TMATH 124: Quiz 5

You may use any work of yours that you made from last week. This includes, practice book problems and worked out WebAssign problems. This does not include photocopies of notes from the book or tutorials shown on WebAssign. Graphing calculators are also not allowed. In short, you are only allowed to use work that you created.

Show all your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [2 each] Differentiate each of the following. Please do not simplify for this problem.

   \[ y = \log_2(1 - 3x) \]

   \[
   f(x) = \log_2 x \\
   g(x) = 1 - 3x \\
   f'(x) = \frac{1}{x \ln 2} \\
   g'(x) = -3 \\
   y' = f'(g(x))g'(x) = \frac{1}{(1-3x)\ln 2} = -3 \\
   \]

   \[ y = x^x \]

   \[
   \ln y = \ln x^x \\
   \Rightarrow \ln y = x \ln x \\
   \frac{1}{y}y' = x(\ln x)' + (x)' \ln x \\
   \frac{1}{y}y' = x \cdot \frac{1}{x} + 1 \cdot \ln x \\
   y' = y \left(1 + \ln x\right) \quad \text{or} \quad x^x \left(1 + \ln x\right) \\
   \]

   \[ \text{got it} + \frac{4.5}{4} \]
2. [3] Find an equation of the tangent line to the curve \( y = x^2e^{-x} \) when \( x = 1 \).

\[
\begin{align*}
y' &= x^2(e^{-x})' + (x)'e^{-x} \\
y' &= x^2e^{-x} + 2xe^{-x} \\
y' &= xe^{-x} \\
\ln y &= 2\ln x + \ln e^{-x} \\
\frac{dy}{dx} &= \frac{2}{x} - 1 \\
\Rightarrow y &= y\left(\frac{2}{x} - 1\right) \\
\text{So } m &= y'(1) = \left(\frac{2}{1} - 1\right)e^{-1} = \frac{1}{e} \\
\text{Looking for } y = mx+b \text{ from the above way we have } m = \frac{1}{e} \\
The line passes through \( (1, (1)^2e^{-1}) = (1, \frac{1}{e}) \) \\
\frac{1}{e} = \frac{1}{e} \cdot 1 + b \\
\Rightarrow b &= 0 \\
y &= \frac{1}{e} \cdot x
\end{align*}
\]

3. [3] Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm²?

\[
\begin{align*}
dx/dt &= 6 \text{ cm/s} \\
A &= x^2 \\
dA/dt &= 2x \cdot dx/dt \\
\text{found what } x \text{ was when } A = 16 \\
A &= 48 \\
\text{get } A = 16.
\end{align*}
\]