Chain Rule Practice

Recall the chain rule: If $f$ and $g$ are both differentiable then $f \circ g$ is differentiable and

$$\frac{d}{dx}(f \circ g)(x) = f'(g(x))g'(x)$$

For each of the problems below complete the following:

1. Define function $f$ and $g$ so that:
   - both functions are easily differentiable and
   - the given function has the rule $f \circ g$.
2. Find $f'$ and $g'$.
3. Find $\frac{d}{dx}(f \circ g)(x)$

<table>
<thead>
<tr>
<th>Function</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
<th>$\frac{d}{dx}(f \circ g)(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{x^3 - 5}$</td>
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<td>$(x^3 - 1)^{100}$</td>
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<td>$e^{3x^2 - x}$</td>
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<tr>
<td>$\sin^5(x)$</td>
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4. Notice that we can use the chain rule in conjunction with the previous rules we already learned. Use the work you did on the previous page to find the derivative of the following functions:

(a) \( \pi e^{3x^2-x} + 2^x \)

(b) \( \sin^5(x)\sqrt{x^3 - 5} \)

5. The chain rule can also be used in conjunction with itself. That is, we can use the chain rule to work on a derivative, but when trying to find the \( g'(x) \), we may need to use the chain rule again.

\( \sqrt{47 + 3e^{3x^2-x}} \)

\( \sin^2(x^3) \)