Disclaimer: I have not proof read this at all!!

1. Define carefully the following terms:
   (a) Linearly Independent
   (b) Spanning Set
   (c) Basis
   (d) Vector Space
   (e) Subspace
   (f) Linear Transformation/Funtion/Operator

2. Complete the following for each of the given matrices $A$.
   (a) Find the reduced row echelon form for $A$.
   (b) Are the columns of $A$ linearly independent?
   (c) Do the columns of $A$ span $\mathbb{R}^m$ where $m$ is the number of rows in $A$?
   (d) Find the domain and codomain for the linear transformation $T_A$.
   (e) Is the linear transformation $T_A$ one-to-one?
   (f) Is the linear transformation $T_A$ onto?
   (g) Find a basis for the column space of $A$
   (h) Find a basis for the null space of $A$.
   (i) What is the rank of $A$?
   (j) Find a basis for the span of $T_A$.
   (k) Find a basis for the kernel of $T_A$.
   (l) If possible, compute the determinant of $A$.
   (m) If possible, determine if $A$ is invertible, and then find $A^{-1}$.
   (n) Is $\vec{0}$ an eigenvector of $A$?
   (o) If possible, find the characteristic polynomial of $A$?
   (p) If possible, find the eigenvalues of $A$?

3. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have at least one common point of intersection? Explain.

The three planes have one point in common.
4. Suppose a $3 \times 5$ coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

Yes. The system is consistent because with three pivots, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echelon form cannot contain a row of the form $[0 \ 0 \ 0 \ 0 \ 1]$.

5. Suppose a system of linear equations has a $3 \times 5$ augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

6. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent.

7. In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

<table>
<thead>
<tr>
<th>Velocity (100 ft/sec)</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (100lb)</td>
<td>0</td>
<td>2.9</td>
<td>14.8</td>
<td>39.6</td>
<td>74.3</td>
<td>119</td>
</tr>
</tbody>
</table>

Find an interpolating polynomial for the data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec. Use $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$. What happens if you try to use a polynomial of degree less than 5?

8. Use the following figure to write $\vec{a}$, $\vec{b}$, and $\vec{c}$ as a linear combination of $\vec{u}$ and $\vec{v}$. Is ever vector in $\mathbb{R}^2$ a linear combination of $\vec{u}$ and $\vec{v}$?

Label the following vectors in the above figure. $\vec{d} = \vec{v} - 2 \vec{u}$, $\vec{e} = 2 \vec{u} - 2 \vec{v}$.

9. A mining company has two mines. One day’s operation at mine #1 produces ore that contains 20 metric tons of copper and 550 kilograms of silver, while one day’s Operation at mine #2 produces ore that contains 30 metric tones of copper and 500 kilograms of silver.
silver. Let \( \vec{v}_1 = [20 \ 550]^\top \) and \( \vec{v}_2 = [30 \ 500]^\top \). Then \( \vec{v}_1 \) and \( \vec{v}_2 \) represent the “output per day” of mine #1 and mine #2 respectively.

(a) What physical interpretation can be given to the vector \( 5\vec{v}_1 \)?

This is the output for 5 days from mine #1

(b) Suppose the company operates mine #1 for \( x_1 \) days and mine #2 for \( x_2 \) days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 150 tons of copper and 2825 kilograms of silver.

The total output is \( x_1\vec{v}_1 + x_2\vec{v}_2 \). So \( x_1 \) and \( x_2 \) should satisfy \( x_1\vec{v}_1 + x_2\vec{v}_2 = [150 \ 2825]^\top \)

(c) Solve the equation you set up above.

1.5 days for mine #1 and 4 days for mine #2.

10. Construct a 3\( \times \)3 matrix, not in echelon form, whose columns span \( \mathbb{R}^3 \). Show that the matrix you constructed has the desires property.

11. Let \( A \) be a 3\( \times \)2 matrix. Explain why the equation \( A\vec{x} = \vec{b} \) cannot be consistent for all \( \vec{b} \) in \( \mathbb{R}^3 \). Generalize your argument to the case of an arbitrary \( A \) with more rows than columns.

12. Find a parametric equation of the line \( M \) through \( \vec{p} = [2 \ -5]^\top \) and parallel to \( \vec{q} = [-3 \ 1]^\top \). Find the parametric equation of the line \( L \) through \( \vec{p} \) and \( \vec{q} \).

\[
L = [2 \ 5]^\top + t[-5 \ 6]^\top
\]

13. Does the equation \( A\vec{x} = \vec{0} \) have a nontrivial solution if

(a) \( A \) is a 3\( \times \)3 matrix with three pivot columns.

When \( A \) is a 3\( \times \)3 matrix with three pivot columns, the equation \( A\vec{x} = \vec{0} \) has no free variables and so has no nontrivial solutions.

(b) \( A \) is a 3\( \times \)2 matrix with two pivot columns.

Each column is a pivot column so the equation \( A\vec{x} = \vec{0} \) has no free variables and so no nontrivial solutions.

14. Construct a 3\( \times \)3 nonzero matrix \( A \) such that the vector \( [1 \ 1 \ 1]^\top \) is a solution of \( A\vec{x} = \vec{0} \).

15. Find the matrix \( A \) so that the corresponding linear operator \( T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \)

(a) rotates all points \( \frac{\pi}{6} \) radians counterclockwise about the origin.

(b) first reflects all points about the \( x \)-axis, then rotates all points \( \frac{\pi}{4} \) radians \textit{clockwise} about the origin.
(c) is such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

16. Given an example of two distinct, nonzero vectors in $\mathbb{R}^3$ that are linearly dependent.