Limit Laws

1. Consider \( f(x) = 4x^7 + 3x^6 - 5.5x^5 + 2.1x^2 - 7x + 2 \).

   (a) Use the limit laws to find \( \lim_{x \to 1} f(x) \). Write all the steps and record which laws you used like we did in class.

   (b) Evaluate \( f(1) \).

   (c) What do you notice about the answers to (a) and (b)?

   (d) Is \( \lim_{x \to -3} f(x) = f(-3) \)? (You do not need to write out all your steps as in (a).)

   (e) Does \( \lim_{x \to a} f(x) = f(a) \) for all real numbers \( a \)?

2. Let \( g(x) = x^{10} - 4x^7 - 4x^5 + 4x^4 - 2.4x + 2 \). Does \( \lim_{x \to a} g(x) = g(a) \) for all real numbers \( a \)? How can you tell?

3. If you were given a polynomial \( p \) and asked to compute \( \lim_{x \to a} p(x) \), how would you go about doing it. Check your answer with the box on page 102.
4. Let \( f(x) = 3x^2 \). Either follow the steps below or compute the difference quotient of \( f \) at 2 directly, that is find:

\[
\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}.
\]

(a) Find \( f(2 + h) \), and simplify.

(b) Find \( f(2) \).

(c) Use (a) and (b) to find \( \frac{f(2 + h) - f(2)}{h} \).

(d) Find \( \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} \).

5. Let \( g(x) = x^2 - 2x \). Either follow the steps below or compute the difference quotient of \( g \) at 1 directly, that is find:

\[
\lim_{h \to 0} \frac{g(1 + h) - g(1)}{h}.
\]

(a) Find \( g(1 + h) \), and simplify.

(b) Use (a) to find \( \frac{g(1 + h) - g(1)}{h} \).

(c) Find \( \lim_{h \to 0} \frac{g(1 + h) - g(1)}{h} \).