Optimization
part two

1. An industrial production process costs $C(q)$ million dollars to produce $q$ million units; these units then sell for $R(q)$ million dollars. If $C(2.1) = 5.1$, $R(2.1) = 6.9$, $MC(2.1) = 0.6$, and $MR(2.1) = 0.7$, find the following:

   (a) The profit earned by producing 2.1 million units.

   (b) Should the company increase or decrease production to maximize profit?

2. Production of an item has fixed costs of $10,000 and each item costs $2 to produce. Assume the relationship between price ($p$) and quantity demanded ($q$) is linear. Market research shows that 10,100 items are sold when the price is $5 and 12,872 items are sold when the price is $4.50.

   (a) Express the cost, $C$, of producing $q$ items.

   (b) Recall that the demand curve is linear. Express $p$, as a function of $q$.

   (c) Recall if you sell $q$ items for $p$, then you will have $p \cdot q$ dollars of revenue. Use the work from (b) to express the revenue, $R$, from selling $q$ items as only a function of $q$.

   (d) Express the profit earned as a function of $q$. Use this to find how many items the company should produce to maximize profit.
3. An offshore oil well is located in the ocean at a point W, which is 5 miles from the closest shorepoint A on a straight shoreline. The oil is to be piped to a shorepoint B that is 8 miles from A by piping it on a straight line under water from W to some shorepoint P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is $100,000 per mile under water and $75,000 per mile over land, where should the point P be located to minimize the cost of laying the pipe?

4. You run a small furniture business. You sign a deal with a customer to deliver up to 400 chairs, the exact number to be determined by the customer later. The price will be $90 per chair up to 300 chairs, and above 300, the price will be reduced by $0.25 per chair (on the whole order) for every additional chair over 300 ordered.

   (a) Write down the revenue as a function of number of chairs sold $q$. Note, you might want a piece-wise defined function here.

   (b) What is the largest and smallest revenues your company can make under this deal?