Some Useful Truth-Table Equivalences

Because of the interdefinability of the connectives, each truth function can be expressed by a number of different FOL sentences. It is useful to be aware of the variant, but equivalent, ways of expressing familiar truth-functions, such as conjunction, disjunction, etc. Each of the following five groups contains five equivalent sentences, all expressing the same truth-function. Use truth-tables, if necessary, to confirm any equivalences you find surprising.

**Group 1: Conjunction**

\[ A \land B \]
\[ B \land A \]
\[ \neg (A \to \neg B) \]
\[ \neg (B \to \neg A) \]

**Group 2: Disjunction**

\[ A \lor B \]
\[ B \lor A \]
\[ \neg (A \land \neg B) \]
\[ \neg (B \land \neg A) \]

**Group 3: Conditional**

\[ A \to B \]
\[ \neg (A \land \neg B) \]
\[ \neg A \lor B \]
\[ \neg B \to \neg A \]

**Group 4: Biconditional**

\[ A \leftrightarrow B \]
\[ B \leftrightarrow A \]
\[ \neg A \leftrightarrow \neg B \]
\[ (A \to B) \land (B \to A) \]
\[ (A \land B) \lor (\neg A \land \neg B) \]

**Group 5: Negation of biconditional**

\[ \neg (A \leftrightarrow B) \]
\[ \neg A \leftrightarrow B \]
\[ A \leftrightarrow \neg B \]
\[ (A \land \neg B) \lor (B \land \neg A) \]
\[ (A \lor B) \land \neg (A \land B) \]

Note the last group, negation of biconditional, expresses exclusive disjunction \((A \text{ xor } B)\) – either \(A\) or \(B\), but not both.