Slopes and Lines

This must be completed in groups of 2 to 3. One copy of the group’s work & answers are to be turned in as part of the WrittenHW. You will likely need to arrange a meeting outside of class, so exchange contact information! As always with group work, make sure each of you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Shape: Let $g(x) = 1 \cdot x$. The graph of $g$ is plotted below and is an example of a line. All the functions on this worksheet will have graphs that are lines.

(a) Let the function $h$ be defined by $h(x) = -2 \cdot x$. The multiplication sign is often suppressed and we write $h(x) = -2x$ instead. Plot the graph of $h$ below.

(b) Plot the graphs of the functions $\alpha(x) = -\frac{1}{2} x$, $\beta(x) = 0x$, and $\gamma(x) = 3x$. You should have four different graphs drawn below by the end of this question.

(c) For each graph above, determine if the graph defines a function or not.

(d) For each graph above, find the intervals that the graph is increasing & decreasing. For example, the graph of $g$ is increasing on the interval $(-\infty, \infty)$.

(e) Let $f$ be the function with the rule $f(x) = mx$ where $m$ be some fixed number (like 2, $\frac{1}{3}$, -4, etc...). Use the above observations to determine if the graph of $f$ is always increasing, decreasing, or doing neither when:

- $m > 0$;
- $m = 0$;
- $m < 0$;
2. Slope:

(a) Given points \((a, b)\) and \((c, d)\) on a line, explicitly find a formula for the slope.

(b) Let \(h(x) = -2x\) be the function from (1a). Find two points that are on the graph of \(h\) and use the formula from 2a to compute the slope of the line defined by \(h\).

(c) If \(f(x) = mx\) where \(m\) is some fixed number (like 2, \(\frac{1}{3}\), -4, etc...), what is the slope of the graph of \(f\)?

3. Let \(\delta\) be a linear function defined by \(\delta(x) = h(x) - 2.5\) where \(h\) was defined in #1. Since \(h(x) = -2x\) we could also write \(\delta(x) = -2x - 2.5\). Identify the \(y\)-intercept of \(\delta\).

4. Let \(\epsilon\) be the linear function defined by \(\epsilon(x) = h(x) + 1\) where \(h(x) = -2x\) again.

(a) Find another way to write the rule of \(\epsilon\) much like we did for the function \(\delta\).

(b) Identify the \(y\)-intercept of \(\epsilon\).

5. Let \(m\) and \(b\) be fixed numbers (like 2, \(\frac{1}{3}\), -4, etc...) and define \(f(x) = mx + b\).

(a) Where is the \(y\)-intercept of \(f\)? (Consider looking at #3 & 4, and generalize!)

(b) The rule of the function \(f\) is a special form called slope intercept form. Why does it have this name?
6. Use the information from #5 to fill in the table below and then graph \( t, r, \) & \( s. \)

<table>
<thead>
<tr>
<th>function ( f(x) )</th>
<th>slope</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t(x) = -2x + 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r(x) = -2x - 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s(x) = \frac{1}{2}x + 3 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Let the graph of the function \( \alpha \) be a line with slope \( \frac{-1}{3} \) and assume the graph crosses the \( y \)-axis at \((0, 7)\). Find the rule of \( \alpha \) without graphing it.

8. Let the graph of the function \( \beta \) be a line with slope \( \frac{1}{2} \) and assume it passes through the point \((3, 4)\). Find the rule of \( \beta \).

Hint: Since you know what \( m \) is, write \( \beta(x) = mx + b \), and try to solve for \( b \) by plugging in points. Otherwise, there is a “point-slope formula” that can be used that is on page 20. Be aware, you will have no books or notes on the exam and quizzes so make sure whatever materials you use to answer this question can be memorized easily.

9. Let the graph of the function \( \gamma \) be a line that passes through the points \((-1, 2)\) and \((2, -5)\), find the rule of \( \gamma \).

10. Consider the graphs of two functions: \( \delta(x) = m_1x + b_1 \) and \( \epsilon(x) = m_2x + b_2 \) with slopes \( m_1 \) and \( m_2 \), respectively. If \(|m_1| > |m_2|\), which graph is steeper?
11. Special Cases: What is the slope of a horizontal line? A vertical one? In each case, is the line the graph of a function?

12. Parallel Lines
   (a) Explain (in English) what parallel lines are.

   (b) If two lines are parallel, what is the relationship between their slopes?

   (c) Find a line that is parallel to $\frac{4}{7}x + \pi$ and passes through the point $(-1, 3)$.

13. Perpendicular Lines
   (a) Explain (in English) what it means for two lines to be perpendicular. Be specific.

   (b) If two lines are perpendicular, what is the relationship between their slopes?

   (c) Find the equation of a line perpendicular to the line that passes through $(-2, -1)$ and $(4, 3)$. Note: There are many right answers.