form of Bed (literally, “what [a] bed is,” \(\delta \varepsilon\sigma\tau\nu\nu \kappa\lambda\iota\nu\eta\)). The crucial move in this “Third Bed Argument” (\textit{TBA}) is a one-over-many move;\textsuperscript{35} as soon as a second Form

\textsuperscript{33} I have been arguing that the \textit{TMA}’s premise set is consistent; hence, I am committed to the consistency of (\textit{IO-M-axiom}). But of course the consistency of this axiom is not independent of the sort of set theory we assume. In particular, the set theory my formalization presupposes cannot include the principle of abstraction—namely, the principle that, for any predicate, there is a set consisting of all and only objects to which that predicate applies—in formal notation:

\[ (\exists x) \ (x \ (x \in \alpha \iff \text{Fx}) ) \]

For if there were such a set (the universal set of \(F\)’s) it would contain no highest-level member (there being no such thing as the \textit{last} Form in the infinite regress) and hence it would not be a set of \textit{any} level (cf. [\textit{D7}]). But then no Form could be immediately over that set (cf. [\textit{D16}]), contradicting (\textit{IO-M-axiom}). Even though the principle of abstraction has its own difficulties (cf. Quine, \textit{Methods of Logic}, p. 249) we may still wish to retain it. In this case (\textit{IO-M-axiom}) would have to be altered to read:

For any set \(\alpha\), if \(\alpha\) is a set of level \(n\), for some \(n\), then there is exactly one Form immediately over \(\alpha\).


\textsuperscript{35} But cf. Vlastos, “Addendum (1963)” (to \textit{TMA I}), p. 263, who cites the \textit{TBA} as an instance where Plato employs the full-strength non-identity
threatens, it is an application of one over many that saves the day. The two beds we thought were both Forms are not Forms after all; it is the Third Bed which is the one Form.

If the TBA shows that one-over-many reasoning does yield the uniqueness thesis, then either the TMA is invalid or my account of it is mistaken. Fortunately, the TBA does not establish the uniqueness thesis; hence it cannot provide an answer to the TMA, although TMA and TBA reasoning will jointly produce a surprising but important conclusion. The TBA shows that there cannot be as many as two Forms of Bed, for the supposition that there are two demands the existence of a Third Bed, which, the TBA assures us, is the Form of Bed. But suppose we add our Third Bed, TMA style, to the beds already collected. The one-over-many principle will produce a Fourth Bed, and it, not the Third, will be the Form. Clearly what the TBA shows is that there is not more than one Form of Bed; it cannot show that there is exactly one unless it can show that the regress described above will stop. But, according to the TMA, this is precisely what it cannot do. So while the TBA shows only that there is not more than one Form, the TMA shows that there is not exactly one Form. And if neither exactly one nor more than one, then none. The surprising conclusion of the TMA together with the TBA is that there are no Forms. But Plato never put the two arguments together in this way, and hence apparently never realized that they produce this conclusion.

assumption; Cherniss, pp. 371-373, who sees Plato here denying self-predication (on the grounds that the $\varepsilon\sigma\tau\iota$ in $\delta\varepsilon\sigma\tau\iota\chi$ means "="); and Strang, p. 157, who correctly points out, contra Cherniss, that (a) if a denial of self-predication is involved in the TBA, Plato cannot have clearly seen it and (b) the TBA is "itself ripe for the TMA treatment."

36 This conclusion can be obtained formally by altering $(D_3)$, in light of the TBA, to read:

$$(D_3') \text{ A Form is an object that is not a particular and does not participate in any object,}$$

and by substituting "object" for "Form" in $(OM\text{-axiom})$. But since every object generated by $(OM\text{-axiom})$ will belong to at least one maximal set, every such object will participate in a higher-level object. Then none of the objects generated by $(OM\text{-axiom})$ is a Form. But from this it follows that

$$(T_3') \text{ For any } n, \text{ if } x \text{ is an object of level } n, \text{ then } x \text{ is not a Form.}$$
THE LOGIC OF THE THIRD MAN

The one-over-many principle will not yield the uniqueness thesis. And the TBA will not safeguard that thesis from the threat of the TMA. But that principle provides only one among many routes to the uniqueness thesis that Plato might have employed. I shall briefly consider one such route, suggested by the language in which the Forms are introduced in the Phaedo.\(^{37}\)

In that dialogue Plato claims that there is something beyond sensible \(F\) things, something he calls "The \(F\) Itself" (74\(a\)11-12).\(^{38}\) The \(F\) Itself is \(F\) without qualification (74\(b\)7ff.);\(^{39}\) it can never seem non-\(F\) (74\(c\)1-3);\(^{40}\) other \(F\) things fall short (ἐνδεικτικόν) of The \(F\) Itself (74\(d\)6-7), they are like (ὁν) it but inferior (φαινόμενον) to it (74\(e\)1-2); they are called by the same name (δύναμις) as The \(F\) Itself (78\(e\)2). Later in the Phaedo, Plato starts calling such things as The Beautiful Itself and The Large Itself "Forms" and says that other things are named after the Forms by participating in them (102\(b\)1-2).

Now this way of referring to a Form (schematically) as "The \(F\) Itself" is striking in several ways. To refer to a Form as "The \(F\) Itself" is, first of all, to name the Form (to say just which Form

\(^{37}\) I owe a number of points both in the remainder of this section and elsewhere in this paper to discussions with Gareth B. Matthews.

\(^{38}\) The sample predicate Plato uses is "equal"; the phrase he uses, ἀνδρὶ τῷ ἀνθ.\(^{39}\)

\(^{39}\) Suggested by Plato's claim that the "sensible equals" (sticks, stones, etc.) may appear to be "equal to this but not to that" (τῷ μὲν ἰόν φαινόμενον, τῷ δὲ ὅθε, 74\(b\)8-9). Presumably, The Equal Itself cannot appear equal to this but not to that; it is just Equal, pure and simple—the qualifications "to this," "not to that" are inappropriate. (On the reading of the datives in ἰόν, cf. G.E.L. Owen, "A Proof in the Peri Ideon," in Allen, op. cit., p. 306, whose interpretation I follow. Even on the traditional reading of the datives as masculine rather than neuter and governed by φαινόμενον rather than ἰόν it is still possible to see Plato here announcing a certain qualification on the F-ness of F particulars which does not apply to The F Itself. But I think Owen's reading is better.) Cf. also Ἰσμ. 211e; Ἱψ. Ἐπ. 289 ff.; Vlastos, "Degrees of Reality in Plato," in New Essays on Plato and Aristotle, ed. by R. Bambrough (London, 1969), pp. 1-19.

\(^{40}\) The question whether ἀνδρὶ τῷ ἀνθ. can ever seem unequal is raised and answered in the negative. But it is a matter of dispute among recent commentators whether the phrase ἀνδρὶ τῷ ἀνθ. ("the equals themselves") does, in fact, refer to the Form—i.e., ἀνδρὶ τῷ ἀνθ. I am assuming that it does, and that the (somewhat unexpected) plural can be satisfactorily explained. Cf. Geach, "The Third Man Again," p. 269; Vlastos, "Postscript to the Third Man Argument: A Reply to Mr. Geach," Philosophical Review, LXV (1956), 83-94, reprinted in Allen, op. cit., pp. 279-291 (esp. pp. 287-288, 291).
it is). But it is also to name the Form in such a way as to make clear how it is that participants in the Form are homonymous instances of it—that is, named after it. Third, and most important, is this: to refer to a Form as "The $F$ Itself" makes it perfectly clear that there is just one Form after which $F$ things are named. After all, it is *The F* Itself. So built into this way of referring to the Forms by their proper names are two other features: that of homonymy—particular $F$'s get their (common) name from the Form's (proper) name—and that of uniqueness—corresponding to the deficient, changeable, qualifiedly $F$ things there is just one thing that is unchangeable and does not fall short of being $F$, which is hence unqualifiedly $F$: *The F* Itself.

What I have been suggesting is not an argument for the uniqueness thesis. That thesis is not so much argued for in the *Phaedo* as simply built into Plato's way of referring to the Forms. To refer to a Form as "The $F$ Itself" does not prove the thesis—but it does, or should, forestall any objections to it. Thus, when Parmenides, at the second step of the *TMA*, claims to have proved the existence of a second Form, what one would expect from Socrates is not a counterargument but a charge of unintelligibility. (*Another The Large Itself? Two The Larges Themselves? Whatever do you mean? That doesn't make any sense!*) But no such charge is to be found in the text. Perhaps, then, Plato's willingness to press on with the *TMA* should indicate to us that the sort of difficulty for the uniqueness thesis which he envisaged was not one which could be palliated by appeal to a way of referring to the Forms. The text seems strongly to support this point, for, despite the fact that the *TMA* includes, *inter alia*, *Phaedo*-style reference to the Forms, Plato seems to take special pains to avoid having to say anything like "another The $F$ Itself." The Form first introduced by the one-over-many principle is referred to canonically at 132a6 as "The Large Itself" (αὐτὸ τὸ μέγα), but the second Form is introduced in a specially cautious way. Parmenides asks (literally) "will not some one [thing] once again large appear?" (οὐχὶ ἐν τι αὖ μέγα φανεραί). It is only after he gets assent to this, which is ambiguous as between The Large Itself making a second appearance and a second (something) making its first appearance, that Parmenides makes clear, for the first time, that the Form which has just appeared
is another Form ("Ἀλλ’ ἔρα ἐιδὸς μεγέθους, α10). According to the reasoning of the Phaedo this should be unintelligible (Another Form of Largeness? How could it be different from the first?) but Plato does not seem to be interested in making that point. Plato, it seems, just turns his back on the sort of reasoning which could save the uniqueness thesis.

VIII

I think it is safe to conclude that Plato in the TMA is interested not so much in the uniqueness thesis per se as in its relation to the one-over-many principle. What the TMA shows is that to keep uniqueness the one-over-many principle will have to be abandoned or modified for it is an application of that principle to the set consisting of large particulars and The Large Itself that generates a second Form. Well, this does not seem too high a price to pay; simply modify the principle in such a way as to make it applicable only to sets of particulars. It will thus generate one Form for each predicate (which we want it to do) but no more than one. But there is no indication that Plato himself ever tried to restrict the principle in this way. We can best understand and

---

41 As Plato himself seems to have done. Cf. Politicus 262a-63d.
42 As some members of the Academy apparently did, restricting the principle, according to Aristotle, to sets of particulars (καθ’ ἐκαστά). Cf. Alex. in Met., 80.8 ff.
43 There is an almost overwhelming temptation to think that the TBA depends upon a restricted one-over-many principle, for it appears that Plato is assuming, in that argument, that anything which requires a Form over it (to make it what it is) is not a Form. And does this not amount to the assumption that the one-over-many principle cannot be applied to Forms? I think this temptation should be resisted. For if the TBA really assumed that the one-over-many principle cannot be applied to Forms, Plato would have had to show that the two alleged Forms of Bed were not really Forms before he could apply the one-over-many principle to deduce the existence of the Third Bed—i.e., the genuine Form of Bed. But how could Plato show this without undercutting his own argument? If the TBA really assumed a restricted one-over-many principle, the argument would collapse. ("Suppose there are two Forms of Bed; since we can’t apply the restricted one-over-many principle to Forms, we can’t deduce the existence of the Third Bed; so we’re stuck with two Forms of Bed!") I conclude that whereas the unrestricted one-over-many principle entails the denial of the uniqueness thesis, the restricted principle is compatible with that thesis but does not entail it.
appreciate his failure thus to restrict the principle, I think, by looking at his most famous formulation of it (at Rep. 596a6-7):^44

We are in the habit of assuming one Form for each set of many things to which we give the same name.

But now recall the Phaedo’s doctrine of the homonymy of Forms and their participants. The things falling under a Form are homonymous instances of it. The general term which is applied to the many is borrowed from the name of the Form: they are called after the Form. That is, what makes it correct to call each particular $F$ “$F$” is that it is correct to call the Form under which the particulars fall “$F$.” So the set of things to which we give the name “$F$” will contain a Form. Yet according to 596a, the principle of collection for a set of many things to which the one-over-many principle is to be applied is that they be things “to which we give the same name.” So it seems inevitable that Plato would ultimately include Forms in sets to which the one-over-many principle is applicable.

It is still possible to read 596a in a harmless way, even if we waive the restriction to particulars: we assume one Form for each set of many things to which we give the same name; and among those will be one thing which does not participate in that Form—namely, the Form itself. Of course, the principle could then no longer be appropriately called the “one-over-many” (perhaps the “one-over-all-but-one-of-the-many” would be more appropriate). This objection is not a frivolous one; for the one-over-many principle is supposed to provide an answer to questions like “What makes it correct for many things all to be called ‘$F$’?” The answer is supposed to be that the many things all stand in a certain relation (participation) to a certain Form—the one over the many. And according to the suggested reading of 596a not all of the many things correctly called “$F$” will stand in that relation. Hence the idea will have to be given up that predicating “$F$” of something is, quite simply, a matter of asserting that a relation obtains between that thing and a certain Form. It is the

---

^44 εἴδος γάρ πού τι ἐν έκαστον εἰσώθωμεν τίθεσαι περὶ έκαστα τὰ πολλά, οἷς ταῦταν ὅνομα ἐπιφέρομεν.
THE LOGIC OF THE THIRD MAN

one-over-many principle which is the metaphysical embodiment of that idea, and in the TMA, I have argued, Plato is pointing out the logical shortcomings of that principle. In so doing he has taken an important step toward liberating himself from an initially compelling but overly simple and ultimately unsatisfactory theory of predication.45

S. Marc Cohen

Indiana University

45 An earlier version of this paper was presented to the philosophy department of the University of Massachusetts in April, 1970, as part of a symposium on Plato's Parmenides, and to the Institute in Greek Philosophy and Science held at Colorado College in July, 1970. Among the many people of whose helpful criticism I have been the beneficiary I wish especially to thank Aryeh Kosman, Gareth B. Matthews, and Gregory Vlastos.